1D, Cartesian, Homogeneous 1 Material Problem Description

PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} k \frac{\partial T}{\partial x} = q$$

Domain/Material Properties

$$\Omega_{x} = [0, 1], \ \rho c_{p} = 10, \ k = 1.5$$

BCs

Left: **Neumann** $-\left.\frac{\partial T}{\partial x}\right|_{x=0} = k \cdot 200t$

Right: **Dirichlet** – T(1, t) = 400

IC

Constant – T(x, 0) = 400

Method of Manufactured Solutions for 1D, XY, Homogeneous Material Problem

Prescribed Solution

$$T(x, t) = (-200x + 200)t + 400$$

Derived Source

$$q=200\,\rho c_p\left(-x+1\right)$$

Interface Level Set Function

$$\phi(x, t) = 1 - (x - 0.04) - 0.2t = 1.04 - x - 0.2t$$

Numerical Parameters

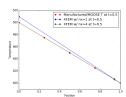
```
19 [GlobalParams]
    order = FIRST
21 family = LAGRANGE
22
24 [Mesh]
   type = GeneratedMesh
    dim = 2
    nx = 1
    ny = 1
    xmin = 0.0
   xmax = 1.0
31 ymin = 0.0
32 ymax = 0.5
    elem_type = OUAD4
34
```

```
147 [Executioner]
     type = Transient
     solve_type = 'PJFNK'
     # petsc_options_iname = '-pc_type -pc_hypre_type'
     # petsc_options_value = 'hypre boomeramg'
     petsc_options_iname = '-pc_type'
153 petsc_options_value = 'lu'
     line search = 'none'
    l tol = 1.0e-6
157 nl_max_its = 15
158  nl_rel_tol = 1.0e-10
     nl_abs_tol = 1.0e-9
160
     start time = 0.0
162 dt = 0.1
163 end_time = 2.0
164 max_xfem_update = 1
```

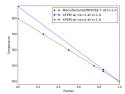
```
[Constraints]
 [./xfem_constraint]
    type = XFEMSingleVariableConstraint
   variable = u
    iump = 0
    jump_flux = 0
    geometric_cut_userobject = 'level_set_cut_uo'
 [.../]
```

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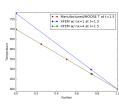
Results Comparison



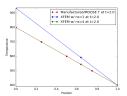
t = 0.5



t = 1.0

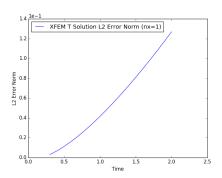


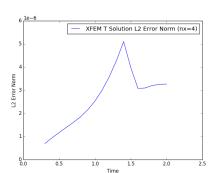
$$t = 1.5$$



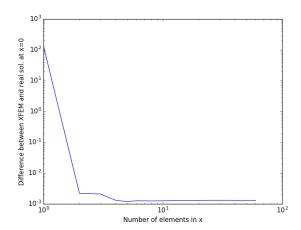
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L2 Error Norms at Each Timestep





Mesh Refinement Effects on Error at x=0



1D, Cartesian, Level Set Dependent Material Problem Description

PDE

$$\rho c_{p} \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_{p} \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} k \frac{\partial T}{\partial x} = q$$

Domain/Material Properties

$$\Omega_{x} = [0, 1], \ \rho c_{p} = 10, \ k = \left(\frac{0.05}{1.04}\right) \phi(x, t) + 1.5 = \frac{0.05}{1.04} \left(-x - 0.2t\right) + 1.55$$

BCs

Left: **Neumann**
$$-\frac{\partial T}{\partial x}\Big|_{x=0} = k(x,t) \cdot 200t$$

Right: **Dirichlet** –
$$T(1, t) = 400$$

ĪC

Constant –
$$T(x,0) = 400$$

Tompkins (TAMU)

