ECON 7130 - Microeconomics III

Spring 2015

Notes for Gibbons and Murphy (*Journal of Political Economy*, 1992)

Question:

- What is the optimal incentive contract when effort and ability are unobservable?
- Do CEO contracts reflect the comparative statics that a model of optimal contracts predicts?
  - E.g., Are explicit incentives are greater for CEO's with more tenure/closer to retirement?

Indentification:

- Test the comparative statics (qualitative predictions) of the model
- Test hypothesis by OLS - see if sign of coefficients is the same as model prediction
  - OLS regressing compensation on job tenure and time until retirement interacted with changes in output
  - Key assumptions: workers don't know their own ability, ability fixed over time

Tools:

- Subgame Perfect Nash Equilibrium
- Rational Expectations Equilibirium
- OLS

Outline of Model

1. Specification of Environment
   (a) Population of agents
      - Employee
      - Employers - competitive

   (b) Preferences
      - Employee has preferences over compensation, w, and effort, a.
        - One period utility: $U(w;a) = -e^{-r[w-g(a)]}$
          * $g(\cdot)$ is convex and increasing in a
          * Utility is NOT separable across time: $U(w_1, ..., w_T; a_1, ..., a_T) = -exp\left(-r\left[\sum_{t=1}^{T} \delta^{t-1}[w_t - g(a_t)]\right]\right)$
          * This assumption implies that agents are indifferent between deterministic wage streams with the same NPV of income
          * This is not a standard assumptions, but works out to be equivalent to standard model if worker can borrow and save without restriction
          * r = coefficient of absolute risk aversion
          * δ = discount rate

   (c) Production technology

  - Workers produce output, $y_t = \eta + a_t + \epsilon_t$, where:
    - η = worker ability, $\eta \sim N(m_0, \sigma_0^2)$
    - a_t = worker effort
    - ϵ_t = noise, $\epsilon \sim N(0, \epsilon_0^2)$

(d) Information technology

  - Asymmetric info regarding worker effort - observed by worker, but not by employer
  - Imperfect, but symmetric info regarding worker ability - neither employer or worker know the worker's ability level

(e) Enforcement technology

  - Labor contracts are one period only
  - Contracts must be linear in output

(f) Matching technology

  - Decentralized, competitive labor market where all firms and workers meet

2. Equilibrium

  - Subgame Perfect Nash Equilibrium

<u>Model outline</u>:

- Employers pay employees in a way that will induce effort
- Workers are risk averse, and don't know their ability level (which will influence compensation), so the contract must provide some insurance to workers
- The contract will optimize effort using both implicit and explicit incentives
  - Explicit incentives are the increases in compensation from more output, which is correlated with effort
  - Implicit incentives are from the belief that the worker is of high ability and so gets paid more in future contracts
    - when output increases, conditional probability that worker is of high type (i.e., has a high η increases), so worker's future contracts will be for more
- Relative importance of implicit vs explicit compensation change over lifetime of worker (i.e., as years in career get less, or get more certain of type, then implicit incentives worth less)

<u>2 Period Model</u>:

- Note: If doing a dynamic model, it's <u>always</u> helpful to work through the simple 2 period model first
  - These models are simple enough to solve by hand, but contain most of the dynamic relationships of interest
- DRAW out timeline: set contract, choose effort, realize output, set contract for period 2...
- Worker utility:

$$U(w_1, w_2; a_1, a_2) = -exp(-r[w_1 - g(a_1) + \delta(w_2 - g(a_2))]) \tag{1}$$

  - with $g'(\cdot) > 0, g''(\cdot) > 0, g'(0) = 0, g'(\infty) = \infty, g'''(0) \geq 0$
  - w, compensation, will be a linear function of worker output in one year: $w_t(y_t) = c_t + b_t y_t$

