# 2D, Cartesian, Homogeneous Material Problem Description

## PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) = q$$

## Domain/Material Properties

$[\Omega_x, \Omega_y] = [[0, 1], [0, 1]]$
$\rho c_p = 10$
$k = 1.5$

# 2D, Cartesian, Homogeneous Material Problem BCs/IC

## BCs

Left: **Neumann** – $\frac{\partial T}{\partial x}\Big|_{x=0} = k \cdot 100t$

Right: **Dirichlet** – $T(1, y, t) = (-100y + 100)t + 400$

Bottom: **Neumann** – $\frac{\partial T}{\partial y}\Big|_{y=0} = k \cdot 100t$

Top: **Dirichlet** – $T(x, 1, t) = (-100x + 100)t + 400$

## ICs

**Constant** – $T(x, y, 0) = 400$

# Method of Manufactured Solutions for 2D, XY, Homogeneous Material Problem

## Prescribed Solution

$$T(x,t) = (-100x - 100y + 200)t + 400$$

## Derived Source

$$q = 100\,\rho c_p\,(-x - y + 2)$$

## Interface Level Set Function

$$\phi(x,y,t) = -0.5(x+y) + 1.04 - 0.2t$$

# Numerical Parameters

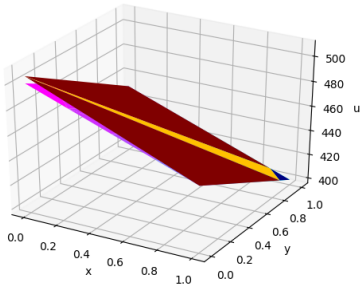
```
[GlobalParams]
  order = FIRST
  family = LAGRANGE
[]

[Mesh]
  type = GeneratedMesh
  dim = 2
  nx = 1
  ny = 1
  xmin = 0.0
  xmax = 1.0
  ymin = 0.0
  ymax = 1.0
  elem_type = QUAD4
[]
```

```
[Executioner]
  type = Transient
  solve_type = 'PJFNK'
  # petsc_options_iname = '-pc_type -pc_hypre_type'
  # petsc_options_value = 'hypre boomeramg'
  petsc_options_iname = '-pc_type'
  petsc_options_value = 'lu'
  line_search = 'none'

  l_tol = 1.0e-6
  nl_max_its = 15
  nl_rel_tol = 1.0e-10
  nl_abs_tol = 1.0e-9

  start_time = 0.0
  dt = 0.1
  end_time = 2.0
  max_xfem_update = 1
[]
```

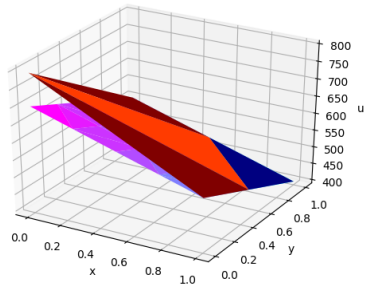
```
[Constraints]
  [./xfem_constraints]
    type = XFEMSingleVariableConstraint
    variable = u
    jump = 0
    jump_flux = 0
    geometric_cut_userobject = 'level_set_cut_uo'
  [../]
[]
```

# Results Comparison

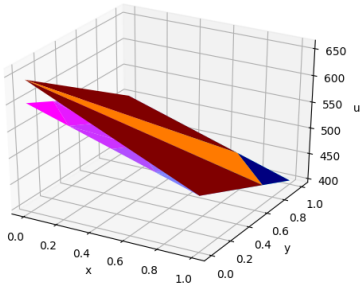
Upper plane is nx=1, ny=1; lower plane is nx=4, ny=4