Method of Manufactured Solutions for 1D, XY, LS Dependent Material Problem

Prescribed Solution

$$T(x, t) = (-200x + 200)t + 400$$

Derived Source

$$q = 200 \rho c_p \left(-x+1\right) - \left(\frac{0.05 \cdot 200t}{1.04}\right)$$

Interface Level Set Function

$$\phi(x, t) = 1 - (x - 0.04) - 0.2t = 1.04 - x - 0.2t$$

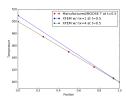


Numerical Parameters

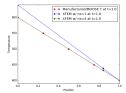
```
[Executioner]
     type = Transient
     solve_type = 'PJFNK'
     # petsc_options_iname = '-pc_type -pc_hypre_type'
     # petsc_options_value = 'hypre boomerama'
     petsc_options_iname = '-pc_type'
     petsc_options_value = 'lu'
     line_search = 'none'
   l_tol = 1.0e-6
155 nl_max_its = 15
     nl_rel_tol = 1.0e-10
     nl.abs.tol = 1.0e-9
     start_time = 0.0
    dt = 0.1
161 end_time = 2.0
     max_xfem_update = 1
```

```
78 [Constraints]
79 [./xfem.constraint]
80 type = XFEMSingleVariableConstraint
81 variable = u
82 jump = 0
83 jump_flux = 0
84 geometric_cut_userobject = 'level_set_cut_uo'
85 [../]
```

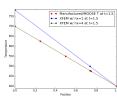
Results Comparison



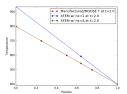
t = 0.5



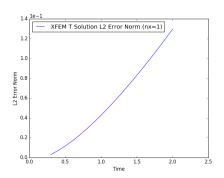
t = 1.0

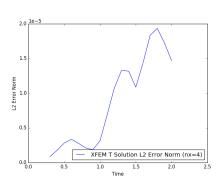


$$t = 1.5$$

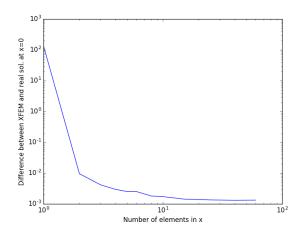


L2 Error Norms at Each Timestep





Mesh Refinement Effects on Error at x=0





1D, Cylindrical, Homogeneous Material Problem Description

PDE

$$\rho c_{p} \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_{p} \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \cdot k \frac{\partial T}{\partial r} \right) = q$$

Domain/Material Properties

$$\Omega_r = [1, 2], \ \rho c_p = 10, \ k = 1.5$$

BCs

Left: **Neumann** $-\left.\frac{\partial T}{\partial r}\right|_{r=1} = k \cdot 200t$

Right: **Dirichlet** – T(2, t) = 400

ICs

Constant - T(r, 0) = 400

Method of Manufactured Solutions for 1D, RZ, Homogeneous Material Problem

Prescribed Solution

$$T(r, t) = (-200r + 400)t + 400$$

Derived Source

$$q = 200 \rho c_p (-x+2) + \frac{200kt}{r}$$

Interface Level Set Function

$$\phi(r,t) = 2 - (r - 0.04) - 0.2t = 2.04 - r - 0.2t$$



1D XY Homog | 1D XY LS Dep | 1D RZ Homog | 1D RZ LS Dep | 2D XY Homog | 2D XY LS Dep | 2D RZ Homog | 2D RZ LS Dep | 2D RZ Homog | 2D RZ LS Dep | 2D RZ LS D

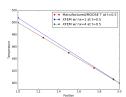
Numerical Parameters

```
11 [GlobalParams]
    order = FIRST
13 family = LAGRANGE
14
16 [Problem]
    coord_type = RZ
18
20 [Mesh]
    type = GeneratedMesh
    dim = 2
    nx = 1
    ny = 1
    xmin = 1.0
    xmax = 2.0
    vmin = 0.0
    vmax = 0.5
    elem_type = QUAD4
30 T
```

