Affine Transformation

An affine transformation is any transformation that preserves collinearity (i.e., all points lying on a line initially still lie on a line after transformation) and ratios of distances (e.g., the midpoint of a line segment remains the midpoint after transformation). In this sense, affine indicates a special class of projective transformations that do not move any objects from the affine space $\mathbb{R}^3$ to the plane at infinity or conversely. An affine transformation is also called an affinity.

Geometric contraction, expansion, dilation, reflection, rotation, shear, similarity transformations, spiral similarities, and translation are all affine transformations, as are their combinations. In general, an affine transformation is a composition of rotations, translations, dilations, and shears.

While an affine transformation preserves *proportions* on lines, it does not necessarily preserve angles or lengths. Any triangle can be transformed into any other by an affine transformation, so all triangles are affine and, in this sense, affine is a generalization of congruent and similar.

A particular example combining rotation and expansion is the rotation-enlargement transformation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = s \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} \tag{1}$$

$$= s \begin{bmatrix} \cos\alpha\,(x - x_0) + \sin\alpha\,(y - y_0) \\ -\sin\alpha\,(x - x_0) + \cos\alpha\,(y - y_0) \end{bmatrix}. \tag{2}$$

Separating the equations,

$$x' = (s\cos\alpha)\,x + (s\sin\alpha)\,y - s\,(x_0\cos\alpha + y_0\sin\alpha) \tag{3}$$

$$y' = (-s\sin\alpha)\,x + (s\cos\alpha)\,y + s\,(x_0\sin\alpha - y_0\cos\alpha). \tag{4}$$

This can be also written as

$$x' = a\,x - b\,y + c \tag{5}$$

$$y' = b\,x + a\,y + d, \tag{6}$$

where

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The scale factor s is then defined by

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$$s \equiv \sqrt{a^2 + b^2}, \qquad (9)$$

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and the rotation angle by

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An affine transformation of $\mathbb{R}^n$ is a map $F : \mathbb{R}^n \to \mathbb{R}^n$ of the form

$$F(\mathbf{p}) = \mathsf{A}\,\mathbf{p} + \mathbf{q} \qquad (11)$$

for all $\mathbf{p} \in \mathbb{R}^n$, where A is a linear transformation of $\mathbb{R}^n$. If $\det(\mathsf{A}) > 0$, the transformation is orientation-preserving; if $\det(\mathsf{A}) < 0$, it is orientation-reversing.

SEE ALSO

Affine, Affine Complex Plane, Affine Equation, Affine Geometry, Affine Group, Affine Hull, Affine Plane, Affine Space, Equiaffinity, Euclidean Motion, Special Affine Transformation

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CITE THIS AS:

Weisstein, Eric W. "Affine Transformation." From *MathWorld*--A Wolfram Web Resource. https://mathworld.wolfram.com/AffineTransformation.html

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