

1D, Cartesian, Homogeneous 1 Material Problem Description

PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} k \frac{\partial T}{\partial x} = q$$

Domain/Material Properties

$$\Omega_x = [0, 1], \quad \rho c_p = 10, \quad k = 1.5$$

BCs

Left: **Neumann** - $\frac{\partial T}{\partial x} \Big|_{x=0} = k \cdot 200t$

Right: **Dirichlet** - $T(1, t) = 400$

IC

Constant - $T(x, 0) = 400$

Method of Manufactured Solutions for 1D, XY, Homogeneous Material Problem

Prescribed Solution

$$T(x, t) = (-200x + 200)t + 400$$

Derived Source

$$q = 200 \rho c_p (-x + 1)$$

Interface Level Set Function

$$\phi(x, t) = 1 - (x - 0.04) - 0.2t = 1.04 - x - 0.2t$$

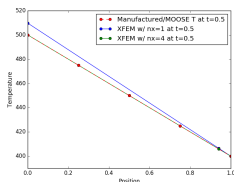
Numerical Parameters

```
19 [GlobalParams]
20   order = FIRST
21   family = LAGRANGE
22   □
23
24 [Mesh]
25   type = GeneratedMesh
26   dim = 2
27   nx = 1
28   ny = 1
29   xmin = 0.0
30   xmax = 1.0
31   ymin = 0.0
32   ymax = 0.5
33   elem_type = QUAD4
34   □
```

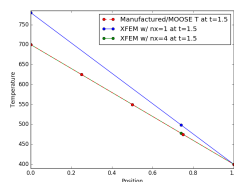
```
147 [Executioner]
148   type = Transient
149   solve_type = 'PJFNK'
150   # petsc_options_iname = '-pc_type -pc_hypre_type'
151   # petsc_options_value = 'hypre boomeramg'
152   petsc_options_iname = '-pc_type'
153   petsc_options_value = 'lu'
154   line_search = 'none'
155
156   l_tol = 1.0e-6
157   nl_max_its = 15
158   nl_rel_tol = 1.0e-10
159   nl_abs_tol = 1.0e-9
160
161   start_time = 0.0
162   dt = 0.1
163   end_time = 2.0
164   max_xfem_update = 1
165   □
```

```
86 [Constraints]
87   [./xfem_constraint]
88     type = XFEMSingleVariableConstraint
89     variable = u
90     jump = 0
91     jump_flux = 0
92     geometric_cut_userobject = 'level_set_cut_uo'
93   [./]
94   □
```

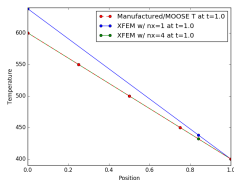
Results Comparison



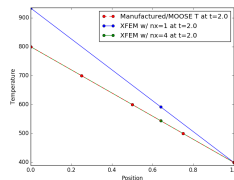
$t = 0.5$



$t = 1.5$

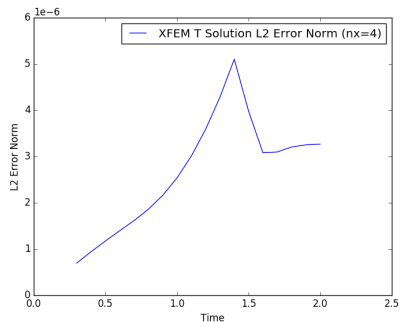
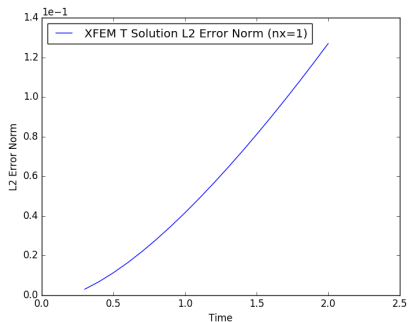


$t = 1.0$



$t = 2.0$

L2 Error Norms at Each Timestep



Mesh Refinement Effects on Error at $x=0$

