Means versus intercepts

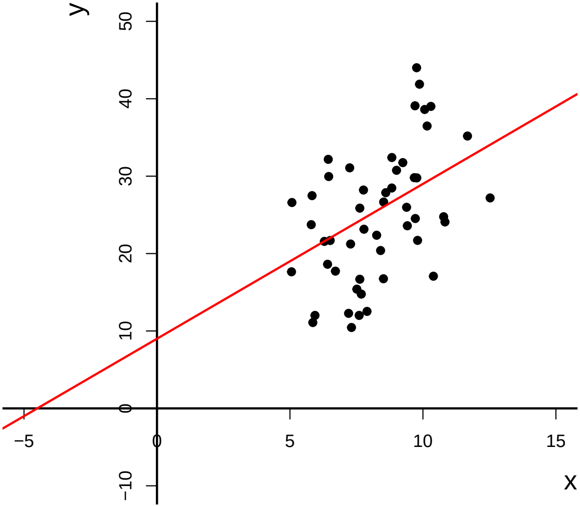
Modeling Intensive Longitudinal Data

Ellen Hamaker

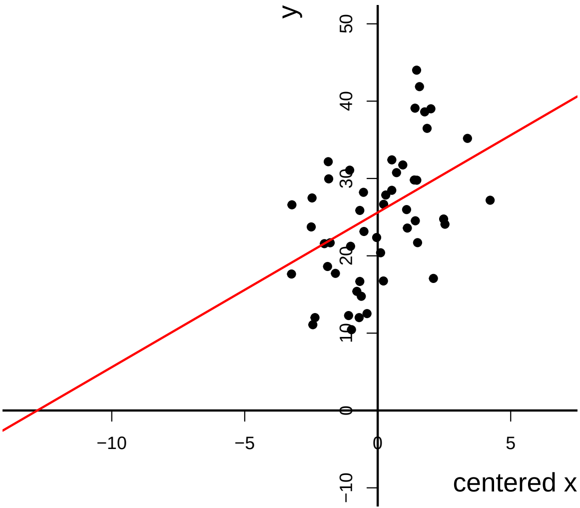
Utrecht University

Sharing science, *shaping tomorrow*

When the mean of X is (not) zero

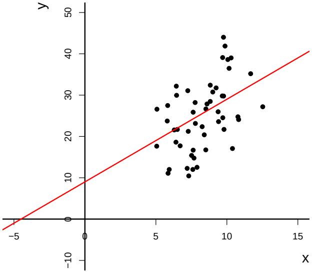


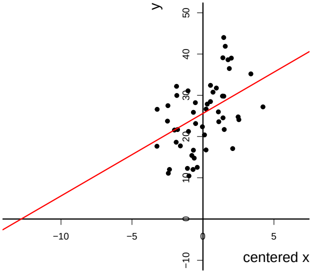
When the mean of X is (not) zero



Utrecht University

When the mean of X is (not) zero





$y = b_0 + b_1 x + e$
where b_0 is the intercept
and b_1 is the slope

$y = b_0^* + b_1^*(x - \bar{x}) + e$
where b_0^* is the mean of y
and b_1^* is the slope

$$b_1 = b_1^*$$
$$b_0 = b_0^* - b_1 \bar{x}$$

Utrecht University

Intercepts and means in an MA model

An MA(1) with an intercept can be expressed as:

$$y_t = c + \epsilon_t - \theta_1 \epsilon_{t-1}$$

The predictor is a previous version of ϵ_t; its mean is zero (by definition).

Hence, the intercept c is identical to the mean of y (i.e., $c = \mu$), and we can thus write:

$$y_t = \mu + \epsilon_t - \theta_1 \epsilon_{t-1}$$

In the more general case of an MA(q), we have

$$y_t = \mu + \epsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q}$$

Intercepts and means in an AR model

An AR(1) with an intercept can be expressed as:

$$y_t = c + \phi_1 y_{t-1} + \epsilon_t$$

The predictor is a previous version of the outcome; hence, the mean of the predictor is identical to the mean of the outcome.

But what is this mean?

$$\begin{aligned} \mu &= c + \phi_1 \mu \\ \mu - \phi_1 \mu &= c \\ (1 - \phi_1)\mu &= c \\ \mu &= c/(1 - \phi_1) \end{aligned}$$

The intercept c is NOT identical to the mean μ!

Intercepts and means in an AR(p) or ARMA(p,q) model

For an AR(p) with an intercept:

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \epsilon_t$$

the mean can be expressed as

$$\mu = c/(1 - \phi_1 - \cdots - \phi_p)$$

For an ARMA(p,q) with an intercept:

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q}$$

the mean can also be expressed as

$$\mu = c/(1 - \phi_1 - \cdots - \phi_p)$$

Utrecht University

Sharing science,
shaping tomorrow