

Examples in README:

$$\lim_{x \rightarrow +0} x \ln(\sin x) = \lim_{x \rightarrow +0} \frac{\ln(\sin x)}{x^{-1}} = \lim_{x \rightarrow +0} \frac{x}{\sin x} \cos x = 0$$

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=0}^{\infty} \frac{1}{n^{\frac{1}{2}}}$$

$$\int_0^{\frac{\pi}{3}} \sqrt{1 + \tan^2 x} \, dx = \int_0^{\frac{\pi}{3}} \frac{1}{\cos x} \, dx = \left[\ln \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \right]_0^{\frac{\pi}{3}} = \ln(2 + \sqrt{3})$$

Calculate the area of the region bounded by the curves $y = e^{\frac{x}{2}} \sin \frac{\sqrt{3}}{2} x$ and x axis:

$$\begin{aligned} S &= \lim_{\alpha \rightarrow -\infty} \int_{\alpha}^0 e^{\frac{x}{2}} \left| \sin \frac{\sqrt{3}}{2} x \right| \, dx \\ &= \lim_{n \rightarrow \infty} \sum_{k=0}^n (-1)^{k-1} \int_{-\frac{2(k+1)}{\sqrt{3}}\pi}^{-\frac{2k}{\sqrt{3}}\pi} e^{\frac{x}{2}} \sin \frac{\sqrt{3}}{2} x \, dx \\ &= \lim_{n \rightarrow \infty} \sum_{k=0}^n (-1)^{k-1} \left[e^{\frac{x}{2}} \sin \left(\frac{\sqrt{3}}{2} x - \frac{\pi}{3} \right) \right]_{-\frac{2(k+1)}{\sqrt{3}}\pi}^{-\frac{2k}{\sqrt{3}}\pi} \\ &= \frac{\sqrt{3}}{2} \lim_{n \rightarrow \infty} \sum_{k=0}^n \left(e^{-\frac{k}{\sqrt{3}}\pi} + e^{-\frac{k+1}{\sqrt{3}}\pi} \right) = \frac{\sqrt{3}}{2} \lim_{n \rightarrow \infty} \sum_{k=0}^n \left(1 + e^{-\frac{\pi}{\sqrt{3}}} \right) e^{-\frac{k}{\sqrt{3}}\pi} \\ &= \frac{\sqrt{3}}{2} \lim_{n \rightarrow \infty} \left(1 + e^{-\frac{\pi}{\sqrt{3}}} \right) \frac{1 - \left(e^{-\frac{\pi}{\sqrt{3}}} \right)^n}{1 - e^{-\frac{\pi}{\sqrt{3}}}} = \frac{\sqrt{3}}{2} \end{aligned}$$

Calculate the surface area of an ellipse:

$$\begin{aligned} S &= 2\pi \int_{-a}^a |y| \sqrt{1 - (|y'|)^2} \, dx \\ &= 2\pi \int_{-a}^a \frac{b}{a} \sqrt{a^2 - x^2} \sqrt{\frac{a^4 - (a^2 - b^2)x^2}{a^2 a^2 - x^2}} \, dx \\ &= 2\pi \int_{-a}^a \frac{b}{a} \sqrt{a^2 - \frac{a^2 - b^2}{a^2} x^2} \, dx \\ &= 4\pi b \int_0^a \sqrt{1 - \frac{e^2}{a^2} x^2} \, dx \quad \left(e^2 = \frac{a^2 - b^2}{a^2} \right) \\ &= 4\pi \frac{ab}{e} \int_0^{\arcsin e} \sqrt{1 - \sin^2 \theta} \cos \theta \, d\theta \quad \left(\theta = \arcsin \frac{e}{a} x \right) \\ &= 4\pi \frac{ab}{e} \int_0^{\arcsin e} \cos^2 \theta \, d\theta = 2\pi \frac{ab}{e} \int_0^{\arcsin e} 1 + \cos 2\theta \, d\theta \\ &= 2\pi \frac{ab}{e} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\arcsin e} = 2\pi \frac{ab}{e} \left[\theta + \sin \theta \sqrt{1 - \sin^2 \theta} \right]_0^{\arcsin e} \\ &= 2\pi \frac{ab}{e} (\arcsin e + e \sqrt{1 - e^2}) \end{aligned}$$