1) Consider two non-zero vectors $x \in \mathbb{C}^n$ and $y \in \mathbb{C}^n$. Suppose the inner product between x and y obeys commutative property (i.e., $x \cdot y = y \cdot x$). Then

- [ ] y must be a conjugate transpose of x
- [x] y is equal to x
- [x] y must be orthogonal to x
- [ ] y must be a scalar (possibly complex) multiple of x

2) The inner product of two distinct vectors x and y that are from $\mathbb{C}^{100}$ is $0.8 - 0.37i$.The vector x is scaled by a scalar $1 - 2i$ to obtain a new vector z, then the inner product between z and y is

- ○ $0.06 - 1.97i$
- ○ $1.54 - 1.23i$
- ● $1.54 + 1.23i$
- ○ $0.8 - 0.37i$
- ○ Not possible to calculate

3) Select the correct statement(s).The Eigenvalue decomposition for the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

- [x] doesn't exist over $\mathbb{R}$ but exists over $\mathbb{C}$
- [ ] doesn't exist over $\mathbb{C}$ but exists over $\mathbb{R}$
- [ ] neither exists over $\mathbb{R}$ nor exists over $\mathbb{C}$
- [ ] exists over both $\mathbb{C}$ and $\mathbb{R}$

4) The matrix $S = \begin{bmatrix} 1 & 1+i & -2-2i \\ 1-i & 1 & -i \\ -2+2i & i & 1 \end{bmatrix}$ is

- ○ Hermitian and Symmetric
- ○ Symmetric but not Hermitian
- ○ Neithet Hermitian nor Symmetric
- ● Hermitian but not Symmetric

5) Suppose that an unitary matrix U is multiplied by a diagonal matrix D with diagonal elements in D denoted as $d_{ii} \in \mathbb{R}$, then the resultant matrix will always be unitary. The statement is

- ● True if $DD^T = I$
- ○ False
- ○ True if $DD^{-1} = I$

6) The eigenvectors of matrix $A = \begin{bmatrix} 3 & 2-i & -3i \\ 2+i & 0 & 1-i \\ 3i & 1+i & 0 \end{bmatrix}$ are

- ● $\begin{bmatrix} -1 \\ 1+2i \\ 1 \end{bmatrix}, \begin{bmatrix} 1-21i \\ 6-9i \\ 13 \end{bmatrix}, \begin{bmatrix} 1+3i \\ -2-i \\ 5 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 \\ 1-2i \\ 1 \end{bmatrix}, \begin{bmatrix} 1-21i \\ 6-9i \\ 13 \end{bmatrix}, \begin{bmatrix} 1+3i \\ -2-i \\ 5 \end{bmatrix}$
- ○ $\begin{bmatrix} -1 \\ 1-2i \\ -1 \end{bmatrix}, \begin{bmatrix} 1-21i \\ 6-9i \\ 13 \end{bmatrix}, \begin{bmatrix} 1+3i \\ -2-i \\ 5 \end{bmatrix}$
- ○ $\begin{bmatrix} -1 \\ 1+2i \\ 1 \end{bmatrix}, \begin{bmatrix} 1-21i \\ 6-9i \\ 13 \end{bmatrix}, \begin{bmatrix} 1-3i \\ 2-i \\ -5 \end{bmatrix}$

7) The matrix $A = \frac{1}{2}\begin{bmatrix} k+i & \sqrt{2} \\ k-i & \sqrt{2}i \end{bmatrix}$ is unitary if k is

- ○ $\frac{1}{2}$
- ● 1
- ○ $-\frac{1}{2}$
- ○ -1
- ○ ± 1
- ○ $\pm\frac{1}{2}$

8) Let $A = \begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$. If A can be factorized as $A = UDU^*$, with U denoting a unitary matrix and D denoting a diagonal matrix. Then, U and D are

- ● $U = \frac{1}{2}\begin{bmatrix} 1+i & -1-i \\ \sqrt{2} & \sqrt{2} \end{bmatrix}, D = \begin{bmatrix} 1+\sqrt{2} & 0 \\ 0 & 1-\sqrt{2} \end{bmatrix}$
- ○ $U = \frac{1}{4}\begin{bmatrix} 1+i & -1-i \\ \sqrt{2} & \sqrt{2} \end{bmatrix}, D = \begin{bmatrix} 1+\sqrt{2} & 0 \\ 0 & 1-\sqrt{2} \end{bmatrix}$
- ○ $U = \frac{1}{2}\begin{bmatrix} -1+i & \sqrt{2} \\ \sqrt{2} & -1-i \end{bmatrix}, D = \begin{bmatrix} 1+\sqrt{2} & 0 \\ 0 & 1-\sqrt{2} \end{bmatrix}$
- ○ $U = \frac{1}{4}\begin{bmatrix} 1+i & \sqrt{2} \\ \sqrt{2} & 1-i \end{bmatrix}, D = \begin{bmatrix} -1+\sqrt{2} & 0 \\ 0 & 1-\sqrt{2} \end{bmatrix}$

9) The matrix $Z = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ has

- ○ only real eigenvalues.
- ● two real and two complex eigenvalue.
- ○ three real and one complex eigenvalues.
- ○ all complex eigenvalues

10) Which of the following matrices is/are unitary?

- [ ] $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$
- [ ] $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
- [x] $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
- [x] $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
- [ ] $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

11) Let U and V be two unitary matrices. Then,

UV is always unitary.
$U+V$ is always unitary.

- ○ Both statements are true.
- ○ Both statements are false.
- ○ 1. is false.
- ● 2. is false.

12) Which of the following is/are eigenvectors of the matrix $A = \begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix}$

- [x] $\begin{bmatrix} -1-i \\ 1 \end{bmatrix}$
- [x] $\begin{bmatrix} -2-2i \\ 2 \end{bmatrix}$
- [x] $\begin{bmatrix} \frac{1+i}{2} \\ 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 1+i \\ 2 \end{bmatrix}$