

1D, Cylindrical, Homogeneous Material Problem Description

PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \cdot k \frac{\partial T}{\partial r} \right) = q$$

Domain/Material Properties

$$\Omega_r = [1, 2], \quad \rho c_p = 10, \quad k = 1.5$$

BCs

Left: **Neumann** – $\left. \frac{\partial T}{\partial r} \right|_{r=1} = k \cdot 200t$

Right: **Dirichlet** – $T(2, t) = 400$

ICs

Constant – $T(r, 0) = 400$

Method of Manufactured Solutions for 1D, RZ, Homogeneous Material Problem

Prescribed Solution

$$T(r, t) = (-200r + 400)t + 400$$

Derived Source

$$q = 200 \rho c_p (-x + 2) + \frac{200kt}{r}$$

Interface Level Set Function

$$\phi(r, t) = 2 - (r - 0.04) - 0.2t = 2.04 - r - 0.2t$$

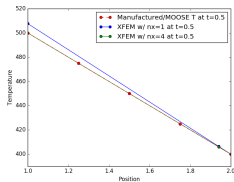
Numerical Parameters

```
11 [GlobalParams]
12   order = FIRST
13   family = LAGRANGE
14   □
15
16 [Problem]
17   coord_type = RZ
18   □
19
20 [Mesh]
21   type = GeneratedMesh
22   dim = 2
23   nx = 1
24   ny = 1
25   xmin = 1.0
26   xmax = 2.0
27   ymin = 0.0
28   ymax = 0.5
29   elem_type = QUAD4
30   □
```

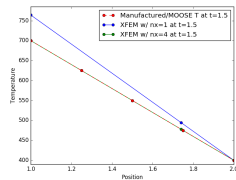
```
143 [Executioner]
144   type = Transient
145   solve_type = 'PJFNK'
146   # petsc_options_iname = '-pc_type -pc_hypre_type'
147   # petsc_options_value = 'hypre boomeramg'
148   petsc_options_iname = '-pc_type'
149   petsc_options_value = 'lu'
150   line_search = 'none'
151
152   l_tol = 1.0e-6
153   nl_max_its = 15
154   nl_rel_tol = 1.0e-10
155   nl_abs_tol = 1.0e-9
156
157   start_time = 0.0
158   dt = 0.1
159   end_time = 2.0
160   max_xfem_update = 1
161   □
```

```
82 [Constraints]
83   [./xfem_constraint]
84     type = XFEMSingleVariableConstraint
85     variable = u
86     jump = 0
87     jump_flux = 0
88     geometric_cut_userobject = 'level_set_cut_uo'
89   [../]
90   □
```

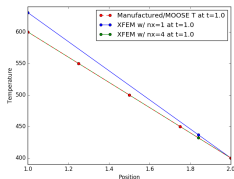
Results Comparison



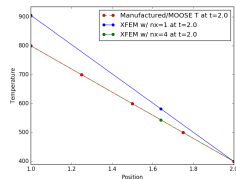
$t = 0.5$



$t = 1.5$

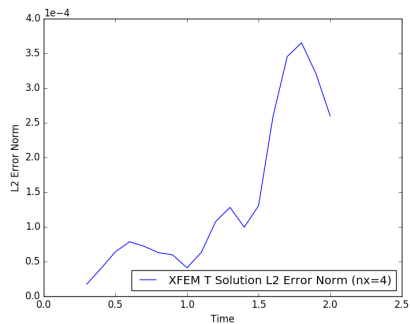
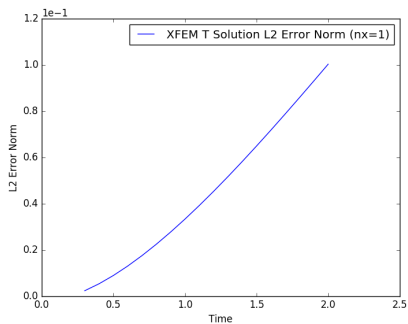


$t = 1.0$



$t = 2.0$

L2 Error Norms at Each Timestep



Mesh Refinement Effects on Error at $x=1$

