1D, Cartesian, Level Set Dependent Material Problem Description

PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} k \frac{\partial T}{\partial x} = q$$

Domain/Material Properties

$$\Omega_x = [0, 1], \ \rho c_p = 10, \ k = \left(\frac{0.05}{1.04}\right) \phi(x, t) + 1.5 = \frac{0.05}{1.04} \left(-x - 0.2t\right) + 1.55$$

BCs

Left: **Neumann** – $\frac{\partial T}{\partial x}\Big|_{x=0} = k(x, t) \cdot 200t$
Right: **Dirichlet** – $T(1, t) = 400$

IC

Constant – $T(x, 0) = 400$

Method of Manufactured Solutions for 1D, XY, LS Dependent Material Problem

Prescribed Solution

$T(x,t) = (-200x + 200)t + 400$

Derived Source

$q = 200\,\rho c_p\,(-x+1) - \left(\frac{0.05 \cdot 200t}{1.04}\right)$

Interface Level Set Function

$\phi(x,t) = 1 - (x - 0.04) - 0.2t = 1.04 - x - 0.2t$

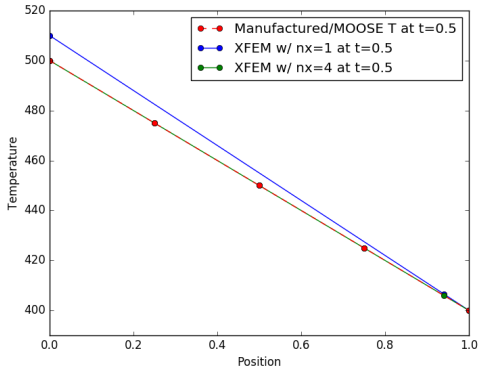
Numerical Parameters

```
[GlobalParams]
  order = FIRST
  family = LAGRANGE
[]

[Mesh]
  type = GeneratedMesh
  dim = 2
  nx = 1
  ny = 1
  xmin = 0.0
  xmax = 1.0
  ymin = 0.0
  ymax = 0.5
  elem_type = QUAD4
[]
```

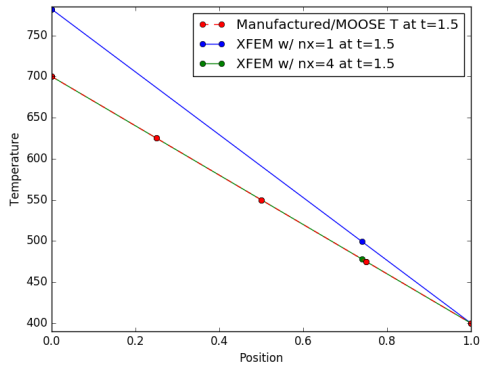
```
[Executioner]
  type = Transient
  solve_type = 'PJFNK'
  # petsc_options_iname = '-pc_type -pc_hypre_type'
  # petsc_options_value = 'hypre boomeramg'
  petsc_options_iname = '-pc_type'
  petsc_options_value = 'lu'
  line_search = 'none'

  l_tol = 1.0e-6
  nl_max_its = 15
  nl_rel_tol = 1.0e-10
  nl_abs_tol = 1.0e-9

  start_time = 0.0
  dt = 0.1
  end_time = 2.0
  max_xfem_update = 1
[]
```

```
[Constraints]
  [./xfem_constraint]
    type = XFEMSingleVariableConstraint
    variable = u
    jump = 0
    jump_flux = 0
    geometric_cut_userobject = 'level_set_cut_uo'
  [../]
[]
```

