

## 171 HW5

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I tried doing as much work in markdown as possible, but it got very math-y so I used latex for part of it. The first set of problems (9.9, 13.8, and 14.1) are done in markdown. The second set of problems (8.8, 8.10, 8.28, 9.20, and 13.13) are done in latex. I couldn't merge the two due to font type problems that were a headache, so I took images of the latex output and pasted them in the markdown. My apologies if a couple problems are out of order.

### 8.8

$Jim \neq George$   
 $Spouse(Jim, Laura)$   
 $\forall X : X \neq Jim \wedge Spouse(X, Laura)$   
therefore  $Spouse(George, Laura)$

### 8.10

a)  $Occupation(Emily, Surgeon) \vee Occupation(Emily, Lawyer)$   
b)  $Occupation(Joe, Actor) \wedge \exists! X : X \neq Actor \wedge Occupation(Joe, X)$   
c)  $\forall X : Occupation(X, Surgeon) \Rightarrow Occupation(X, Doctor)$   
d)  $\forall X : Occupation(X, Lawyer) \Rightarrow Customer(Joe, X)$   
e)  $\exists! X : Boss(X, Emily) \Rightarrow Occupation(X, Lawyer)$   
f)  $\exists X, \forall Y (Occupation(X, Lawyer) \Rightarrow (Customer(Y, X) \Rightarrow Occupation(Y, Doctor)))$   
g)  $\forall X, \exists Y : Occupation(X, Surgeon) \Rightarrow Customer(X, Y)$

### 8.28

b)  $\neg Wrote(Gershwin, "Eleanor Rigby.")$   
d)  $\exists X : Wrote(Joe, X)$   
g)  $\forall X, \exists Y : Sings(Y, X, Revolver) \Rightarrow Y \neq Gershwin$   
h)  $\forall X, \exists Y : Wrote(Gershwin, X) \Rightarrow Sings(Gershwin, X, Y) \wedge \forall X, \exists Y, \exists Z : Sings(McCartney, X, Y) \Rightarrow (CopyOf(Z, Y) \wedge Owns(Joe, Z))$

### 9.20

a)  $\exists p, \forall q : \neg S(q, q) \iff S(p, q)$

b)

$$\begin{aligned} \exists p, \forall q : \neg S(q, q) &\iff S(p, q) \\ \exists p, \forall q : (\neg S(q, q) \Rightarrow S(p, q)) \wedge (S(p, q) \Rightarrow \neg S(q, q)) \\ \exists p, \forall q : (S(q, q) \vee S(p, q)) \wedge (\neg S(p, q) \vee \neg S(q, q)) \\ \forall q : (S(q, q) \vee S(P, q)) \wedge (\neg S(P, q) \vee \neg S(q, q)) &p = P \\ S(q, q) \vee S(P, q) \wedge (\neg S(P, q) \vee \neg S(q, q)) \end{aligned}$$

c) The CNF form resolves to empty clause so the logic is unsatisfiable

### 13.13

$$\begin{aligned} P(\text{TestA} = \text{pos} \mid \text{Virus} = \text{Present}) &= .95 \\ P(\text{TestA} = \text{pos} \mid \text{Virus} = \text{Absent}) &= .1 \\ P(\text{TestB} = \text{pos} \mid \text{Virus} = \text{Present}) &= .9 \\ P(\text{TestB} = \text{pos} \mid \text{Virus} = \text{Absent}) &= .05 \\ P(\text{Virus} = \text{Present}) &= .01 \\ P(\text{Virus} = \text{Present} \mid \text{TestA} = \text{pos}) \\ &= \frac{P(\text{TestA} = \text{pos} \mid \text{Virus} = \text{Present}) \cdot P(\text{Virus} = \text{Present})}{P(\text{TestA} = \text{pos} \mid \text{Virus} = \text{Present}) \times P(\text{Virus} = \text{Present}) + P(\text{TestA} = \text{pos} \mid \text{Virus} = \text{Absent}) \times P(\text{Virus} = \text{absent})} \\ &= \frac{.0095}{.1085} = .088 \\ P(\text{Virus} = \text{Present} \mid \text{TestB} = \text{pos}) \\ &= \frac{P(\text{TestB} = \text{pos} \mid \text{Virus} = \text{Present}) \cdot P(\text{Virus} = \text{Present})}{P(\text{TestB} = \text{pos} \mid \text{Virus} = \text{Present}) \times P(\text{Virus} = \text{Present}) + P(\text{TestB} = \text{pos} \mid \text{Virus} = \text{Absent}) \times P(\text{Virus} = \text{absent})} \\ &= \frac{.009}{.0585} = .15 \end{aligned}$$

