2D, Cylindrical, Level Set Dependent Material Problem Description

PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot k \frac{\partial T}{\partial r}\right) - \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) = q$$

Domain/Material Properties

$[\Omega_r, \Omega_z] = [[1,2],[1,2]]$

$\rho c s_p = 10$

$k(r,z,t) = \left(\frac{0.05}{2.04}\right)\phi(r,z,t) + 1.5 = -\frac{0.025}{2.04}(r+z) + 1.55 - \frac{0.01t}{2.04}$

2D, Cylindrical, Level Set Dependent Material Problem BCs/IC

BCs

Left: **Neumann** – $\frac{\partial T}{\partial r}\Big|_{r=1} = k(r,z,t)\cdot 100t$
Right: **Dirichlet** – $T(2,z,t) = (-100z+200)t+400$
Bottom: **Neumann** – $\frac{\partial T}{\partial z}\Big|_{z=1} = k(r,z,t)\cdot 100t$
Top: **Dirichlet** – $T(r,2,t) = (-100r+200)t+400$

ICs

Constant – $T(r,z,0) = 400$

Method of Manufactured Solutions for 2D, RZ, LS Dependent Material Problem

Prescribed Solution

$$T(x,t) = (-100r - 100z + 400)t + 400$$

Derived Source

$$q = 100\,\rho c_p\,(-r - z + 4) + t\left(-\frac{2.5}{2.04}\frac{z}{r} + 155\frac{1}{r} + \frac{1}{2.04}\frac{t}{r} - \frac{7.5}{2.04}\right)$$

Interface Level Set Function

$$\phi(x,y,t) = -0.5(x+y) + 2.04 - 0.2t$$

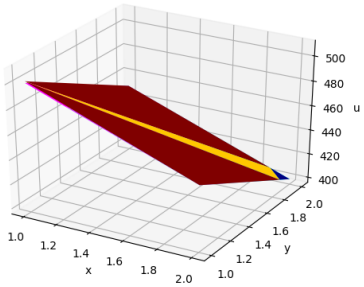
Numerical Parameters

```
[GlobalParams]
  order = FIRST
  family = LAGRANGE
[]

[Problem]
  coord_type = RZ
[]

[Mesh]
  type = GeneratedMesh
  dim = 2
  nx = 1
  ny = 1
  xmin = 1.0
  xmax = 2.0
  ymin = 1.0
  ymax = 2.0
  elem_type = QUAD4
[]
```

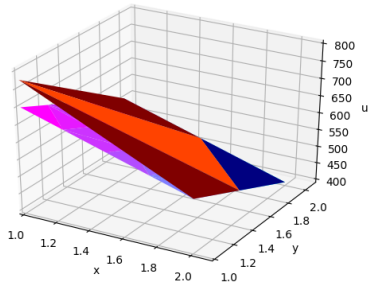
```
[Executioner]
  type = Transient
  solve_type = 'PJFNK'
  # petsc_options_iname = '-pc_type -pc_hypre_type'
  # petsc_options_value = 'hypre boomeramg'
  petsc_options_iname = '-pc_type'
  petsc_options_value = 'lu'
  line_search = 'none'

  l_tol = 1.0e-6
  nl_max_its = 15
  nl_rel_tol = 1.0e-10
  nl_abs_tol = 1.0e-9

  start_time = 0.0
  dt = 0.1
  end_time = 2.0
  max_xfem_update = 1
[]
```

```
[Constraints]
  [./xfem_constraint]
    type = XFEMSingleVariableConstraint
    variable = u
    jump = 0
    jump_flux = 0
    geometric_cut_userobject = 'level_set_cut_uo'
  [../]
[]
```

