# Stationarity

## Modeling Intensive Longitudinal Data

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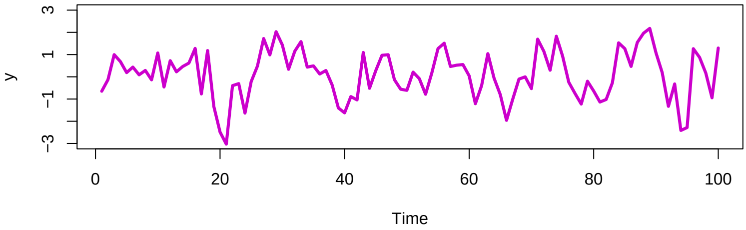
# Stationarity

Stationarity implies that **all moments** (i.e., means, variances, covariances, lagged covariances, etc.) are **invariant over time**.

A time series is stationary when:

- the **mean is stable** over time (i.e, no trend or seasonality)
- the **variances and autocovariances are stable** over time
- (and all high-order moments are stable over time)



# Non-stationarity

**Reasons** for nonstationarity:

- the process has a **unit root** (e.g., a random walk)
- there is a **trend over time** (e.g., growth curve, day-of-week effect)
- the **parameters** (e.g., autoregressive parameter) **change over time**

**Different forms** of nonstationarity require:

- different ways to **detect** them (e.g., specific tests, model features)
- different ways to **handle** them (e.g., detrending versus differencing)

Nonstationarity is discussed as a **property of**:

- the **model** (i.e., the data-generating mechanism)
- the **observed data**

# Stationarity of an AR(1)

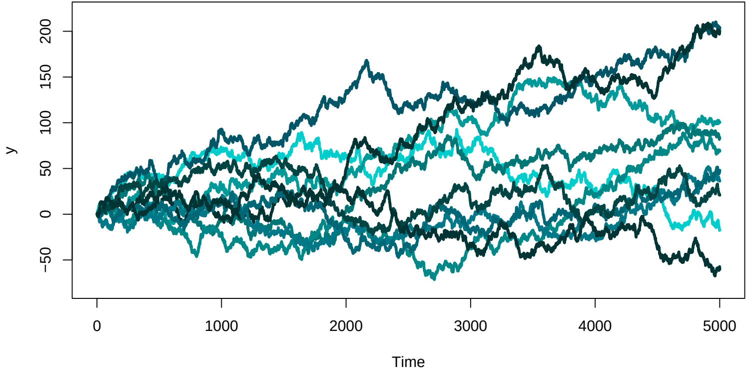
For an AR(1) to be stationary, $|\phi| < 1$.

To see why, we rewrite the AR(1) as an MA($\infty$):

$$
\begin{aligned}
y_t &= \phi_1 y_{t-1} + \epsilon_t \\
&= \phi_1 \big(\phi_1 y_{t-2} + \epsilon_{t-1}\big) + \epsilon_t \\
&= \phi_1^2 y_{t-2} + \phi_1 \epsilon_{t-1} + \epsilon_t \\
&= \phi_1^2 \big(\phi_1 y_{t-3} + \epsilon_{t-2}\big) + \phi_1 \epsilon_{t-1} + \epsilon_t \\
&= \phi_1^3 y_{t-3} + \phi_1^2 \epsilon_{t-2} + \phi_1 \epsilon_{t-1} + \epsilon_t \\
&= \dots \\
&= \sum_{k=0}^{\infty} \phi_1^k \epsilon_{t-k}
\end{aligned}
$$