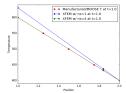
```
43 [Executioner]
    type = Transient
    solve_type = 'PJFNK'
    # petsc_options_iname = '-pc_type -pc_hypre_type'
    # petsc_options_value = 'hypre boomeramg'
    petsc_options_iname = '-pc_type'
    petsc_options_value = 'lu'
    line search = 'none'
152 l_tol = 1.0e-6
nl_rel_tol = 1.0e-10
    nl_abs_tol = 1.0e-9
    start time = 0.0
158 dt = 0.1
159 end_time = 2.0
   max_xfem_update = 1
```

```
82 [Constraints]
83 [./xfem_constraint]
84 type = XFEMSingleVariableConstraint
85 variable = u
86 jump = 0
87 jump_flux = 0
88 geometric_cut_userobject = 'level_set_cut_uo'
89 [../]
```

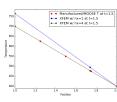
Results Comparison



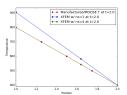
t = 0.5



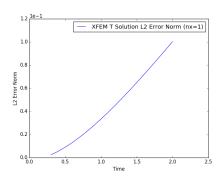
t = 1.0

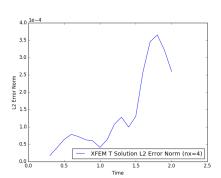


$$t = 1.5$$

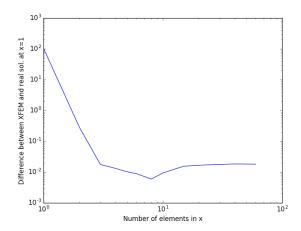


L2 Error Norms at Each Timestep





Mesh Refinement Effects on Error at x=1



1D, Cylindrical, Level Set Dependent Material Problem Description

PDE

$$\rho c_{p} \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_{p} \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \cdot k \frac{\partial T}{\partial r} \right) = q$$

Domain/Material Properties

$$\Omega_r = [1, 2], \quad \rho c_p = 10, \quad k = \left(\frac{0.05}{2.04}\right) \phi(x, t) + 1.5 = \frac{0.05}{2.04} \left(-x - 0.2t\right) + 1.55$$

BCs

Left: **Neumann**
$$-\frac{\partial T}{\partial r}\Big|_{r=1} = k(r,t) \cdot 200t$$

Right: **Dirichlet** –
$$T(2, t) = 400$$

ICs

Constant –
$$T(r, 0) = 400$$

Method of Manufactured Solutions for 1D, RZ, LS Dependent Material Problem

Prescribed Solution

$$T(r,t) = (-200r + 400)t + 400$$

Derived Source

$$q = 200 \rho c_p \left(-x+2\right) + \frac{1}{r} \left(310t - \frac{10rt}{1.02} - \frac{t^2}{1.02}\right)$$

Interface Level Set Function

$$\phi(r,t) = 2 - (r - 0.04) - 0.2t = 2.04 - r - 0.2t$$

1D XY Homog 1D XY LS Dep 1D RZ Homog 1D RZ LS Dep 2D XY Homog 2D XY LS Dep 2D RZ Homog 2D RZ LS Dep 000000

Numerical Parameters

```
[GlobalParams]
    order = FIRST
    family = LAGRANGE
16
18 [Problem]
    coord_type = RZ
20
22 [Mesh]
    type = GeneratedMesh
    dim = 2
    nx = 1
    ny = 1
    xmin = 1.0
    xmax = 2.0
    vmin = 0.0
    ymax = 0.5
    elem_type = QUAD4
32
```

