SUBJECT NAME : **Discrete Mathematics**

SUBJECT CODE : **MA 3354**

MATERIAL NAME : **University Questions**

Unit – I (Logic and Proofs)

• Simplification by Truth Table and without Truth Table

1. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (N/D 2012)

2. Without using the truth table, prove that $\neg p \to (q \to r) \equiv q \to (p \vee r)$.

   (N/D 2010)

3. Prove $\left((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))\right) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ is a tautology.

   (N/D 2013),(A/M 2015),(A/M 2020)

4. Prove that $(P \to Q) \wedge (R \to Q) \Rightarrow (P \vee R) \to Q$. (M/J 2013)

5. Show that $(\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R)) \Leftrightarrow \mathrm{R}$, without using truth table.

   (A/M 2018)

6. Prove that $(p \to q) \wedge (q \to r) \Rightarrow (p \to r)$. (M/J 2014)

• PCNF and PDNF

1. Obtain the PDNF and PCNF of $(P \wedge Q) \vee (\neg P \wedge R)$. (N/D 2020)

2. Find the PCNF of $(P \vee R) \wedge (P \vee \neg Q)$. Also find its PDNF, without using truth table.

(A/M 2018)

3. Without using truth table find the PCNF and PDNF of
$P \rightarrow (Q \wedge P) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$. (A/M 2020)

4. Find the principal disjunctive normal form of the statement,
$(q \vee (p \wedge r)) \wedge \sim ((p \vee r) \wedge q)$. (N/D 2012)

5. Obtain the principal conjunctive normal form and principal disjunctive normal form of
$(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ by using equivalences. (M/J 2016),(A/M 2017)

6. Show that $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P) = (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge$
$(P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$. (M/J 2019)

- **Theory of Inference**

1. Prove that the following argument is valid: $p \rightarrow \neg q,\ r \rightarrow q,\ r \Rightarrow \neg p$.

(M/J 2012)

2. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$,
$P \rightarrow M$, $\neg M$. (N/D 2016)

3. Show that $J \wedge S$ logically follows from the premises $P \rightarrow Q$, $Q \rightarrow \neg R$, R, $P \vee (J \wedge S)$.
(N/D 2019)

4. Show that $(p \rightarrow q) \wedge (r \rightarrow s)$, $(q \rightarrow t) \wedge (s \rightarrow u)$, $\neg(t \wedge u)$ and $(p \rightarrow r) \Rightarrow \neg p$.

(A/M 2015)

5. Prove that the premises $P \rightarrow Q$, $Q \rightarrow R$, $R \rightarrow S$, $S \rightarrow \sim R$ and $P \wedge S$ are inconsistent.
(N/D 2014)

6. Prove that the premises $a \to (b \to c)$, $d \to (b \wedge \neg c)$ and $(a \wedge d)$ are inconsistent.

(N/D 2010)

7. Using indirect method, show that $R \to \neg Q$, $R \vee S$, $S \to \neg Q$, $P \to Q \Rightarrow \neg P$.

(A/M 2019)

8. Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q.

9. Show that using rule C.P $\neg P \vee Q, \neg Q \vee R, R \to S \Rightarrow P \to S$. (A/M 2018)

10. Show that the hypothesis, "It is not sunny this afternoon and it is colder than yesterday", "we will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip" and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". (N/D 2012),(N/D 2020)

11. Show that "It rained" is a conclusion obtained from the statements.

"If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded". (M/J 2016)

12. Prove that $\sqrt{2}$ is irrational by giving a proof using contradiction.

(N/D 2011),(M/J 2013),(N/D 2013),(M/J 2016)

- ## Quantifiers

1. Show that $(\forall x)(P(x) \to Q(x)),\ (\exists y)P(y) \Rightarrow\ (\exists x)Q(x)$. (M/J 2020)

2. Use the indirect method to prove that the conclusion $\exists z Q(z)$ follows form the premises $\forall x(P(x) \to Q(x))$ and $\exists y P(y)$. (N/D 2019)

3. Show that $(x)[P(x) \to Q(x)] \wedge (x)[Q(x) \to R(x)] \Rightarrow (x)[P(x) \to R(x)]$.

(N/D 2020)

4. Prove that $\forall x\big(P(x) \to Q(x)\big),\ \forall x\big(R(x) \to \neg Q(x)\big) \ \Rightarrow\ \forall x\big(R(x) \to \neg P(x)\big)$.