# Random walk examples

$$y_t = y_{t-1} + \epsilon_t = \sum_{k=0}^{\infty} \epsilon_{t-k}$$

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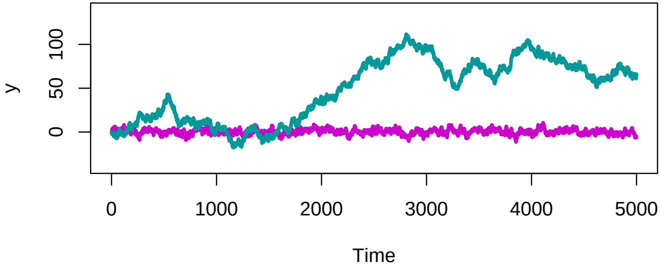
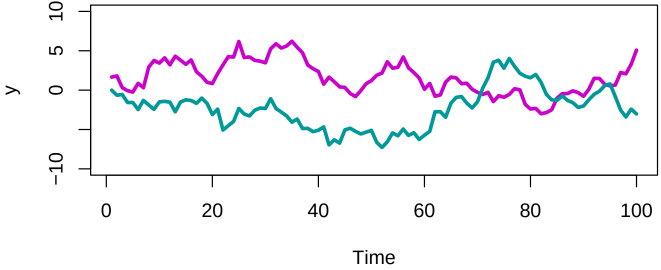
# AR(1) versus random walk

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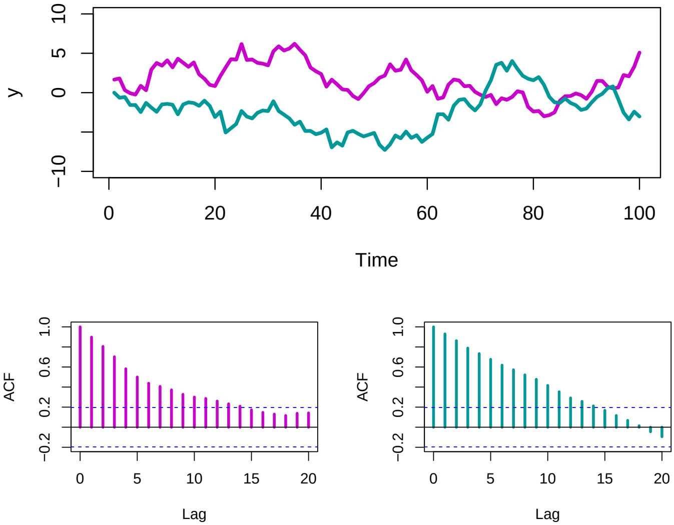
# AR(1) versus random walk

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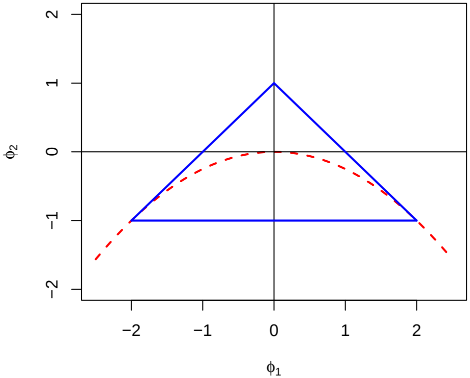
# Stationarity of an AR(2)

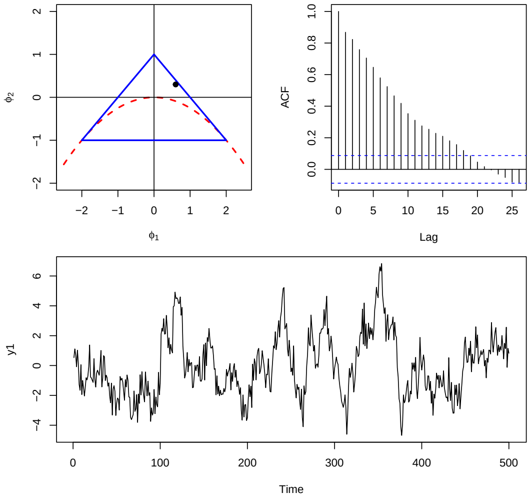
For an AR(2) process to be **stationary** we need:

