# 2D, Cartesian, Level Set Dependent Material Problem Description

## PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) = q$$

## Domain/Material Properties

$[\Omega_x, \Omega_y] = [[0,1],[0,1]]$

$\rho c_p = 10$

$k(x,y,t) = \left(\frac{0.05}{1.04}\right)\phi(x,y,t) + 1.5 = \left(\frac{0.01}{1.04}\right)(-2.5x - 2.5y - t) + 1.55$

# 2D, Cartesian, Level Set Dependent Material Problem BCs/IC

## BCs

Left: **Neumann** – $\frac{\partial T}{\partial x}\Big|_{x=0} = k(x,y,t)\cdot 100t$

Right,Top: **Dirichlet** – $T(1,y,t) = (-100y+100)t+400$

Bottom: **Neumann** – $\frac{\partial T}{\partial y}\Big|_{y=0} = k(x,y,t)\cdot 100t$

Top: **Dirichlet** – $T(x,1,t) = (-100x+100)t+400$

## ICs

**Constant** – $T(x,y,0) = 400$

# Method of Manufactured Solutions for 2D, XY, LS Dependent Material Problem

## Prescribed Solution

$$T(x,t) = (-100x - 100y + 200)t + 400$$

## Derived Source

$$q = 100\,\rho c_p\,(-x - y + 2) - \frac{5t}{1.04}$$

## Interface Level Set Function

$$\phi(x,y,t) = -0.5(x+y) + 1.04 - 0.2t$$

# Numerical Parameters

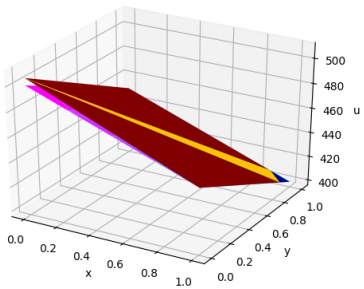
```
11 [GlobalParams]
12   order = FIRST
13   family = LAGRANGE
14 []
15
16 [Mesh]
17   type = GeneratedMesh
18   dim = 2
19   nx = 1
20   ny = 1
21   xmin = 0.0
22   xmax = 1.0
23   ymin = 0.0
24   ymax = 1.0
25   elem_type = QUAD4
26 []
```

```
163 [Executioner]
164   type = Transient
165   solve_type = 'PJFNK'
166   # petsc_options_iname = '-pc_type -pc_hypre_type'
167   # petsc_options_value = 'hypre boomeramg'
168   petsc_options_iname = '-pc_type'
169   petsc_options_value = 'lu'
170   line_search = 'none'
171
172   l_tol = 1.0e-6
173   nl_max_its = 15
174   nl_rel_tol = 1.0e-10
175   nl_abs_tol = 1.0e-9
176
177   start_time = 0.0
178   dt = 0.1
179   end_time = 2.0
180   max_xfem_update = 1
181 []
```

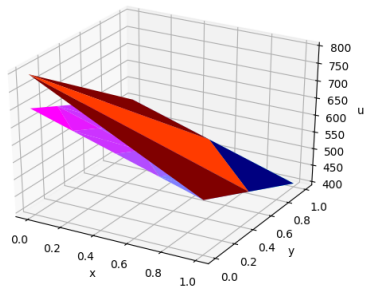
```
78 [Constraints]
79   [./xfem_constraints]
80     type = XFEMSingleVariableConstraint
81     variable = u
82     jump = 0
83     jumpflux = 0
84     geometric_cut_userobject = 'level_set_cut_uo'
85   [../]
86 []
```

# Results Comparison

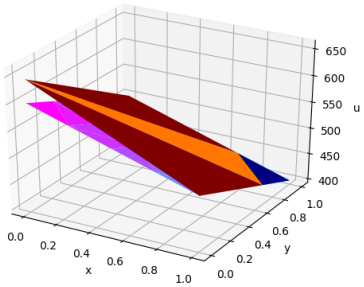
Upper plane is nx=1, ny=1; lower plane is nx=4, ny=4