(N/D 2010)

5. Use indirect method of proof to prove that $(\forall x)\big(P(x) \vee Q(x)\big) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$.
(A/M 2011),(N/D 2011),(A/M 2015)

6. Show that $(\exists x)\big(P(x) \wedge Q(x)\big) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$. Is the converse true?

(N/D 2013)

7. Show that $(\exists x)P(x) \to \forall x Q(x) \Rightarrow (x)\big(P(x) \to Q(x)\big)$. (M/J 2014)

8. Show that the premises “One student in this class knows how to write programs in JAVA” and “Everyone who knows how to write programs in Java can get a high paying job imply a conclusion “Someone in this class can get a high paying job”.(N/D 2019)

9. Use rules of inferences to obtain the conclusion of the following arguments:

   “Babu is a student in this class, knows how to write programmes in JAVA”. “Everyone who knows how to write programmes in JAVA can get a high-paying job”. Therefore, “someone in this class can get a high-paying job”. (A/M 2017)

10. Write the symbolic form and negate the following statements: (A/M 2019)

   (i) Every one who is healthy can do all kinds of work.

   (ii) Some people are not admired by every one.

   (iii) Every one should help his neighbors, or his neighbors will not help him.

   (iv) Every one agrees with some one and some one agrees with every one.

Unit – II (Combinatorics)

- **Mathematical Induction and Strong Induction**

1. Prove by the principle of mathematical induction, for $'n'$ a positive integer,

   $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$. (M/J 2017),(A/M 2019)

2. Using mathematical induction show that $\sum_{r=1}^{n} 3^r = \frac{3^{n+1}-1}{2}$. (M/J 2016),(A/M 2017)

3. Using mathematical induction to show that $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}} > \sqrt{n},\ n \geq 2$.

   (N/D 2011),(N/D 2016)

4. Prove by mathematical induction that $6^{n+2}+7^{2n+1}$ is divisible by 43 for each positive integer n. (N/D 2013)

5. Prove, by mathematical induction, that for all $n \geq 1,\ n^3+2n$ is a multiple of 3.

   (N/D 2010),(N/D 2015)

6. Prove that the number of subsets of set having n elements is 2^n. (M/J 2019)

7. State the Strong Induction (the second principle of mathematical induction). Prove that a positive integer greater than 1 is either a prime number or it can be written as product of prime numbers. (M/J 2018)

- ## Pigeonhole Principle

1. Let m any odd positive integer. Then prove that there exists a positive integer n such that m divides $2^n - 1$. (M/J 2017)

2. Prove that in a group of six people, atleast three must be mutual friends or atleast three must be mutual strangers. (N/D 2020)

3. What is the maximum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade if there are five possible grades A, B, C, D and E? (N/D 2021)

• Permutations and Combinations

1. How many positive integers n can be formed using the digits 3, 4, 4, 5, 5, 6, 7 if n has to exceed 5000000? (N/D 2010)

2. Find the number of distinct permutations that can be formed from all the letters of each word (1) RADAR (2) UNUSUAL. (M/J 2012)

3. From a club consisting of six men and seven women, in how many ways we select a committee of (1) 3 men and four women? (2) 4 person which has at least one women? (3) 4 person that has at most one man? (4) 4 persons that has children of both sexes?

   (N/D 2019)

4. There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if (1) they can be male or female, (2) two must be men and two women, (3) they must all are of the same sex. (M/J 2016),(A/M 2017)

5. Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members form the mathematics department and four from the computer science department? (N/D 2021)

6. A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if (1) They can be any colour (2) Two must be white and two red (3) They must all be the same colour. (A/M 2021)

• Solving recurrence relations by generating function

1. Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ given that $a_0 = 5,\ a_1 = 9$ and $a_2 = 15$. (M/J 2014)

2. Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5,\ a_1 = -9$ and $a_2 = 15$.

