1D, Cylindrical, Level Set Dependent Material Problem Description

PDE

$$\rho c_p \frac{\partial T}{\partial t} - \nabla k \nabla T = \rho c_p \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \cdot k \frac{\partial T}{\partial r} \right) = q$$

Domain/Material Properties

$$\Omega_r = [1, 2], \quad \rho c_p = 10, \quad k = \left(\frac{0.05}{2.04} \right) \phi(x, t) + 1.5 = \frac{0.05}{2.04} \left(-x - 0.2t \right) + 1.55$$

BCs

Left: **Neumann** – $\frac{\partial T}{\partial r} \big|_{r=1} = k(r, t) \cdot 200t$
Right: **Dirichlet** – $T(2, t) = 400$

ICs

Constant – $T(r, 0) = 400$

Method of Manufactured Solutions for 1D, RZ, LS Dependent Material Problem

Prescribed Solution

$T(r, t) = (-200r + 400)t + 400$

Derived Source

$q = 200 \, \rho c_p \left(-x + 2 \right) + \frac{1}{r} \left(310t - \frac{10rt}{1.02} - \frac{t^2}{1.02} \right)$

Interface Level Set Function

$\phi(r, t) = 2 - (r - 0.04) - 0.2t = 2.04 - r - 0.2t$

Numerical Parameters

```
13 [GlobalParams]
14   order = FIRST
15   family = LAGRANGE
16 □
17
18 [Problem]
19   coord_type = RZ
20 □
21
22 [Mesh]
23   type = GeneratedMesh
24   dim = 2
25   nx = 1
26   ny = 1
27   xmin = 1.0
28   xmax = 2.0
29   ymin = 0.0
30   ymax = 0.5
31   elem_type = QUAD4
32 □
```

```
149 [Executioner]
150   type = Transient
151   solve_type = 'PJFNK'
152   # petsc_options_iname = '-pc_type -pc_hypre_type'
153   # petsc_options_value = 'hypre boomeramg'
154   petsc_options_iname = '-pc_type'
155   petsc_options_value = 'lu'
156   line_search = 'none'
157
158   l_tol = 1.0e-6
159   nl_max_its = 15
160   nl_rel_tol = 1.0e-10
161   nl_abs_tol = 1.0e-9
162
163   start_time = 0.0
164   dt = 0.1
165   end_time = 2.0
166   max_xfem_update = 1
167 □
```
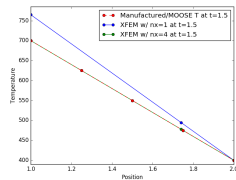
```
84 [Constraints]
85   [./xfem_constraint]
86     type = XFEMSingleVariableConstraint
87     variable = u
88     jump = 0
89     jump_flux = 0
90     geometric_cut_userobject = 'level_set_cut_uo'
91   [../]
```
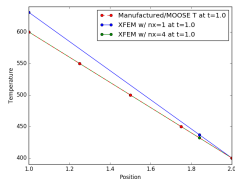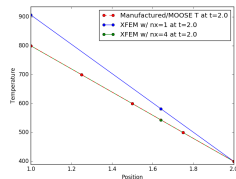
Results Comparison
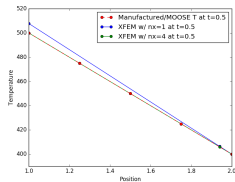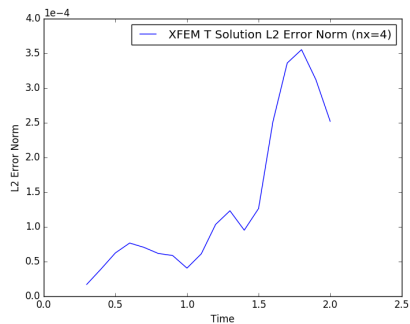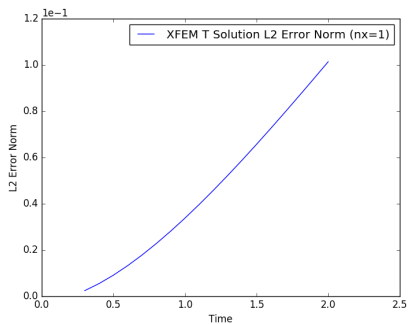

$t = 0.5$

$t = 1.5$

$t = 1.0$

$t = 2.0$

L2 Error Norms at Each Timestep

Mesh Refinement Effects on Error at x=0