* GM assume only linear contracts - because only interested in comparative static, so slope of contract is what is of interest
    * GM assume only one-period contracts, which amounts to contracts being renegotiation proof (like sub-game perfect)
    * But note that $t > 1$ contracts depend on $\{y_t\}_{t=1}^{t-1}$ implicitly because these output levels provide some info regarding worker ability

- output: $y_t = \eta + a_t + \epsilon_t$
  - η= worker ability
  - a_t = worker effort in period t
  - ϵ_t = noise
- The worker's problem is:

$$\max_{a_1,a_2} -E_{\eta,\epsilon_1,\epsilon_2}\left\{exp\left(-r\left[c_1 + b_1(\eta + a_1 + \epsilon_1) - g(a_1) + \delta[c_2 + b_2(\eta + a_2 + \epsilon_2) - g(a_2)]\right]\right)\right\} \tag{2}$$

- To solve, use backwards induction:
  - In period 2, a_1 has been chosen and y_1 observed
  - Problem in period 2 is:

$$\max_{a_2} -E\left\{exp\left(-r\left[c_2 + b_2(\eta + a_2 + \epsilon_2) - g(a_2)\right]\right)|y_1\right\} \tag{3}$$

  - FOC w.r.t. a_2:

$$E\left\{e^{-r[\cdot]}\left[b_2 - g'(a_2)\right]\right\} = 0 \tag{4}$$

  - $\Rightarrow b_2 = g'(a_2)$, which defines the optimal choice of a_2 given b_2 - call this a_2^*
  - Note that a increases with b (know this b/c $g''(\cdot) > 0$)
- To solve for the optimal second period contract, use fact that employers are completive, so expected profits must be zero
  - $E\{[c_2 + b_2(\underbrace{\eta + a_2 + \epsilon_2}_{E\{y_2|y_1\}})]|y_1\} = E\{y_2|y_1\}$
  - $\Rightarrow c_2 + b_2 E\{y_2|y_1\} = E\{y_2|y_1\}$
  - $\Rightarrow c_2 = (1 - b_2)E\{y_2|y_1\} \rightarrow c_2(b_2)$ solves this
- Note that $c_2(b_2)$ relies on $E\{y_2|y_1\}$
  - $E\{y_2|y_1\} = E\{\eta + a_2 + \epsilon_2|y_1\} = E\{\eta|y_1\} + E\{a_2|y_1\}$
  - This depends on $E\{\eta|y_1\}$- how compute this?
  - Use distributional assumptions on type and noise and find conditional distribution of η
  - Use conjecture about a_1: $\hat{a}_1$ - this is what the firm believes about effort in period 1
    * In Rational Expectations Equilibrium, $\hat{a}_1 = a_1$
  - The conditional distribution of η is normal with mean:

$$m_1(y_1; \hat{a}_1) = \frac{\sigma_\epsilon^2 m_0 + \sigma_0^2(\overbrace{y_1 - \hat{a}_1}^{=\eta+\epsilon_1})}{\underbrace{\sigma_\epsilon^2 + \sigma_0^2}_{\text{Conditional variance of } \eta+\epsilon_1}} \tag{5}$$

  and variance:

$$\sigma_1^2 = \frac{\sigma_0^2\sigma_\epsilon^2}{\sigma_0^2 + \sigma_\epsilon^2} \tag{6}$$

- To find the optimal second period contract, need to find the optimal b_2. To do so, solve:

$$\max_{b_2} -E\{exp(-r[c_2(b_2) + b_2[\eta + a_2^*(b_2) + \epsilon_2] - g(a_2^*(b_2))])|y_1\} \tag{7}$$

  - Note from above that $c_2(b_2) = (1-b_2)E\{y_2|y_1\} = (1-b_2)[E\{\eta|y_1\}+a_2^*(b_2)] = (1-b_2)[m_1(y_1, \hat{a}_1)+a_2^*(b_2)]$

  Thus, the problem can be written as:

$$\max_{b_2} -E\{exp(-r[m_1(y_1, \hat{a}_1) + a_2^* + b_2\epsilon_2 - g(a_2^* b_2)])|y_1\} \tag{8}$$