(N/D 2014)

3. Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, with the initial conditions $a_0 = 2,\ a_1 = 5$ and $a_2 = 15$. (N/D 2019)

4. Find the generating function of Fibonacci sequence. (N/D 2013)

5. Solve using generating function: $S(n) + 3S(n-1) - 4S(n-2) = 0$; $n \geq 2$ given $S(0) = 3$, $S(1) = -2$. (A/M 2018)

6. Using generating function, solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 0$ where $n \geq 2,\ \ a_0 = 0$ and $a_1 = 1$. (M/J 2019)

7. Using generating function solve $y_{n+2} - 5y_{n+1} + 6y_n = 0,\ \ n \geq 0$ with $y_0 = 1$ and $y_1 = 1$.

   (A/M 2021)

8. Solve the recurrence relation $a_n - 7a_{n-1} + 6a_{n-2} = 0$, for $n \geq 2$ with initial conditions $a_0 = 8$ and $a_1 = 6$, using generating function. (A/M 2017)

9. Use generating functions to solve the recurrence relation $a_n + 3a_{n-1} - 4a_{n-2} = 0,\ \ n \geq 2$ with the initial condition $a_0 = 3,\ a_1 = -2$. (N/D 2012),(N/D 2015)

10. Solve the recurrence relation $a_n = 3a_{n-1} + 2,\ \ n \geq 1$, with $a_0 = 1$ by the method of generating functions. (M/J 2019)

11. Use the method of generating function to solve the recurrence relation $a_n = 3a_{n-1} + 1,\ \ n \geq 1$ given that $a_0 = 1$. (A/M 2015)

12. Use generating function to solve the recurrence relation $S(n+1) - 2S(n) = 4^n$ with $S(0) = 1$, $n \geq 0$. (M/J 2016)

13. Using method of generating function to solve the recurrence relation
$a_n = 4a_{n-1} - 4a_{n-2} + 4^n;\ \ n \geq 2$, given that $a_0 = 2$ and $a_1 = 8$. (N/D 2021)

14. Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n,\ \ n \geq 0,\ a_0 = 3$. (N/D 2011)

- **Inclusion and Exclusion**

1. Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2, 3, 5, 7. (N/D 2020),(M/J 2016)

2. Find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2, 3, 5 and 7. (A/M 2015),(N/D 2016),(A/M 2018)

3. Find the number of integers between 1 and 500 that are not divisible by any of the integers 2, 3, 5 and 7. (A/M 2017)

4. Determine the number of positive integers $n,\ \ 1 \leq n \leq 2000$ that are not divisible by 2, 3 or 5 but are divisible by 7. (M/J 2013)

5. Find the number of positive integers ≤ 1000 and not divisible by any of $3, 5, 7$ and 22.

   (M/J 2014)

6. There are 2500 students in a college, of these 1700 have taken a course in C, 1000 have taken a course Pascal and 550 have taken a course in Networking. Further 750 have taken courses in both C and Pascal. 400 have taken courses in both C and Networking, and 275 have taken courses in both Pascal and Networking. If 200 of these students have taken courses in C, Pascal and Networking.

   (1) How many of these 2500 students have taken a course in any of these three courses C, Pascal and Networking?

   (2) How many of these 2500 students have not taken a course in any of these three courses C, Pascal and Networking? (A/M 2011)

7. A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken atleast one of Spanish, French and Russian, how many students have taken a course in all three languages? (N/D 2013)

Unit – III (Graphs)

- ## Drawing graphs from given conditions

1. Draw the complete graph K_5 with vertices A, B, C, D, E. Draw all complete sub graph of K_5 with 4 vertices. (N/D 2010)

2. Draw the graph with 5 vertices, A, B, C, D, E such that $\deg(A) = 3$, B is an odd vertex, $\deg(C) = 2$ and D and E are adjacent. (N/D 2020)

