ECON 7130 - Microeconomics III
Spring 2015
Notes for Bresnahan (*Journal of Industrial Economics*, 1987)

Question:

- Was the increase in quantity and decrease in price of autos sold in 1955 due to a price war (i.e. a break down in collusion)?

- DRAW supply and demand - show that only a shift in supply will increase quantity and lower price - so know a supply side effect

- Note: Causal effect - break down in collusion causes prices to drop and quantity to increase

Identification:

- Estimate structural model of equilibrium prices and quantities under different assumptions about market structure.

- Identify parameters in the model via maximum likelihood

    - The MLE finds model parameters that minimize the distance between the equilibrium prices and quantities from the model and those in the data
    - Weight the distances between the model predictions of prices and quantities and those observed in the data by the density function of the normal distribution
    - Key assumption: auto model quality is exogenous

Tools:

- Nash equilibrium

- Maximum likelihood estimation

Outline of Model

1. Specification of Environment

    (a) Population of agents
        - Continuum of consumers, defined by preferences $\nu_i \sim U[0, V_{max}]$ with density δ
        - F firms with J products
            - Each firm produces a suset $\mathcal{F}_f$ of the J products
    (b) Preferences
        - Consumer utility from purchasing an auto with quality x is $U(x, Y, \nu) = \nu x + Y - P$
            - $x =$ quality of auto
            - $Y =$ income
            - $P =$ price
            - ν is the consumers taste for quality (i.e. willingness to pay for quality)
        - Binary decision - buy or not
        - Utility if not buy is $U(x, Y, \nu) = \nu \gamma + Y - E$
            - $\gamma =$ "quality" of outside option (e.g. a used car)

- E is the price of the outside option

(c) Production technology
- Auto manufacturer cost function is: $c(x, q) = A(x) + mc(x)q$
  - Will assume $mc(x) = \mu e^x$ (want mc to be increasing and convex)

(d) Information technology
- Perfect info

(e) Enforcement technology
- N/A

(f) Matching technology
- Decentralized market where all buyers and sellers meet

2. Equilibrium

- Non-cooperative equilibirum: Bertrand-Nash Equilibrium and a Cooperative Equilibrium

Demand:

- Utility function (and discrete choice) $\Rightarrow$ choose type j to max:

$$U(x_j, Y, \nu) = \nu x_j - P_j + Y \tag{1}$$

- This implies that type j will solve

$$\min P_j - \nu x_j \tag{2}$$

- We can thus determine demand by finding where a consumer is indifferent between 2 options, say h and i, where $x_i > x_h$ (note we'll rank-order all options to that those with higher letters correspond to higher quality)

  - Consumer ν_{hi} is indifferent between h and i iff:

$$P_i - x_i \nu_{hi} = P_h - x_h \nu_{hi} \tag{3}$$

  which implies:

$$\nu_{hi} = \frac{P_i - P_h}{x_i - x_h} \tag{4}$$

  - if $\nu < \nu_{hi}$, then prefer h
  - if $\nu > \nu_{hi}$ then prefer i

- We can do the same for products i and j $(x_j > x_i) \Rightarrow$:

$$\nu_{ij} = \frac{P_j - P_i}{x_j - x_i} \tag{5}$$

- So and individual with $\nu \in [\nu_{hi}, \nu_{ij}]$ will buy i

- $\Rightarrow$ aggregate demand for i is:

$$q_i = \delta[\nu_{ij} - \nu_{hi}] = \delta \left[\frac{P_j - P_i}{x_j - x_i} - \frac{P_i - P_h}{x_i - x_h} \right] \tag{6}$$

- DRAW the graph with three demand curves - for h, i, j - and show this range...

