2D, Cylindrical, Level Set Dependent Material Problem Description

PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot k \frac{\partial T}{\partial r}\right) - \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) = q$$

Advection PDE for Level Set

$$\frac{\partial \phi}{\partial t} + u(r,z,t) \cdot \nabla \phi = 0\ ,\ \ \frac{\partial \phi}{\partial t} + u(r,z,t) \cdot \left(\frac{\partial \phi}{\partial r} + \frac{\partial \phi}{\partial z}\right) = 0$$

Domain/Material Properties

$$[\Omega_r, \Omega_z] = [[1,2],[1,2]]$$
$$\rho cs_p = 10$$
$$k(r,z,t) = \left(\frac{0.05}{2.04}\right)\phi(r,z,t) + 1.5$$

2D, Cylindrical, Level Set Dependent Material Problem BCs/IC

BCs

Left: **Neumann** – $\frac{\partial T}{\partial r}\Big|_{r=1} = k(r,z,t)\cdot 100t$
Right: **Dirichlet** – $T(2,z,t) = (-100z+200)t+400$
Bottom: **Neumann** – $\frac{\partial T}{\partial z}\Big|_{z=1} = k(r,z,t)\cdot 100t$
Top: **Dirichlet** – $T(r,2,t) = (-100r+200)t+400$

ICs

Constant – $T(r,z,0) = 400$
Function – $\phi(r,z,0) = -0.5(x+y)+1.7$

Provided PDE Terms

Provided T Source

$$q = 100\,\rho c_p\,(-r - z + 4) + t\left(-\frac{2.5}{2.04}\frac{z}{r} + 155\frac{1}{r} + \frac{1}{2.04}\frac{t}{r} - \frac{7.5}{2.04}\right)$$

Provided ϕ velocity term, $u(r, z, t)$

$$u(r, z, t) = -\frac{0.07}{200}\,(T - 410) - 0.2$$

Note about Problem

This is a problem that fully couples the T and ϕ solutions: k is dependent on ϕ, and $u(r, z, t)$ is dependent on T. We have provided values to make the problem similar to previous MMS Verification problems, but still need to develop a version of this problem with an analytical solution in mind. This acts as more of a *proof of concept* than true verification problem.