3. Draw the graph with 5 vertices A, B, C, D and E such that $\deg(A) = 3$, B is an odd vertex, $\deg(C) = 2$ and D and E are adjacent. (A/M 2011)

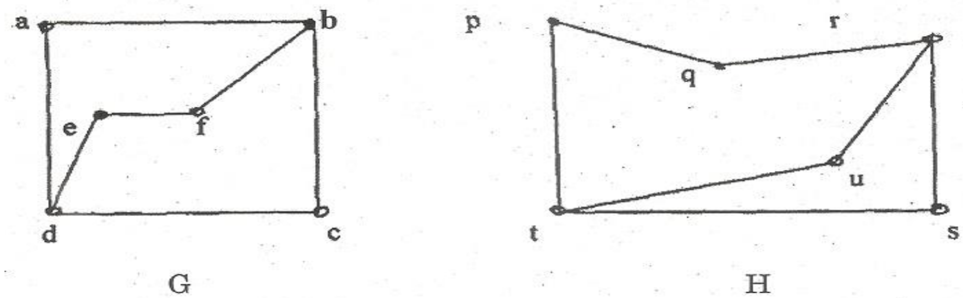
4. Given an example of a graph which is (N/D 2016)

   (i) Euleran but not Hamiltonian (ii) Hamiltonian but not Eulerian

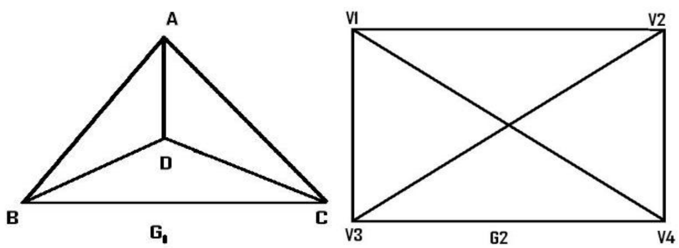
   (iii) Hamiltonian and Eulerian (iv) Neither Hamiltonian nor Eulerian

- ## Isomorphism of graphs

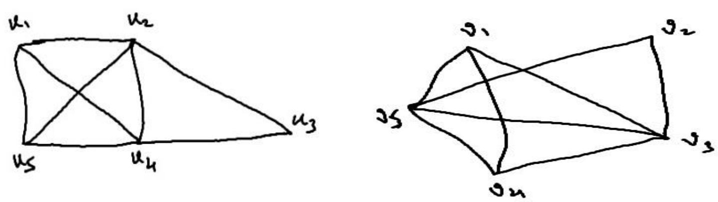
1. Determine whether the graphs G and H given below are isomorphic. (N/D 2012)



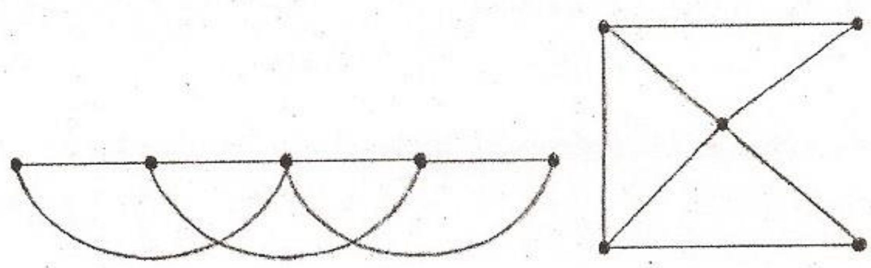
2. Using circuits, examine whether the following pairs of graphs G_1, G_2 given below are isomorphic or not: (N/D 2011),(N/D 2016)



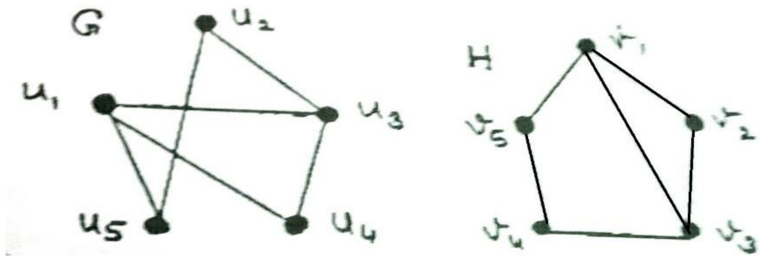
3. Examine whether the following pair of graphs are isomorphic. If not isomorphic, give the reasons. (A/M 2011)



