2D, Cylindrical, Homogeneous Material Problem Description

Transient, 2D, RZ Heat Transfer PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot k \frac{\partial T}{\partial r}\right) - \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) = q$$

Advection PDE for Level Set

$$\frac{\partial \phi}{\partial t} + u(r,z,t) \cdot \nabla \phi = 0 \; , \; \frac{\partial \phi}{\partial t} + u(r,z,t) \cdot \left(\frac{\partial \phi}{\partial r} + \frac{\partial \phi}{\partial z}\right) = 0$$

Domain/Material Properties

$[\Omega_r, \Omega_z] = [[1,2],[1,2]]$
$\rho c_p = 10$
$k = 1.5$

2D, Cylindrical, Homogeneous Problem BCs/IC

BCs

Left: **Neumann** – $\frac{\partial T}{\partial r}\Big|_{r=1} = k \cdot 100t$
Right: **Dirichlet** – $T(2, z, t) = (-100z + 200)t + 400$
Bottom: **Neumann** – $\frac{\partial T}{\partial z}\Big|_{z=1} = k \cdot 100t$
Top: **Dirichlet** – $T(r, 2, t) = (-100r + 200)t + 400$

ICs

Constant – $T(r, z, 0) = 400$
Function – $\phi(r, z, 0) = -0.5(x + y) + 2.04$

Method of Manufactured Solutions for 2D, RZ, Homogeneous Material Problem

Prescribed T Solution

$$T(x,t) = (-100r - 100z + 400)t + 400$$

Derived T Source

$$q = 100\,\rho c_p\,(-r - z + 4) + \frac{100kt}{r}$$

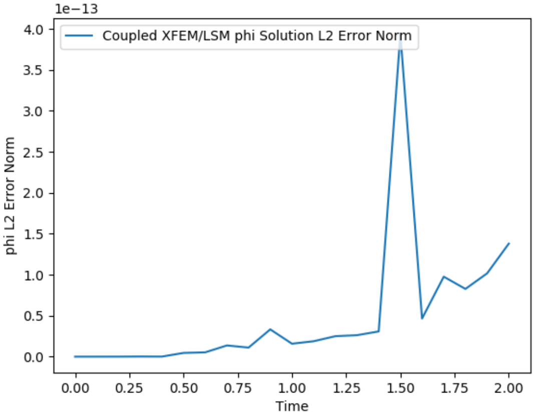
Prescribed ϕ Interface Level Set Solution (& ϕ Initial Condition)

$$\phi(x,y,t) = -0.5(x+y) + 2.04 - 0.2t$$

Derived ϕ velocity term, $u(r,z,t)$

$$u(r,z,t) = -0.2$$

ϕ Solution L2 Error Norms at Each Timestep



T Solution L2 Error Norms at Each Timestep