- $\phi_2 - \phi_1 < 1$
- $\phi_2 + \phi_1 < 1$
- $|\phi_2| < 1$



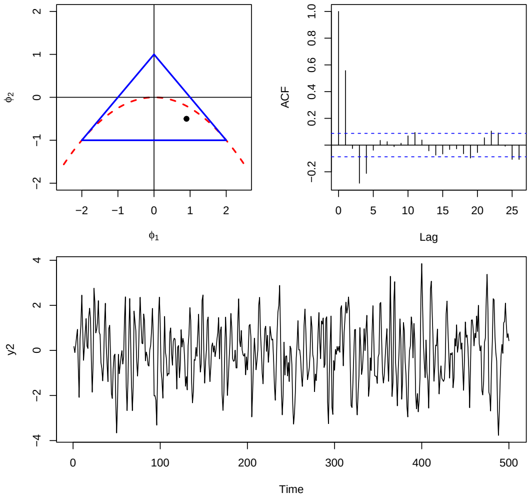
# AR(2) example: Slow decay in ACF

$$y_t = 0.6y_{t-1} + 0.3y_{t-2} + \epsilon_t$$



# AR(2) example: Oscillating

$$y_t = 0.9y_{t-1} - 0.5y_{t-2} + \epsilon_t$$

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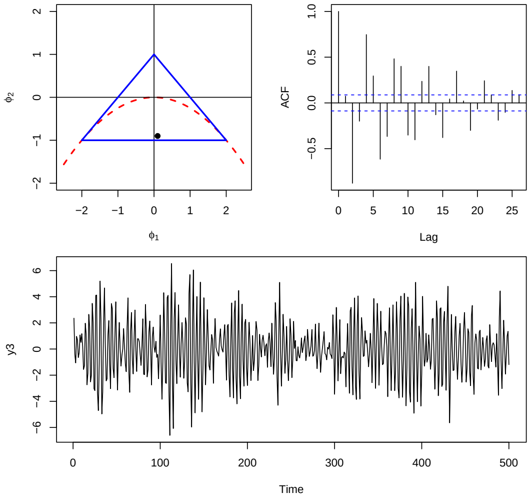
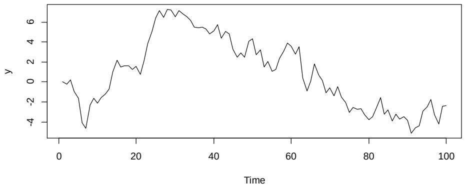
# AR(2) example: Oscillating

$$y_t = 0.1y_{t-1} - 0.9y_{t-2} + \epsilon_t$$



# Random walk as part of the ARIMA model

A **random walk**: $y_t = y_{t-1} + \epsilon_t$




Taking the **difference** gives **white noise**: $\Delta y_t = y_t - y_{t-1} = \epsilon_t$

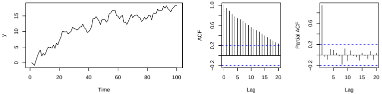
**ARIMA(p,d,q) process**: Autoregressive integrated moving average model with:

- $p$ number of AR components
- $d$ number of times the data need to be differenced to be stationary
- $q$ number of MA components

# Trends over time

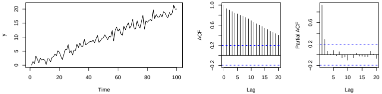
**Random walk with drift:**

$$y_t = y_{t-1} + \delta + \epsilon_t$$




**Deterministic time trend, e.g.:**

$$y_t = \alpha + \beta t + \epsilon_t$$



# Three useful unit root tests

**The (augmented) Dickey-Fuller test for a unit root:**

H0: a unit root

H1: stationarity

Use adf.test(data) in R

**Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for level:**

H0: mean-stationarity

H1: not mean-stationary

Use kpss.test(data) in R

**Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for trend:**

H0: trend-stationarity (but note: only a linear trend!)

H1: not trend-stationary

Use kpss.test(data, null = "Trend") in R

# Accounting for deterministic trends and seasonality

To account for **deterministic trends**, we can:

- detrend the data and model the residuals
- include the trend as part of the model

**Seasonality** may refer to:

- time-of-day effects in ESM
- day-of-week effects in daily diary
- monthly and annual patterns

**Seasonality** may be handled by:

- including dummies: flexible but "expensive"
- smoother trends (e.g., a sine wave)

# Conclusion

- Many popular time series models assume stationarity
- There are different reasons for nonstationarity
- There are tests, but these only test for one particular form of nonstationarity
- Depending on the kind of nonstationarity, different remedies can be used