4. Check whether the two graphs given are isomorphic or not. (M/J 2013)



5. Determine whether the following graphs G and H are isomorphic. (N/D 2013)



6. Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic? (M/J 2016),(A/M 2017)

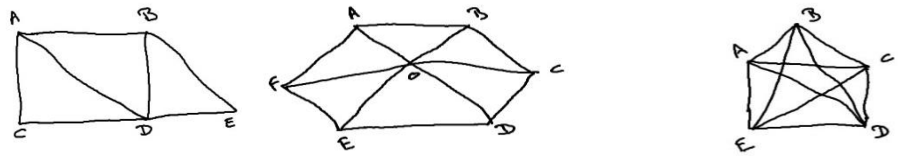
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

7. The adjacency matrices of two pairs of graph as given below. Examine the isomorphism of G and H by finding a permutation matrix. $A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

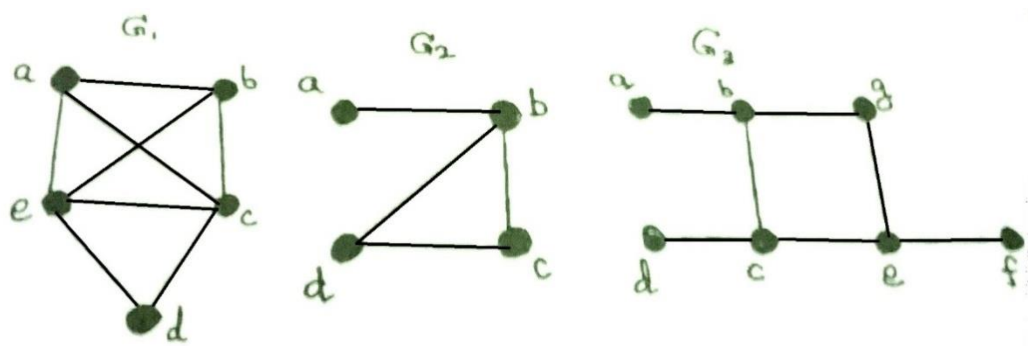
(N/D 2010)

- ## General problems in graphs

1. Find an Euler path or an Euler circuit, if it exists in each of the three graphs below. If it does not exist, explain why? (N/D 2011),(A/M 2015)



2. Which of the following simple graphs have a Hamilton circuit or, if not, a Hamilton path?



- ## Theorems

1. Prove that an undirected graph has an even number of vertices of odd degree.

   (N/D 2012),(M/J 2014)

2. Prove that the number of vertices of odd degree in any graph is even.

   (A/M 2015),(N/D 2015),(M/J 2016),(A/M 2017)

3. State and prove hand shaking theorem. Also prove that maximum number of edges in a connected graph with n vertices is $\frac{n(n-1)}{2}$. (N/D 2016),(A/M 2018)

4. Prove that the complement of a disconnected graph is connected. (A/M 2017)

5. Show that if a graph with n vertices is self-complementary then $n \equiv 0 \text{ or } 1 \pmod 4$.

   (M/J 2013),(M/J 2016)

6. Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. (N/D 2011),(A/M 2019),(N/D 2015)

7. Let G be a graph with exactly two vertices has odd degree. Then prove that there is a path between those two vertices. (N/D 2016)

8. Show that graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty subsets V_1 and V_2 such that there exists no edge is G whose one end vertex is in V_1 and the other in V_2. (M/J 2012),(A/M 2018)

• Theorems based on Euler and Hamilton graph

1. Prove that a connected graph G is Eulerian if and only if all the vertices are of even degree. (M/J 2012),(N/D 2013),(M/J 2014),(N/D 2015),(A/M 2019)