### 9.9

**note:** I'll be referring to goals and axioms as Gx and Ax where x is the identifier.

- a) G1:  $7 \leq 3 + 9$ ; A8{x1 / 7, z1 / (3 + 9)}
- G2:  $7 \leq y1$ ; A4{x2 / 7, y1 / (7 + 0)} (success)
- G3:  $7 + 0 \leq 3 + 9$ ; A8{x3 / (7 + 0), z3 / (3 + 9)}
- G4:  $7 + 0 \leq y3$ ; A6{x4 / 7, y4 / 0, y3 / (0 + 7)} (success)
- G5:  $0 + 7 \leq 3 + 9$ ; A7{w5 / 0, x5 / 7, y5 / 3, z5 / 9}
- $0 \leq 3$ ; A1 (success)
- $7 \leq 9$ ; A2 (success)
- G1, G3, G5 (success)
- b) infer from A1, A2, A7 {w / 0, x / 7, y / 3, z / 9}
- A9:  $0 + 7 \leq 3 + 9$
- infer from A9, A6, A8 {x1 / 0, y1 / 7, x2 / (0 + 7), y2 / (7 + 0), z2 / (3 + 9)}
- A10:  $7 + 0 \leq 3 + 9$
- infer from A4, A10, A8
- {x3 / 7, x4 / 7, y4 / (7 + 0), z4 / (3 + 9)}
- A11:  $|7 \leq 3 + 9$

### 13.8

$$\begin{aligned} \text{a) } P(\text{Toothache}) &= 0.108 + 0.012 + 0.016 + 0.064 \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} \text{b) } P(\text{Cavity}) &= \langle 0.2, 0.8 \rangle \\ &= 0.108 + 0.012 + 0.072 + 0.008 \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} \text{c) } P(\text{Toothache}|\text{Cavity}) &= \langle (.108+.012)/.2, (0.072+0.008)/0.2 \rangle \\ &= \langle 0.6, 0.4 \rangle \\ &= .6 \end{aligned}$$

$$\begin{aligned} \text{d) } P(\text{Cavity} \vee \text{Catch}) &= 0.108 + 0.012 + 0.016 + 0.064 + 0.072 + 0.144 \\ &= 0.416 \end{aligned}$$

therefore,

$$\begin{aligned} P(\text{Cavity}|\text{Toothache} \vee \text{Catch}) &= \langle (0.108 + 0.012 + 0.072) / 0.416, (0.016 + 0.064 + 0.144) / 0.416 \rangle \\ &= \langle 0.4615, 0.5384 \rangle \end{aligned}$$

### 14.1

a) let RV C have domain {a, b, c}

In the network, C is the root and X1, X2, X3 are the children.

CPT for C:

C	P(C)
a	1/3
b	1/3
c	1/3

CPT for Xi

C	X1	P(C)
a	heads	.2
b	heads	.6
c	heads	.8

b)  $P(X1=\text{tails}, X2=\text{heads}, X3=\text{heads} \mid C=a)$

$$= .8 * .2 * .2 = .032$$

$$P(2 \text{ heads}, 1 \text{ tails} \mid C=a) = 3 * .032 = .096$$

$$P(" " " " \mid C=b) = .432$$

$$P(" " " " \mid C=c) = .384$$

therefore, b is most likely to be drawn