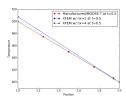
```
149 [Executioner]
     type = Transient
     solve_type = 'PJFNK'
     # petsc_options_iname = '-pc_type -pc_hypre_type'
     # petsc_options_value = 'hypre boomeramg'
     petsc_options_iname = '-pc_type'
     petsc_options_value = 'lu'
     line search = 'none'
   l_tol = 1.0e-6
    nl_max_its = 15
    nl_rel_tol = 1.0e-10
161
     nl_abs_tol = 1.0e-9
    start_time = 0.0
164 dt = 0.1
165 end_time = 2.0
    max_xfem_update = 1
167
```

```
[Constraints]
  [./xfem_constraint]
    type = XFEMSingleVariableConstraint
    variable = u
    jump = 0
    iump_flux = 0
    geometric_cut_userobject = 'level_set_cut_uo'
```

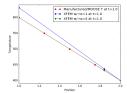
XFEM Moving Interface Verification

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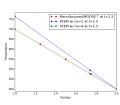
Results Comparison



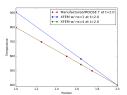
t = 0.5



t = 1.0

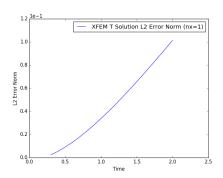


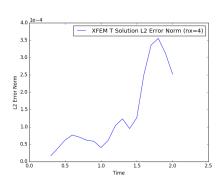
t = 1.5



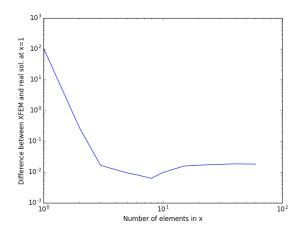
 $\leftarrow \rightarrow t = 2.0$

L2 Error Norms at Each Timestep





Mesh Refinement Effects on Error at x=1



2D, Cartesian, Homogeneous Material Problem Description

PDE

$$\rho c_{p} \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_{p} \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) = q$$

Domain/Material Properties

$$[\Omega_x, \Omega_y] = [[0, 1], [0, 1]]$$

 $\rho c_p = 10$
 $k = 1.5$

2D, Cartesian, Homogeneous Material Problem BCs/IC

BCs

Left: **Neumann** – $\frac{\partial T}{\partial x}\Big|_{x=0} = k \cdot 100t$

Right: **Dirichlet** – T(1, y, t) = (-100y + 100)t + 400

Bottom: **Neumann** $-\left.\frac{\partial T}{\partial y}\right|_{y=0} = k \cdot 100t$

Top: **Dirichlet** – T(x, 1, t) = (-100x + 100)t + 400

ICs

Constant – T(x, y, 0) = 400

Method of Manufactured Solutions for 2D, XY, Homogeneous Material Problem

Prescribed Solution

$$T(x, t) = (-100x - 100y + 200)t + 400$$

Derived Source

$$q = 100 \rho c_p \left(-x - y + 2 \right)$$

Interface Level Set Function

$$\phi(x, y, t) = -0.5(x + y) + 1.04 - 0.2t$$

Numerical Parameters

```
[GlobalParams]
    order = FIRST
16 family = LAGRANGE
17
19 [Mesh]
    type = GeneratedMesh
    dim = 2
    nx = 1
   ny = 1
    xmin = 0.0
    xmax = 1.0
   ymin = 0.0
   ymax = 1.0
    elem_type = OUAD4
```

```
L62 [Executioner]
    type = Transient
    solve_type = 'PJFNK'
    # petsc_options_iname = '-pc_type -pc_hypre_type'
    # petsc_options_value = 'hypre boomeramg'
    petsc_options_iname = '-pc_type'
    petsc_options_value = 'lu'
    line_search = 'none'
    1 \text{ tol} = 1.0e-6
172 nl_max_its = 15
nl_abs_tol = 1.0e-9
    start_time = 0.0
    dt = 0.1
   end_time = 2.0
    max_xfem_update = 1
```

```
81 [Constraints]
    [./xfem_constraints]
      type = XFEMSinaleVariableConstraint
      variable = u
      jump = 0
      jump_flux = 0
      geometric_cut_userobject = 'level_set_cut_uo'
    [.../]
89
```