2. Show that the complete graph with n vertices K_n has a Hamiltonian circuit whenever $n \geq 3$. (N/D 2012)

3. Prove that if G is a simple graph with at least three vertices and $\delta(\mathrm{G}) \geq \frac{|V(G)|}{2}$ then G is Hamiltonian. (M/J 2013)

4. Let G be a simple undirected graph with n vertices. Let u and v be two non adjacent vertices in G such that $\deg(u)+\deg(v) \geq n$ in G. Show that G is Hamiltonian if and only if $G+uv$ is Hamiltonian. (A/M 2011)

5. If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle.

(M/J 2016),(A/M 2017)

Unit – IV (Algebraic Structures)

- ## Group, Subgroup and Normal Subgroup

1. Prove that $G = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$ forms an abelian group under matrix multiplication. (N/D 2015)

2. Show that M_2, the set of all 2×2 non-singular matrices over R is a group under usual matrix multiplication. Is it abelian? (A/M 2015)

3. If $(G,*)$ is an abelian group, show that $(a*b)^2 = a^2 * b^2$. (N/D 2010)

4. In any group $\langle G,* \rangle$, show that $(a*b)^{-1} = b^{-1} * a^{-1}$, for all $a,b \in G$. (M/J 2016)

5. Show that $(Q,*)$ is an abelian group, where $*$ is defined by $a*b = \dfrac{ab}{2}, \forall a,b \in Q^+$.

   (N/D 2016),(A/M 2018)

6. If $*$ is a binary operation on the set R of real numbers defined by $a*b = a+b+2ab$,

   (1) Find $\langle R,* \rangle$ is a semi group

   (2) Find the identity element if it exists

   (3) Which elements has inverse and what are they? (A/M 2011)

7. If $S = N \times N$, the set of ordered pairs of positive integers with the operation $*$ defined by $(a,b)*(c,d) = (ad+bc, bd)$ and if $f:(S,*) \to (Q,+)$ is defined by $f(a,b) = \dfrac{a}{b}$, show that f is a semigroup homomorphism. (A/M 2015)

8. Find the left cosets of the subgroup $H = \{[0],[3]\}$ of the group $[z_6, +_6]$.

(M/J 2014)

9. State and prove Lagrange's theorem. (N/D 2020),(A/M 2011),(M/J 2012),(N/D 2013),(M/J 2014),(A/M 2015),(M/J 2016),(N/D 2016),(A/M 2017)

10. Prove that the order of a subgroup of a finite group divides the order of the group.

(N/D 2011),(M/J 2013),(A/M 2018)

11. Find all the subgroups of $\left(z_9, +_9\right)$. (M/J 2014)

12. Prove the theorem: Let $\langle G, *\rangle$ be a finite cyclic group generated by an element $a \in G$. If G is of order n, that is, $|G| = n$, then $a^n = e$, so that $G = \left\{a, a^2, a^3, \ldots, a^n = e\right\}$. Further more n is a least positive integer for which $a^n = e$. (N/D 2011)

13. Prove that intersection of any two subgroups of a group $(G, *)$ is again a subgroup of $(G, *)$. (N/D 2013)

14. Prove that intersection of two normal subgroups of a group $(G, *)$ is a normal subgroup of a group $(G, *)$. (M/J 2013),(N/D 2016),(A/M 2018)

15. Show that the union of two subgroups of a group G is a subgroup of G if and only if one is contained in the other. (A/M 2015)

16. Prove that every cyclic group is an abelian group. (N/D 2013)

17. Prove that every subgroup of a cyclic group is cyclic. (M/J 2016),(A/M 2017)

18. Prove that the necessary and sufficient condition for a non empty subset H of a group $\{G, *\}$ to be a sub group is $a, b \in H \Rightarrow a * b^{-1} \in H$. (N/D 2012),(N/D 2019)

19. If $*$ is the operation defined on $S = Q \times Q$, the set of ordered pairs of rational numbers and given by $(a,b)*(x,y) = (ax, ay+b)$, show that $(S,*)$ is a semi group. Is it commutative? Also find the identity element of S. (N/D 2012)

20. Define the Dihedral group $\langle D_4, * \rangle$ and give its composition table. Hence find the identify element and inverse of each element. (A/M 2011)