- Note how demand for i is only a function of autos with quality h and $j \to$ only "nearby" competitors matter

- To determine demand for the lowest and highest quality autos:

  - Assume those with the highest willingness to pay buy the highest quality auto, J:

$$q_J = \delta \left[V_{max} - \frac{P_J - P_{J-1}}{x_J - x_{J-1}} \right] \tag{7}$$

  - Assume that the outside option defined by E and γ has lower quality than all new autos: $\gamma < x_1$ (e.g. it's a used car):

$$q_1 = \delta \left[\frac{P_2 - P_1}{x_2 - x_1} - \frac{P_1 - E}{x_1 - \gamma} \right] \tag{8}$$

Supply:

- Cost function $\Rightarrow$ profits:

$$\pi_i = P_i q_i - mc(x_i) q_i - A(x_i) \tag{9}$$

- Assume Bertrand (price) competition

- Thus, if not collude with neighbors (or have similar products in own firm), then:

$$\frac{\partial \pi_i}{\partial P_i} = q_i + P_i \frac{\partial q_i}{\partial P_i} - mc(x_i) \frac{\partial q_i}{\partial P_i} = 0 \tag{10}$$

  which can be written as:

$$\frac{\partial \pi_i}{\partial P_i} = q_i + \underbrace{[P_i - mc(x_i)]}_{\text{markup}} \frac{\partial q_i}{\partial P_i} = 0 \tag{11}$$

- and if collude with $i+1$ then:

$$\frac{\partial \pi_i}{\partial P_i} = q_i + [P_i - mc(x_i)] \frac{\partial q_i}{\partial P_i} + [P_{i+1} - mc(x_{i+1})] \underbrace{\frac{\partial q_{i+1}}{\partial P_i}}_{\text{the cross-price elasticity matters}} = 0 \tag{12}$$

  - Note that collusion is modeled as the firm jointly maximizing the profits of itself and its neighbor
  - Also, note that cross-price elasticities of non-neighbors are zero - so only need to consider the neighbor in the collusive framework
    * This is a result of how consumer preferences and choices (buy/not buy) are specified

- Bresnahan introduces H, a symmetric, $J \times J$ matrix to summarize collusion (either across firms or within firms)

  - $H_{ij} = \begin{cases} 1, & \text{if products } i \text{ and } j \text{ collude.} \\ 0, & \text{otherwise} \end{cases}$

  - This implies the FOC can be written as the following:

$$\frac{\partial \pi_i}{\partial P_i} = q_i + [P_i - mc(x_i)] \frac{\partial q_i}{\partial P_i} + H_{i,i+1}[P_{i+1} - mc(x_{i+1})] \frac{\partial q_{i+1}}{\partial P_i} + H_{i,i-1}[P_{i-1} - mc(x_{i-1})] \frac{\partial q_{i-1}}{\partial P_i} = 0 \tag{13}$$

- From the demand curves we know:

$$\frac{\partial q_j}{\partial P_j} = \delta \left[\frac{1}{x_j - x_{j+1}} + \frac{1}{x_{j-1} - x_j} \right], \quad \leq 0 \tag{14}$$

and

$$\frac{\partial q_j}{\partial P_k} = \begin{cases} \delta \left(\frac{1}{|x_j - x_k|} \right), k \in \{j-1, j+1\}, & \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

- an implication of this is that if H_{ij} goes from 0 to 1, for any i, j neighbors, then $P_k - mc_k \uparrow \forall k$
- Think about this - prices go up more since now value spill over of demand to neighbors colluding with

Equilibrium:

- To solve for the equilibrium, plug the q_i and $\frac{\partial q_i}{P_k}, \forall i, k$ into the J FOCs and get the J length vectors for equilibrium prices and quantities:

  - $P = P^*(x, H, \gamma, V_{max}, \delta, \mu)$
  - $q = Q^*(x, H, \gamma, V_{max}, \delta, \mu)$

- e.g., for autos of quality 3:

  - FOC:

$$\delta \left[\frac{P_4 - P_3}{x_4 - x_3} - \frac{P_3 - P_2}{x_3 - x_2} \right] + (P_3 - \mu e^{x_3}) \delta \left[\frac{1}{x_3 - x_4} + \frac{1}{x_2 - x_3} \right] + H_{3,4}(P_4 - \mu e^{x_4}) \delta \left[\frac{1}{x_4 - x_3} \right]$$
$$+ H_{2,3}(P_2 - \mu e^{x_2}) \delta \left[\frac{1}{x_3 - x_2} \right] = 0 \quad (16)$$

  - Note that there are J FOCs like this
  - Will use these J FOCs to solve for the J prices (these are the unknowns - remember that quality, x is assumed exogenous)
  - Then plug the J prices into the J demand equations to derive the equilibrium quantities

The next step is to take the model to the data...