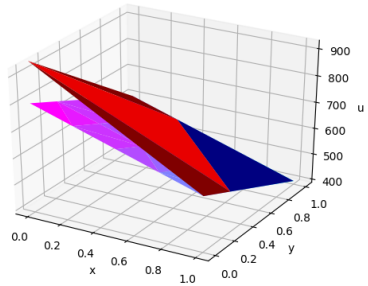

$t = 0.5$




$t = 1.5$

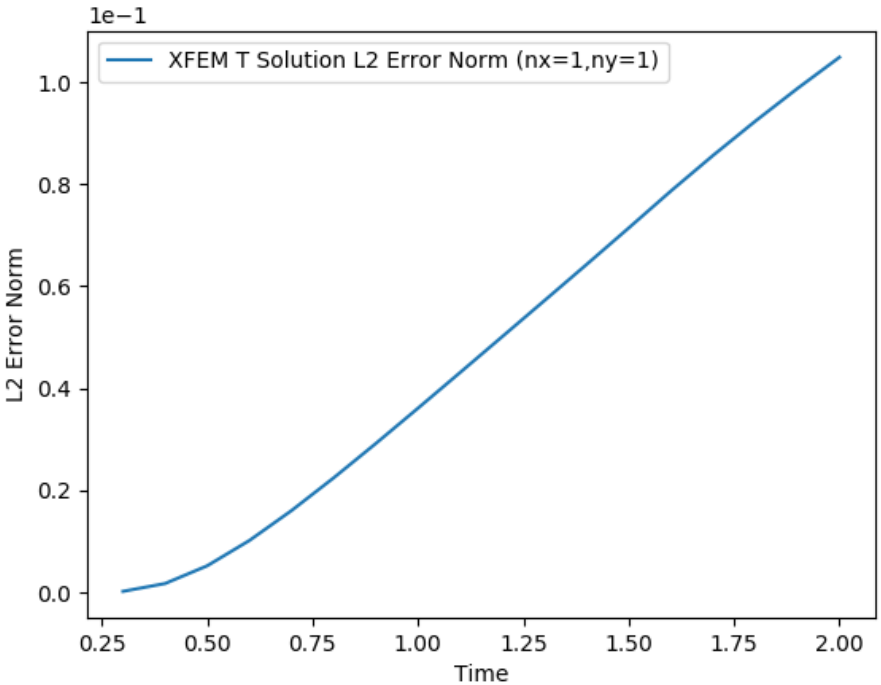


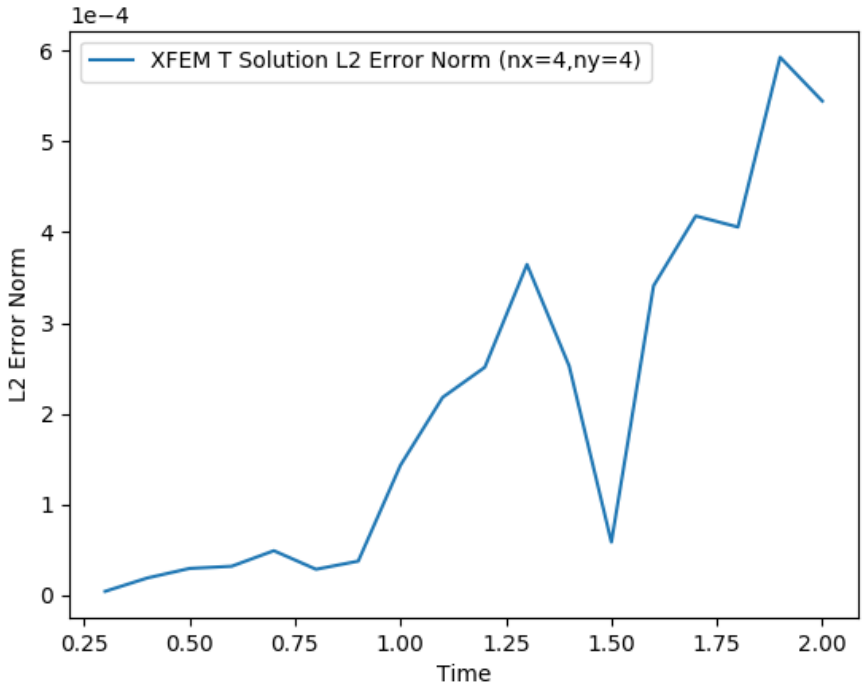

$t = 1.0$




$t = 2.0$

# L2 Error Norms at Each Timestep



# Mesh Refinement Effects on Error at x=0,y=0