  Which equals:

$$\max_{b_2} -exp\left(-r\left[m_1(y_1, \hat{a}_1) + a_2^* + g(a_2^*(b_2)) - \frac{1}{2}rb_2^2\Sigma_2^2\right]\right) \tag{9}$$

  where $\Sigma_2^2 \equiv \sigma_1^2 + \sigma_\epsilon^2$
  (the expectation was calculated using the rule: $E(e^{-k\mu}) = e^{(-k\mu + \frac{1}{2}k^2\sigma^2)}$)

- The FOC w.r.t. b_2 is:

$$-e^{-r[\cdot]}\left[\frac{\partial a_2^*(b_2)}{b_2} - g'(a_2^*(b_2))\frac{\partial a_2^*(b_2)}{b_2} - \frac{2}{2}rb_2\Sigma_2^2\right] = 0 \tag{10}$$

$$\Rightarrow \frac{\partial a_2^*(b_2)}{b_2} - g'(a_2^*(b_2))\frac{\partial a_2^*(b_2)}{b_2} = \frac{2}{2}rb_2\Sigma_2^2 \tag{11}$$

  - Recall, from employee FOC: $g'(a_2^*) = b_2$
  - Differentiating this implies: $g''(a_2^*)\frac{\partial a_2^*}{b_2} = 1 \Rightarrow \frac{\partial a_2^*}{b_2} = \frac{1}{g''(a_2^*)}$

$$\Rightarrow \frac{1}{g''(a_2^*)} - \frac{b_2}{g''(a_2^*)} = rb_2\Sigma_2^2 \tag{12}$$

$$\Rightarrow \frac{1}{g''(a_2^*)} = \frac{b_2}{g''(a_2^*)} + rb_2\Sigma_2^2 \tag{13}$$

$$\Rightarrow \frac{1}{g''(a_2^*)} = b_2\left(\frac{1}{g''(a_2^*)} + r\Sigma_2^2\right) \tag{14}$$

$$\Rightarrow b_2 = \frac{1}{1 + r\Sigma_2^2 g''(a_2^*)} \tag{15}$$

  - This equation defines b_2^*
  - Note that it does NOT depend on y_1
    * Because there are no wealth effects, the optimal incentives depend upon the variance, not the mean
  - NOTE: the contract will still depend on y_1 because c_2 does
    * $c_2(b_2^*) = (1 - b_2^*)E\{y_2|y_1\} = (1 - b_2^*)[m_1(y_1, \hat{a}_1) + a_2^*(b_2^*)]$
    * So y_1 affects base pay, but not the incentive pay

- To solve for the stage 1 contract, proceed in a similar manner:
  - Find effort level as s function of incentive pay
  - Find incentive pay that maximizes utility given effort function
  - Find base compensation that results in zero profits

- Worker in period 1 solves:

$$\max_{a_1} -E\{exp(-r[c_1 + b_1(\eta + a_1 + \epsilon_1) - g(a_1) + \delta[c_2(b_2^*) + b_2^*(\eta + a_2^*(b_2^*) + \epsilon_2) - g(a_2^*(b_2^*))]])\} \tag{16}$$

  - We can write the above w/ $c_2(b_2^*)$ written out...
  - The FOC is then:

$$E\{e^{-r[\cdot]}\}[b_1 - g'(a_1) + \delta\frac{\partial c_2}{\partial a_1}] = 0 \tag{17}$$

$$\Rightarrow b_1 + \delta\frac{\partial c_2}{\partial a_1} = g'(a_1) = B_1 \rightarrow a_1^*(b_1) \text{ solves this} \tag{18}$$