• Homomorphism and Isomorphism

1. Prove that the group homomorphism preserves the identity element. (N/D 2021)

2. Prove that every finite group of order n is isomorphic to a permutation group of order n. (N/D 2011),(M/J 2013)

3. State and prove the fundamental theorem of group homomorphism. (N/D 2013)

4. Let $f : G \to G'$ be a homomorphism of groups with Kernel K. Then prove that K is a normal subgroup of G and G/K is isomorphic to the image of f. (M/J 2012)

5. Let $(G,*)$ and (H,Δ) be two groups and $g:(G,*) \to (H,\Delta)$ be group homomorphism. Prove that the Kernel of g is normal subgroup of $(G,*)$.

   (M/J 2013),(M/J 2016),(A/M 2017),(N/D 2019)

6. Show that the Kernel of a homomorphism of a group $\langle G,* \rangle$ into an another group $\langle H,\Delta \rangle$ is a subgroup of G. (A/M 2011),(A/M 2018)

7. If $f : G \to G'$ is a group homomorphism from $\{G,*\}$ to $\{G',\Delta\}$ then prove that for any $a \in G,\ f\left(a^{-1}\right) = \left[f(a)\right]^{-1}$. (N/D 2012)

• Ring and Fields

1. Show that $(Z,+,\times)$ is an integral domain where Z is the set of all integers.

(N/D 2010)

2. Prove that the set $Z_4=\{[0],[1],[2],[3]\}$ is a commutative ring with respect to the binary operation addition modulo and multiplication modulo $+_4$ and $\times_4$.

(N/D 2012),(N/D 2015)

Unit – V (Lattices and Boolean algebra)

• Partially Ordered Set (Poset)

1. Show that $(N,\le)$ is a partially ordered set where N is set of all positive integers and $\le$ is defined by $m\le n$ iff $n-m$ is a non-negative integer. (N/D 2010),(A/M 2018)

2. Draw the Hasse diagram for (1) $P_1=\{2,3,6,12,24\}$ (2) $P_2=\{1,2,3,4,6,12\}$ and $\le$ is a relation such $x\le y$ if and only is $x\mid y$. (A/M 2011)

3. Draw the Hasse diagram representing the partial ordering $\{(A,B):A\subseteq B\}$ on the power set $P(S)$ where $S=\{a,b,c\}$. Find the maximal, minimal, greatest and least elements of the poset. (N/D 2012)

4. Consider the Lattice D_{105} with partial ordered relation divides, then (N/D 2016)

   (i) Draw the Hasse diagram of D_{105} (ii) Find the complement of each elements of D_{105}
   (iii) Find the set of atoms of D_{105} (iv) Find the number of sub algebras of D_{105}

5. Let D_{30} with D if and only if x divides y. Find the following (A/M 2018)

   i) All lower bounds of 10 and 15

   ii) GLB of 10 and 15

iii) All upper bounds of 10 and 15

iv) LUB of 10 and 15

v) Draw the Hasse diagram for D_{30}

- **Lattices**

1. In a distributive lattice prove that $a*b=a*c$ and $a\oplus b=a\oplus c$ imply $b=c$.

(M/J 2014),(A/M 2021)

2. Let L be lattice, where $a*b=\mathrm{glb(a,b)}$ and $a\oplus b=\mathrm{lub(a,b)}$ for all $a,b\in L$. Then both binary operations $*$ and $\oplus$ defined as in L satisfies commutative law, associative law, absorption law and idempotent law. (M/J 2013)

3. Show that every ordered lattice $\{L,\vee,\wedge\}$ satisfies the following properties of the algebraic lattice (i) idempotent (ii) commutative (iii) associative (iv) absorption.

