

2D, Cylindrical, Level Set Dependent Material Problem Description

PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \cdot k \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) = q$$

Domain/Material Properties

$$[\Omega_r, \Omega_z] = [[1, 2], [1, 2]]$$

$$\rho c_p = 10$$

$$k(r, z, t) = \left(\frac{0.05}{2.04} \right) \phi(r, z, t) + 1.5 = -\frac{0.025}{2.04} (r + z) + 1.55 - \frac{0.01t}{2.04}$$

2D, Cylindrical, Level Set Dependent Material Problem

BCs/IC

BCs

Left: **Neumann** - $\frac{\partial T}{\partial r} \Big|_{r=1} = k(r, z, t) \cdot 100t$

Right: **Dirichlet** - $T(2, z, t) = (-100z + 200)t + 400$

Bottom: **Neumann** - $\frac{\partial T}{\partial z} \Big|_{z=1} = k(r, z, t) \cdot 100t$

Top: **Dirichlet** - $T(r, 2, t) = (-100r + 200)t + 400$

ICs

Constant - $T(r, z, 0) = 400$

Method of Manufactured Solutions for 2D, RZ, LS Dependent Material Problem

Prescribed Solution

$$T(x, t) = (-100r - 100z + 400)t + 400$$

Derived Source

$$q = 100 \rho c_p (-r - z + 4) + t \left(-\frac{2.5}{2.04} \frac{z}{r} + 155 \frac{1}{r} + \frac{1}{2.04} \frac{t}{r} - \frac{7.5}{2.04} \right)$$

Interface Level Set Function

$$\phi(x, y, t) = -0.5(x + y) + 2.04 - 0.2t$$

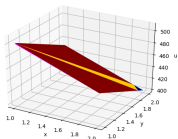
Numerical Parameters

```
11 [GlobalParams]
12   order = FIRST
13   family = LAGRANGE
14   □
15
16 [Problem]
17   coord_type = RZ
18   □
19
20 [Mesh]
21   type = GeneratedMesh
22   dim = 2
23   nx = 1
24   ny = 1
25   xmin = 1.0
26   xmax = 2.0
27   ymin = 1.0
28   ymax = 2.0
29   elem_type = QUAD4
30   □
```

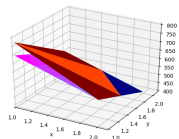
```
168 [Executioner]
169   type = Transient
170   solve_type = 'PJFNK'
171   # petsc_options_iname = '-pc_type -pc_hype_type'
172   # petsc_options_value = 'hype boomeramg'
173   petsc_options_iname = '-pc_type'
174   petsc_options_value = 'lu'
175   line_search = 'none'
176
177   l_tol = 1.0e-6
178   nl_max_its = 15
179   nl_rel_tol = 1.0e-10
180   nl_abs_tol = 1.0e-9
181
182   start_time = 0.0
183   dt = 0.1
184   end_time = 2.0
185   max_xfem_update = 1
186   □
```

```
82 [Constraints]
83   [./xfem_constraint]
84   type = XFEMSingleVariableConstraint
85   variable = u
86   jump = 0
87   jump_flux = 0
88   geometric_cut_userobject = 'level_set_cut_uo'
89   [../]
```

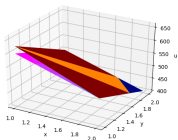
Results Comparison



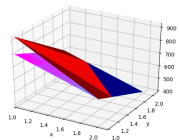
$t = 0.5$



$t = 1.5$

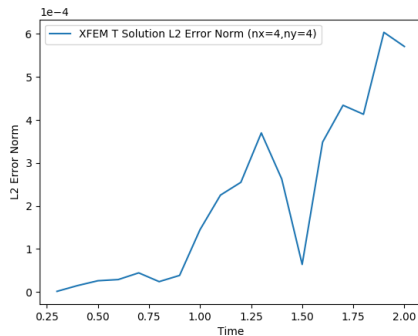
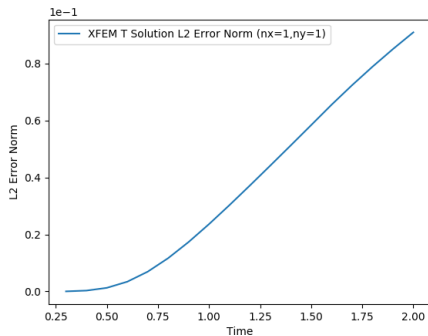


$t = 1.0$



$t = 2.0$

L2 Error Norms at Each Timestep



Mesh Refinement Effects on Error at $x=1, y=1$

