

2D, Cylindrical, Homogeneous Material Problem Description

PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \cdot k \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) = q$$

Domain/Material Properties

$$[\Omega_r, \Omega_z] = [[1, 2], [1, 2]]$$

$$\rho c_p = 10$$

$$k = 1.5$$

2D, Cylindrical, Homogeneous Problem BCs/IC

BCs

Left: **Neumann** – $\frac{\partial T}{\partial r} \Big|_{r=1} = k \cdot 100t$

Right: **Dirichlet** - $\tilde{T}(2, z, t) = (-100z + 200)t + 400$

Bottom: **Neumann** - $\frac{\partial T}{\partial z} \Big|_{z=1} = k \cdot 100t$

Top: **Dirichlet** - $T(r, 2, t) = (-100r + 200)t + 400$

ICs

Constant – $T(r, z, 0) = 400$

Method of Manufactured Solutions for 2D, RZ, Homogeneous Material Problem

Prescribed Solution

$$T(x, t) = (-100r - 100z + 400)t + 400$$

Derived Source

$$q = 100 \rho c_p (-r - z + 4) + \frac{100kt}{r}$$

Interface Level Set Function

$$\phi(x, y, t) = -0.5(x + y) + 2.04 - 0.2t$$

Numerical Parameters

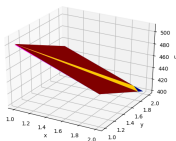
```
11 [GlobalParams]
12   order = FIRST
13   family = LAGRANGE
14   □
15
16 [Problem]
17   coord_type = RZ
18   □
19
20 [Mesh]
21   type = GeneratedMesh
22   dim = 2
23   nx = 1
24   ny = 1
25   xmin = 1.0
26   xmax = 2.0
27   ymin = 1.0
28   ymax = 2.0
29   elem_type = QUAD4
30   □
```

```
163 [Executioner]
164   type = Transient
165   solve_type = 'PJFNK'
166   # petsc_options_iname = '-pc_type -pc_hypre_type'
167   # petsc_options_value = 'hypre boomeramg'
168   petsc_options_iname = '-pc_type'
169   petsc_options_value = 'lu'
170   line_search = 'none'
171
172   l_tol = 1.0e-6
173   nl_max_its = 15
174   nl_rel_tol = 1.0e-10
175   nl_abs_tol = 1.0e-9
176
177   start_time = 0.0
178   dt = 0.1
179   end_time = 2.0
180   max_xfem_update = 1
181   □
```

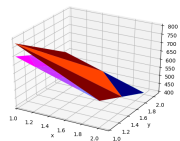
```
82 [Constraints]
83   [./xfem_constraints]
84     type = XFEMSingleVariableConstraint
85     variable = u
86     jump = 0
87     jump_flux = 0
88     geometric_cut_userobject = 'level_set_cut_uo'
89   [../]
```

Results Comparison

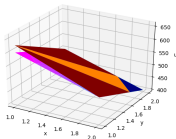
Upper plane is $n_x=1$, $n_y=1$; lower plane is $n_x=4$, $n_y=4$



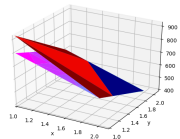
$t = 0.5$



$t = 1.5$

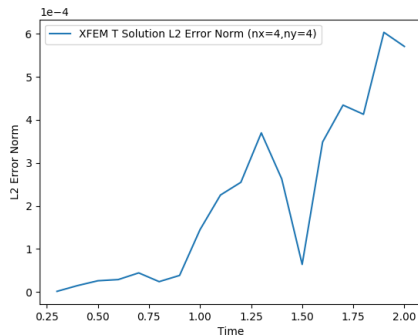
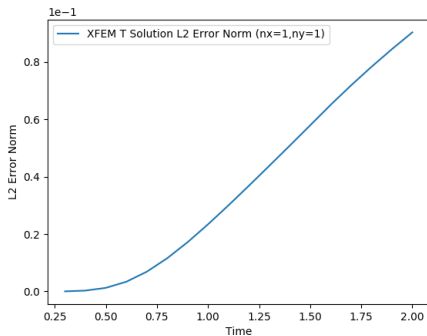


$t = 1.0$



$t = 2.0$

L2 Error Norms at Each Timestep



Mesh Refinement Effects on Error at $x=1, y=1$