Data:

- Need data on prices and quantities that are predicted from the model:
  - prices $\rightarrow$ list prices for new auto models
  - quantities $\rightarrow$ sales = quantity produced
- Note: no need to observe marginal cost directly
  - assumed functional form will allow identification

Identification:

- Assume that quality is a function of characteristics of the auto model:

  - $x_j = \sqrt{\beta_0 + \sum_k \beta_k z_{jk}}$
  - z_{jk} = characteristic k of model j (e.g., a V8 engine)

- Let (P^*, Q^*) be the model equilibrium

  - With the above assumption, these vectors are only a function of parameters: $\{\beta, H, \gamma, V_{max}, \delta, \mu\}$
  - H will depend upon the assumed market structure

- Assume measurement error in the data:

  - $P_j = P_j^* + \varepsilon_j^p$
  - $Q_j = Q_j^* + \varepsilon_j^q$
  - $\varepsilon_j^p \sim N(0, \sigma_p^2)$
  - $\varepsilon_j^q \sim N(0, \sigma_q^2)$
  - Note: the structural model is "wrong" without these error terms (it's not going to hit the data exactly, so we build in this error as you might do with OLS)

- The assumed distribution of the error terms imply the form of the likelihood function

  - Because of the normal distributions for measurement error, we can write the likelihood of observing the observed prices and quantities as:

$$\mathcal{L} = \prod_{j=1}^{J} \frac{1}{\sqrt{2\pi\sigma_p^2}} exp\left[\frac{-(P_j - P_j^*)^2}{2\sigma_p^2}\right] \frac{1}{\sqrt{2\pi\sigma_q^2}} exp\left[\frac{-(Q_j - Q_j^*)^2}{2\sigma_q^2}\right] \tag{17}$$

  - Note that Bresnahan makes an adjustment to the above to account for the the amount of autos sold varying across years (it amounts to controlling for heteroskedasticity as the error terms will be larger if more autos are sold)

- Bresnahan finds the parameter vector that maximizes the likelihood for each year (1954, 1955, 1956) and each of four assumptions about market structure;

  1. Collusion: $H = ones(J, J)$
  2. Nash, no collusion: H has blocks of ones (for companies with multiple products)
  3. Products: H=Identity matrix (i.e., compete with even other models from same firm)
  4. Hedonic: different structure than is summarized by H ("continuous product differentiation", no optimal behavior by firms)

- Each of the 12 market outcomes (3 years $\times$ 4 models) are compared

  - Use Cox test to find which market structure fits each year best
    * Formal test of model fit
    * Test statistic if the likelihood ratio from two models (so implicitly assuming one of 4 models is correct)
    * Using test statistic that is distributed χ^2
  - Check to see how parameter estimates vary across years - idea is that if the model is correct, "deep" parameters estimated through structural estimation should not vary much (any) across years
  - Finds collusion in 1954 and 1956, breakdown in collusion in 1955 (Nash, no collusion)

Strong points of paper:

- Parsimonious estimation of structural model of supply and demand using only observations of quantity and price

- Good example of structural estimation

- Clear identification

Weak points of paper:

- Cox tests require that some alternative wins, even if none correct

- No errors in function determining quality (does this seem reasonable? Likely omitted variables, measurement error here)

- Demand only allows substitution to neighboring products

- Is quality really exogenous?

- Cars are durable goods - what about dynamics?