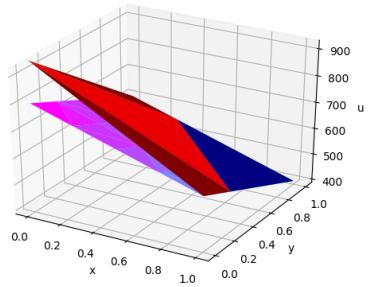

$t = 0.5$




$t = 1.5$

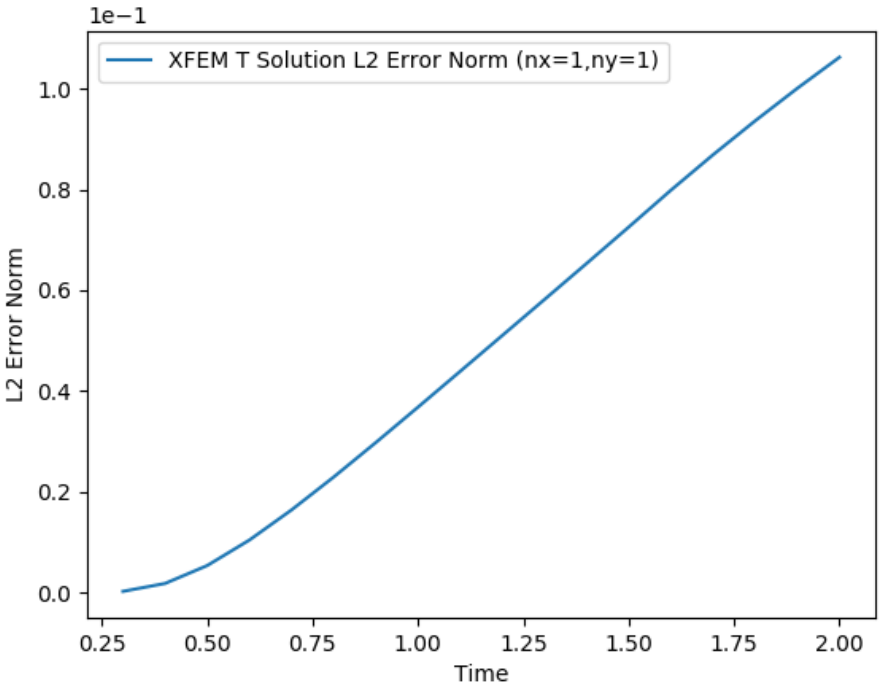


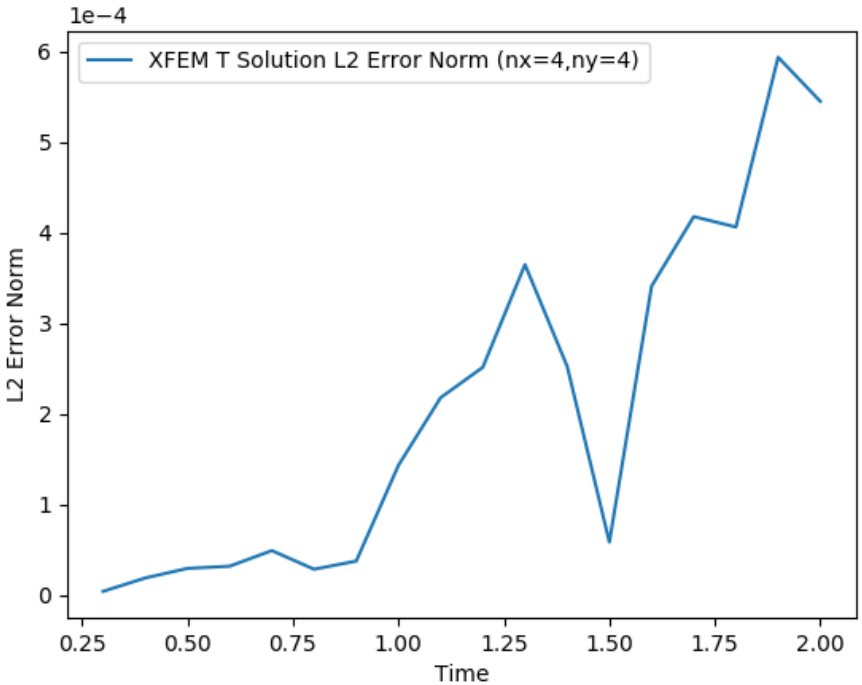