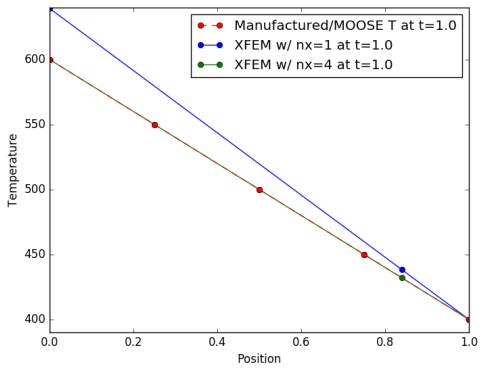
Results Comparison



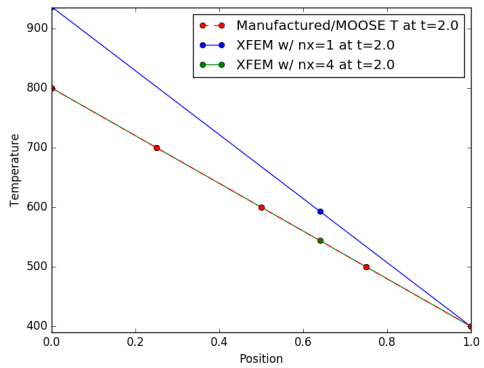
$t = 0.5$



$t = 1.5$

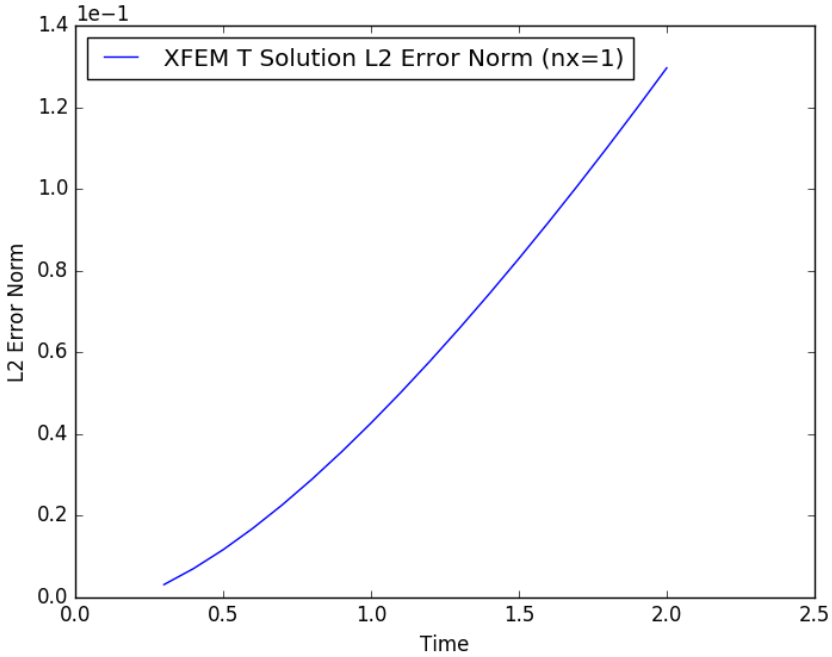


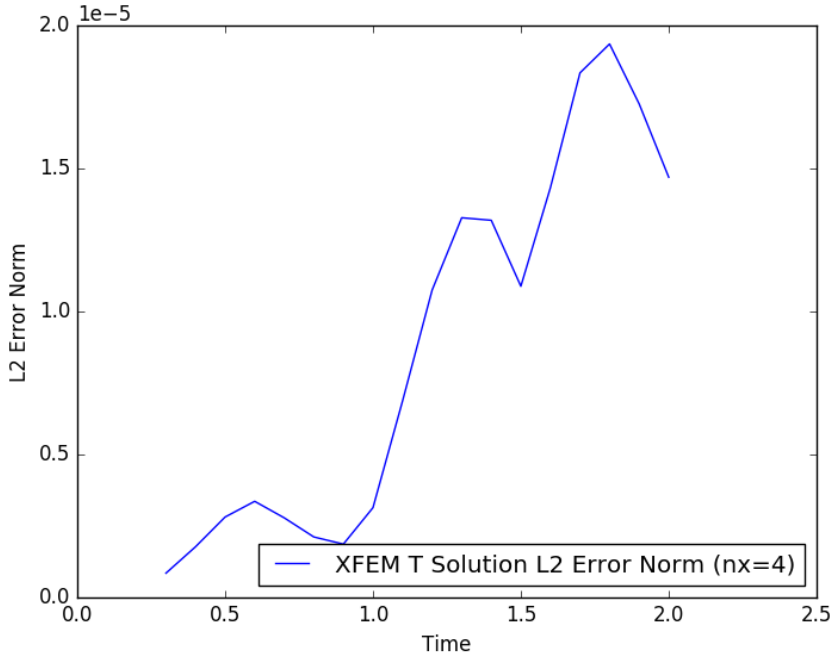
$t = 1.0$



$t = 2.0$

L2 Error Norms at Each Timestep





Mesh Refinement Effects on Error at x=0