  - This means that $B_1 = b_1 + \delta\left[(1 - b_2^*)\frac{\sigma_0^2}{\sigma_0^2 + \sigma_\epsilon^2}\right]$
    * Note that b_1 is the explicit incentive
    * The second term are the implicit, career concern incentives - i.e. how effort increase future compensation through increases in the believed type
    * B_1 is :
      - > 0 b/c $0 < b_1 < 1$
      - $\uparrow$ as $\sigma_0^2 \uparrow$ (b/c as variance of type increases, need more "insurance")
      - $\downarrow$ as $\sigma_\epsilon^2 \uparrow$ (b/c learn less about type with more noise)
  - Note that R.E. $\Rightarrow \hat{a}_1 = a_1^*(b_1)$
  - The zero profit condition will imply:

$$c_1(b_1) = (1 - b_1)E(y_1) = (1 - b_1)[m_0 + a_1^*(b_1)] \tag{19}$$

    * $\rightarrow$ substitute a_1^* and $c_1(b_1)$ into employee utility and find the FOC w.r.t. b_1
    * $\Rightarrow b_1^* = \frac{1}{1 + r\Sigma_1^1 g''(a_1^*(b_1))} - \delta(1 - b_2^*)\frac{\sigma_0^2}{\sigma_0^2 + \sigma_\epsilon^2} - \frac{r\delta b_2^* \sigma_0^2 g''(a_1^*(b_1^*))}{1 + r\Sigma_1^2 g''(a_1^*(b_1^*))}$
    * in the above, $\Sigma_1^2 = \sigma_0^2 + \sigma_\epsilon^2$
    * Can show: $b_1^* < b_2^*$
    * 3 terms in b_1^*:
      1. Noise reduction effect: learning about ability $\Rightarrow \downarrow$ in conditional variance so trade off between insurance and incentives more towards incentives over time. So this increases explicit incentive compensation.
      2. Career concerns effect: lower pay for performance when career concerns high b/c want to put forth effort for high pay later. i.e., less explicit incentives because implicit incentives are high.
      3. Human capital insurance effect: want to insure against low type (b/c employee risk averse). The decreases pay for performance because performance a function of ability. This effect can drive $B_1 < 0$.

- GM then generalize the model to T periods
  - Same 3 effects present
  - $b_{t-1} < b_t \forall t < T$
    * explicit incentive increase as near retirement (b/c career concerns decline)
    * this will be the key hypothesis they test
    * Comparative static is $\frac{\partial b_t}{t} > 0$

<u>Data</u>:

- Need data on compensation and employee output

- Hard to get either of these for rank and file employees (administrative data from SSA or IRS may give compensation, but output measures near impossible)
- ⇒ use CEO's b/c compensation must be reported on financial statements and can see output since CEOs should materially affect stock price
  - Also, agency problem between CEO and shareholders is clear
- Data:
  - Compensation: Execucomp
    - Charlie has used
    - From public company financials
    - Total compensation for CEOs including stock options
  - Output: Stock price from Compustat
    - Use change in stock value

Hypotheses:

- H1: Holding tenure constant, explicit incentives increase as near retirement
- H2: Holding constant time until retire, explicit incentives increase as tenure increases
- H3, H4: Have to do with changes in expected output over career, the first having to do with effort as near retirement, the second having to do with more precise estimates of ability as tenure increases

Identification:

- OLS w/ panel data
- $\Delta ln(w_{it}) = \alpha_0 + \alpha_1(\text{few years left})_{it} + \sum_{n=1972}^{1988} \alpha_n(\text{nth year dummy})_{it} +$ $\left[\beta_0 + \beta_1(\text{few years left})_{it} + \sum_{n=1972}^{1988} \beta_n(\text{nth year dummy})_{it}\right] \times \Delta ln(V_{it})$
  - Use elasticities b/c better with firms of different sizes
  - Interested in the β's - how incentive structure changes over career
- Some other, similar, specifications are run
- Support is found for H1 (β increases as years left falls). H2 is rejected, but authors propose alternative model that is consistent with empirical results (i.e., ability that evolves are a random walk with drift)