Results Comparison

Upper plane is nx=1, ny=1; lower plane is nx=4, ny=4



t = 0.5



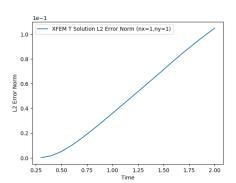
t = 1.0

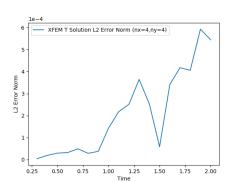


t = 1.5

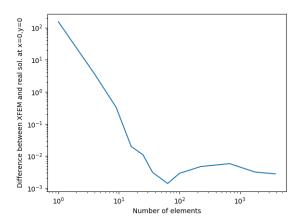


L2 Error Norms at Each Timestep





Mesh Refinement Effects on Error at x=0,y=0



2D, Cartesian, Level Set Dependent Material Problem Description

PDE

$$\rho c_{p} \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_{p} \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) = q$$

Domain/Material Properties

$$\begin{split} & [\Omega_x, \Omega_y] = [[0, 1], [0, 1]] \\ & \rho c_p = 10 \\ & k(x, y, t) = \left(\frac{0.05}{1.04}\right) \phi(x, y, t) + 1.5 = \left(\frac{0.01}{1.04}\right) (-2.5x - 2.5y - t) + 1.55 \end{split}$$



2D, Cartesian, Level Set Dependent Material Problem BCs/IC

BCs

Left: **Neumann** $-\frac{\partial T}{\partial x}\Big|_{x=0} = k(x, y, t) \cdot 100t$

Right, Top: **Dirichlet** -T(1, y, t) = (-100y + 100)t + 400

Bottom: **Neumann** $-\left.\frac{\partial T}{\partial y}\right|_{y=0} = k(x,y,t) \cdot 100t$

Top: **Dirichlet** – T(x, 1, t) = (-100x + 100)t + 400

ICs

Constant – T(x, y, 0) = 400



Method of Manufactured Solutions for 2D, XY, LS Dependent Material Problem

Prescribed Solution

$$T(x, t) = (-100x - 100y + 200)t + 400$$

Derived Source

$$q = 100 \rho c_p \left(-x - y + 2 \right) - \frac{5t}{1.04}$$

Interface Level Set Function

$$\phi(x, y, t) = -0.5(x + y) + 1.04 - 0.2t$$

Numerical Parameters

```
11 [GlobalParams]
12 order = FIRST
13 family = LAGRANGE
14 ]
15
16 [Mesh]
17 type = GeneratedMesh
18 dim = 2
19 nx = 1
20 ny = 1
21 xmin = 0.0
22 xmax = 1.0
23 ymin = 0.0
24 ymax = 1.0
25 elem_type = QUAD4
```

```
[Executioner]
  type = Transient
  solve_type = 'PJFNK'
 # petsc_options_iname = '-pc_type -pc_hypre_type'
 # petsc_options_value = 'hypre boomerama'
 petsc_options_iname = '-pc_type'
 petsc_options_value = 'lu'
 line_search = 'none'
 l_tol = 1.0e-6
 nl max its = 15
 nl_rel_tol = 1.0e-10
 nl_abs_tol = 1.0e-9
 start_time = 0.0
 dt = 0.1
 end_time = 2.0
 max_xfem_update = 1
```

```
78 [Constraints]
79 [./xfem_constraints]
80 type = XFEMSingleVariableConstraint
81 variable = u
82 jump = 0
83 jumpflux = 0
84 geometric_cut_userobject = 'level_set_cut_uo'
85 [../]
```