(A/M 2017)

4. In a distributive Lattice $\{L,\vee,\wedge\}$ if an element $a\in L$ a complement then it is unique.

(N/D 2012),(N/D 2016),(A/M 2018)

5. Show that every chain is a lattice. (M/J 2013)

6. Prove that every chain is modular. (M/J 2016)

7. Prove that every chain is a distributive lattice.

(N/D 2013),(A/M 2015),(M/J 2016),(N/D 2016),(A/M 2017)

8. Prove that every distributive lattice is modular. Is the converse true? Justify your claim.

(A/M 2011)

9. Show that in a distributive and complemented lattice $a\le b \Leftrightarrow a*b'=0 \Leftrightarrow a'\oplus b=1 \Leftrightarrow b'\le a'$. (M/J 2013)

10. In a distributive complemented lattice. Show that the following are equivalent.

(i) $a \le b$ (ii) $a \wedge \overline{b} = 0$ (iii) $\overline{a} \vee b = 1$ (iv) $\overline{b} \le \overline{a}$ (M/J 2016),(N/D 2016),(A/M 2017)

11. Show that in a lattice if $a \le b \le c$, then (N/D 2013)

(i) $a \oplus b = b * c$

(ii) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$

12. Show that the direct product of any two distributive lattices is a distributive lattice.

(A/M 2011) ,(M/J 2012)

13. If $P(S)$ is the power set of a set S and $\cup, \cap$ are taken as join and meet, prove that $\langle P(S), \subseteq \rangle$ is a lattice. Also, prove the modular inequality of a Lattice $\langle L, \le \rangle$ for any $a,b,c \in L$, $a \le c \Leftrightarrow a \vee (b \wedge c) \le (a \vee b) \wedge c$. (N/D 2011)

14. Prove that Demorgan's laws hold good for a complemented distributive lattice $\langle L, \wedge, \vee \rangle$, viz $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$. (N/D 2011),(M/J 2013)

15. State and prove De Morgan's laws in a complemented, distributive lattice.

(M/J 2019),(A/M 2015)

16. If S_{42} is the set all divisors of 42 and D is the relation "divisor of" on S_{42}, prove that $\{S_{42}, D\}$ is a complemented Lattice. (N/D 2010)

17. If S_n is the set of all divisors of the positive integer n and D is the relation of 'division', prove that $\{S_{30}, D\}$ is a lattice. Find also all the sub lattices of $\{S_{30}, D\}$ that contains 6 or more elements. (A/M 2015)

• Boolean Algebra

1. In a Boolean algebra, prove that $(a \wedge b)' = a' \vee b'$. (N/D 2020)

2. Prove that in a Boolean algebra $(a \vee b)' = a' \wedge b'$. (N/D 2015)

3. Show that the De Morgan's laws hold in a Boolean algebra. (N/D 2014),(M/J 2016)

4. In any Boolean algebra, show that $ab' + a'b = 0$ if and only if $a = b$. (N/D 2011)

5. In any Boolean algebra, prove that the following statements are equivalent:

   (1) $a + b = b$ (2) $a \bullet b = a$ (3) $a' + b = 1$ and (4) $a \bullet b' = 0$ (N/D 2011)

6. In a Boolean algebra, prove that $a.(a + b) = a$, for all $a, b \in B$. (N/D 2012)

7. Simplify the Boolean expression $a'.b'.c + a.b'.c + a'.b'.c'$ using Boolean algebra identities. (N/D 2020)

8. In any Boolean algebra, show that $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$.

   (N/D 2013),(M/J 2014)

9. Let B be a finite Boolean algebra and let A be the set of all atoms of B. Then prove that the Boolean algebra B is isomorphic to the Boolean algebra $P(A)$, where $P(A)$ is the power set of A. (M/J 2012)

10. If $P(S)$ is the power set of a non-empty S, prove that $\{P(S), \cup, \cap, \backslash, \phi, S\}$ is a Boolean algebra. (N/D 2019)

11. Prove that D_{110}, the set of all positive divisors of a positive integer 110, is a Boolean algebra and find all its sub algebras. (A/M 2021)

12. If $a,b \in S\{1,2,3,6\}$ and $a+b = LCM(a,b)$, $a \bullet b = GCD(a,b)$ and $a' = \dfrac{6}{a}$, show that $\{S,+,\bullet,',1,6\}$ is a Boolean algebra. (A/M 2015)