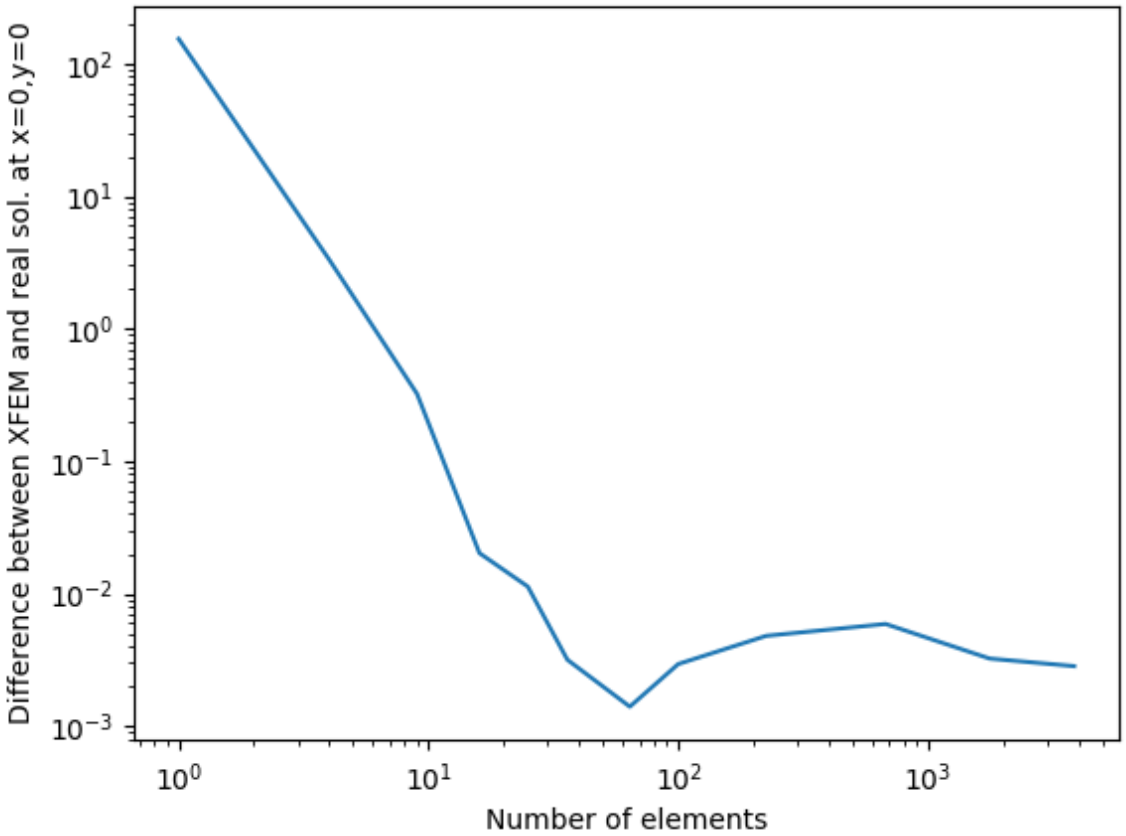

$t = 1.0$




$t = 2.0$

# L2 Error Norms at Each Timestep






# Mesh Refinement Effects on Error at x=0