Results Comparison

Upper plane is nx=1, ny=1; lower plane is nx=4, ny=4



t = 0.5



t = 1.0

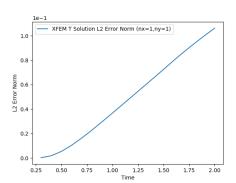


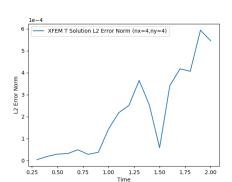
t = 1.5



$$t = 2.0$$

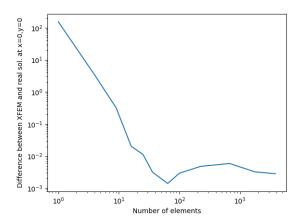
L2 Error Norms at Each Timestep







Mesh Refinement Effects on Error at x=0



2D, Cylindrical, Homogeneous Material Problem Description

PDE

$$\rho c_{p} \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_{p} \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \cdot k \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) = q$$

Domain/Material Properties

$$[\Omega_r, \Omega_z] = [[1, 2], [1, 2]]$$

 $\rho c_p = 10$
 $k = 1.5$

2D, Cylindrical, Homogeneous Problem BCs/IC

BCs

Left: **Neumann** $-\frac{\partial T}{\partial r}\Big|_{r=1} = k \cdot 100t$

Right: **Dirichlet** – T(2, z, t) = (-100z + 200)t + 400

Bottom: **Neumann** $-\frac{\partial T}{\partial z}\Big|_{z=1} = k \cdot 100t$

Top: **Dirichlet** -T(r, 2, t) = (-100r + 200)t + 400

ICs

Constant -T(r, z, 0) = 400

Method of Manufactured Solutions for 2D, RZ, Homogeneous Material Problem

Prescribed Solution

$$T(x, t) = (-100r - 100z + 400)t + 400$$

Derived Source

$$q = 100 \rho c_p (-r - z + 4) + \frac{100kt}{r}$$

Interface Level Set Function

$$\phi(x, y, t) = -0.5(x + y) + 2.04 - 0.2t$$

1D XY Homog 1D XY LS Dep 1D RZ Homog 1D RZ LS Dep 2D XY Homog 2D XY LS Dep 2D RZ Homog 2D RZ LS Dep 0000000

Numerical Parameters

```
11 [GlobalParams]
                                                 [Executioner]
    order = FIRST
                                                   type = Transient
13 family = LAGRANGE
                                                   solve_type = 'PJFNK'
14
                                                   # petsc_options_iname = '-pc_type -pc_hypre_type'
                                                   # petsc_options_value = 'hypre boomerama'
16 [Problem]
                                                   petsc_options_iname = '-pc_type'
    coord_type = RZ
                                                   petsc_options_value = 'lu'
18
                                               170 line_search = 'none'
20 [Mesh]
                                               72 l_tol = 1.0e-6
21 type = GeneratedMesh
                                               73 nl max its = 15
    dim = 2
                                                   nl rel tol = 1.0e-10
   nx = 1
                                                   nl_abs_tol = 1.0e-9
   nv = 1
   xmin = 1.0
                                                   start_time = 0.0
   xmax = 2.0
                                               178 dt = 0.1
   ymin = 1.0
                                               179 end_time = 2.0
   ymax = 2.0
                                               180 max_xfem_update = 1
    elem_type = QUAD4
```

```
[Constraints]
  \/xfem_constraints
    type = XFEMSingleVariableConstraint
    variable = u
    jump = 0
    jump_flux = 0
    geometric_cut_userobject = 'level_set_cut_uo'
  [.../I
```