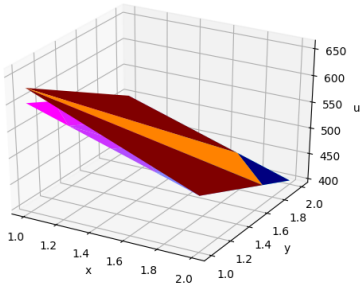
Results Comparison



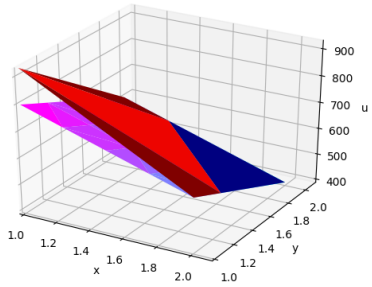
$t = 0.5$



$t = 1.5$

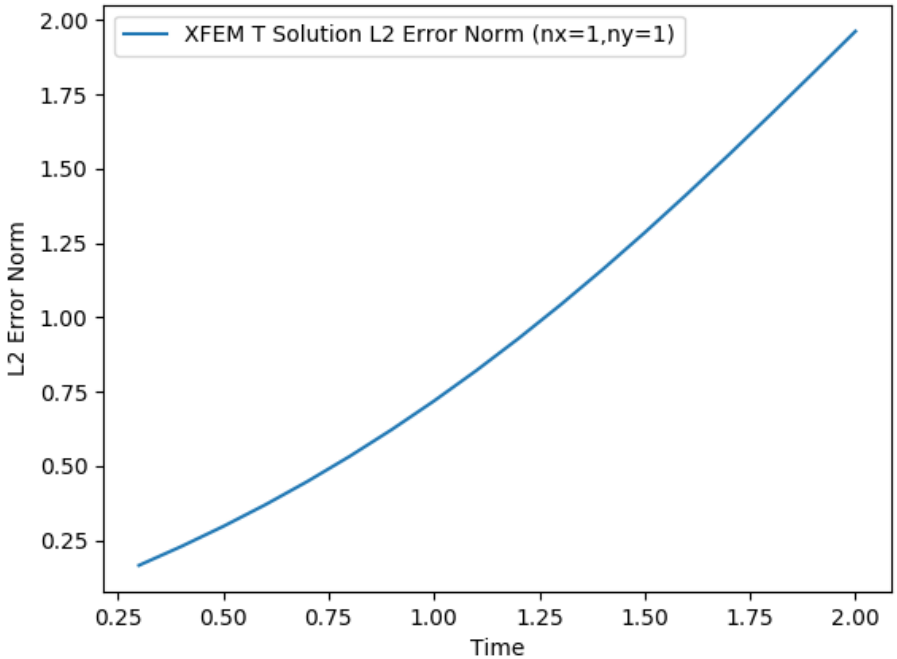


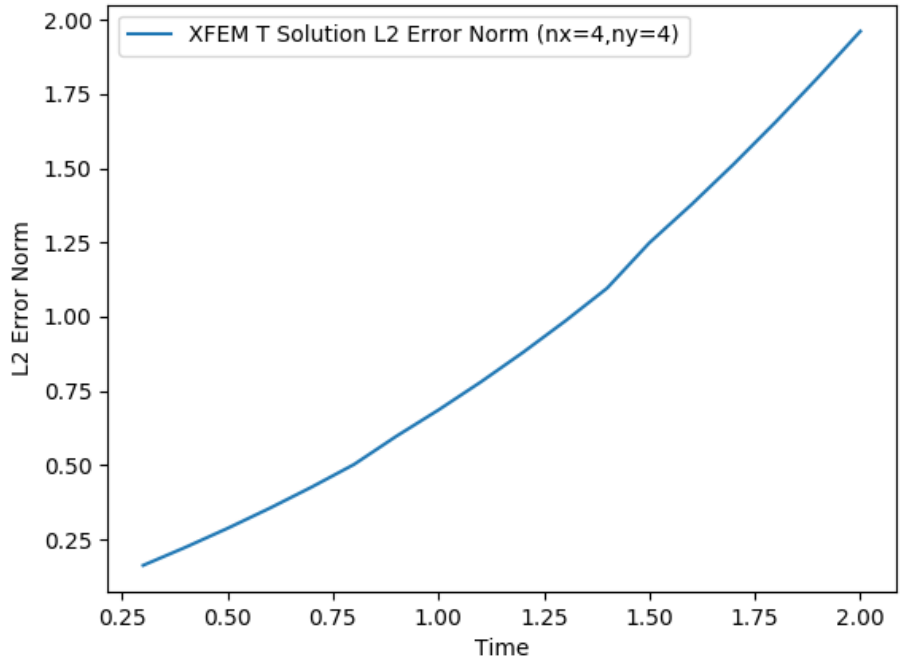
$t = 1.0$



$t = 2.0$

L2 Error Norms at Each Timestep





Mesh Refinement Effects on Error at x=1, y=1