# 2D, Cylindrical, Level Set Dependent Material Problem Description

## PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot k \frac{\partial T}{\partial r}\right) - \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) = q$$

## Advection PDE for Level Set

$$\frac{\partial \phi}{\partial t} + u(r,z,t) \cdot \nabla \phi = 0 \ , \ \frac{\partial \phi}{\partial t} + u(r,z,t) \cdot \left(\frac{\partial \phi}{\partial r} + \frac{\partial \phi}{\partial z}\right) = 0$$

## Domain/Material Properties

$[\Omega_r, \Omega_z] = [[1,2],[1,2]]$

$\rho cs_p = 10$

$k(r,z,t) = \left(\frac{0.05}{2.04}\right)\phi(r,z,t) + 1.5 = -\frac{0.025}{2.04}(r+z) + 1.55 - \frac{0.01t}{2.04}$

# 2D, Cylindrical, Level Set Dependent Material Problem BCs/IC

## BCs

Left: **Neumann** – $\left.\frac{\partial T}{\partial r}\right|_{r=1} = k(r,z,t) \cdot 100t$
Right: **Dirichlet** – $T(2,z,t) = (-100z + 200)t + 400$
Bottom: **Neumann** – $\left.\frac{\partial T}{\partial z}\right|_{z=1} = k(r,z,t) \cdot 100t$
Top: **Dirichlet** – $T(r,2,t) = (-100r + 200)t + 400$

## ICs

**Constant** – $T(r,z,0) = 400$
**Function** – $\phi(r,z,0) = -0.5(x+y) + 2.04$

# Method of Manufactured Solutions for 2D, RZ, LS Dependent Material Problem

## Prescribed $T$ Solution

$$T(x,t) = (-100r - 100z + 400)t + 400$$

## Derived $T$ Source

$$q = 100\,\rho c_p\,(-r - z + 4) + t\left(-\frac{2.5}{2.04}\frac{z}{r} + 155\frac{1}{r} + \frac{1}{2.04}\frac{t}{r} - \frac{7.5}{2.04}\right)$$

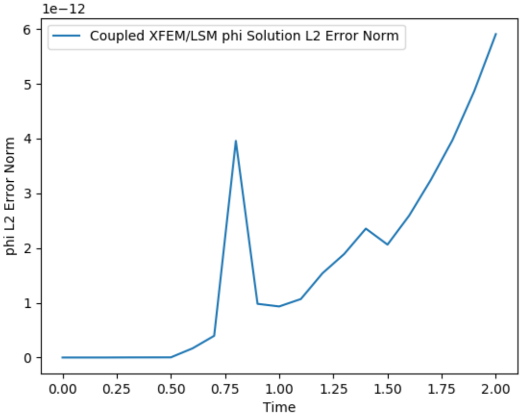
## Prescribed $\phi$ Interface Level Set Solution

$$\phi(x,y,t) = -0.5(x+y) + 2.04 - 0.2t$$

## Derived $\phi$ velocity term, $u(r,z,t)$

$$u(r,z,t) = -0.2$$

# $\phi$ Solution L2 Error Norms at Each Timestep



# $T$ Solution L2 Error Norms at Each Timestep