30

Results Comparison

Upper plane is nx=1, ny=1; lower plane is nx=4, ny=4



t = 0.5



t = 1.0

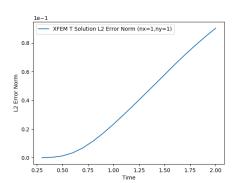


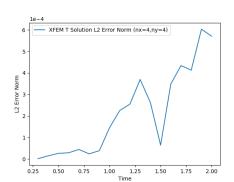
t = 1.5



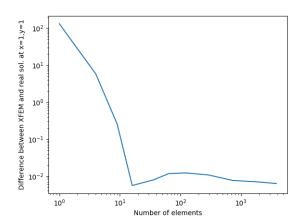
t = 2.0

L2 Error Norms at Each Timestep





Mesh Refinement Effects on Error at x=1, y=1



2D, Cylindrical, Level Set Dependent Material Problem Description

PDE

$$\rho c_{p} \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_{p} \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \cdot k \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) = q$$

Domain/Material Properties

$$\begin{split} &[\Omega_r, \Omega_z] = [[1, 2], [1, 2]] \\ &\rho c s_p = 10 \\ &k(r, z, t) = \left(\frac{0.05}{2.04}\right) \phi(r, z, t) + 1.5 = -\frac{0.025}{2.04}(r + z) + 1.55 - \frac{0.01t}{2.04} \end{split}$$

2D, Cylindrical, Level Set Dependent Material Problem BCs/IC

BCs

Left: Neumann – $\frac{\partial T}{\partial r}\Big|_{r=1} = k(r, z, t) \cdot 100t$

Right: **Dirichlet** – T(2, z, t) = (-100z + 200)t + 400

Bottom: **Neumann** – $\frac{\partial T}{\partial z}\Big|_{z=1} = k(r, z, t) \cdot 100t$

Top: Dirichlet -T(r, 2, t) = (-100r + 200)t + 400

ICs

Constant – T(r, z, 0) = 400

Method of Manufactured Solutions for 2D, RZ, LS Dependent Material Problem

Prescribed Solution

$$T(x, t) = (-100r - 100z + 400)t + 400$$

Derived Source

$$q = 100 \rho c_p \left(-r - z + 4\right) + t \left(-\frac{2.5}{2.04} \frac{z}{r} + 155 \frac{1}{r} + \frac{1}{2.04} \frac{t}{r} - \frac{7.5}{2.04}\right)$$

Interface Level Set Function

$$\phi(x, y, t) = -0.5(x + y) + 2.04 - 0.2t$$

Numerical Parameters

```
11 [GlobalParams]
   order = FIRST
13 family = LAGRANGE
14
16 [Problem]
    coord_type = RZ
18
20 [Mesh]
   type = GeneratedMesh
22 dim = 2
   nx = 1
   nv = 1
   xmin = 1.0
   xmax = 2.0
   ymin = 1.0
28 \quad ymax = 2.0
    elem_type = OUAD4
30
```

```
[Executioner]
     type = Transient
     solve_type = 'PJFNK'
     # petsc_options_iname = '-pc_type -pc_hypre_type'
     # petsc_options_value = 'hypre boomerama'
     petsc_options_iname = '-pc_type'
     petsc_options_value = 'lu'
175 line_search = 'none'
77 l_tol = 1.0e-6
78 nl max its = 15
79 nl rel tol = 1.0e-10
     nl_abs_tol = 1.0e-9
     start_time = 0.0
183 dt = 0.1
184 end_time = 2.0
185 max_xfem_update = 1
L86 🗖
```

```
82 [Constraints]
83    [./xfem_constraint]
84    type = XFEMSingleVariableConstraint
85    variable = u
86    jump = 0
87    jump_flux = 0
88    geometric_cut_userobject = 'level_set_cut_uo'
```

Results Comparison



t = 0.5



t = 1.0

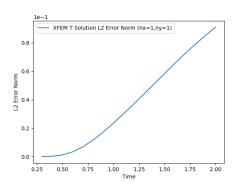


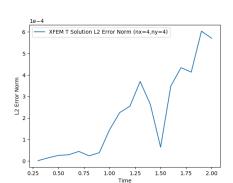
t = 1.5



$$t = 2.0$$

L2 Error Norms at Each Timestep





Mesh Refinement Effects on Error at x=1, y=1

