1) The function $f(x,y) = x^2 + y^2$

- ○ has no stationary point.
- ● has a stationary point at (0, 0).
- ○ has a stationary point at (1, 1).
- ○ has a stationary point at (-1, -1).

$f_x'(x,y) = 2x \quad f_y'(x,y) = 2y$

both functions are 0 only at (0,0)

2) If $A = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ then $A + B$ is a positive definite matrix. Is this statement true?

- ● Yes, it is true
- ○ No, it is not true

$A+B = \begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

Since $a>0 ; 5>0$

$ac > b^2 ; 20 > 9$

hence the matrix is Positive definitive

3) The matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ is

- ● positive definite
- ○ positive semi-definite
- ○ negative definite
- ○ negative semi-definite

4) find eigen values

$\left| \begin{bmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{bmatrix} \right| = 2-\lambda((2-\lambda)^2 - (-1)^2$
$+ (-1)(-1(2-\lambda) - (-1))$
$+ 1(1-(2-\lambda))$

$2-\lambda(4+\lambda^2-4\lambda-1) - (-1)(\lambda-1) + 1(\lambda-1)$

$(2-\lambda)(\lambda^2-4\lambda+3) + 2(\lambda-1)$

$2\lambda^2 - 8\lambda + 6 - \lambda^3 + 4\lambda^2 - 3\lambda + 2\lambda - 2$

$-\lambda^3 + \lambda^2 + 5\lambda^2 - 5\lambda - 4\lambda + 4$

$\underset{a=-1}{-\lambda^3} + \underset{b=6}{6\lambda^2} - \underset{c=-9}{9\lambda} + \underset{d=4}{4} = 0$ — (1)

Since $a+b+c+d=0$
$\lambda = 1$ is a root

Solving the equation

$(\lambda-1)(-\lambda^2+5\lambda-4) = -(\lambda-1)(\lambda^2-5\lambda+4) = -(\lambda-1)(\lambda(\lambda-1)-4(\lambda-1))$

$= 1, 1, 4$ — Since all eigenvalues are +ve its a positive definitive

$f_x = 4x + 2y - 6$; $f_y = 2x + 4y$

f_x & f_y are 0 at $(2, -1)$

$f_x = 4(2) + 2(-1) - 6 = 0$

$f_y = 2(2) + 4(-1) = 0$

4) The function $f(x, y) = 2x^2 + 2xy + 2y^2 - 6x$ has a stationary point at

- ○ (2, 1)
- ○ (1, 2)
- ○ (-1, 2)
- ● (2, -1)

5) The correct representation of $x^2 + y^2 - z^2 - xy + yz + xz$ in the matrix form is

- ● $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
- ○ $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
- ○ $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
- ○ $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

6) Given $f(x, y) = 3x^2 + 4xy + 2y^2$, the point $(0, 0)$ is a ________

- ○ maxima
- ● minima
- ○ saddle Point
- ○ None of these

$f_x = 6x + 4y$; $f_y = 4x + 4y$; $f_{xx} = 6$; $f_{xy} = 4$; $f_{yy} = 4$

Discriminant $D = f_{xx} f_{yy} - (f_{xy})^2$

$= (4 \times 6) - (4)^2 = 24 - 16 = \underline{\underline{8}}$

Since $D > 0$ & $f_{xx} > 0$ $(0, 0)$ is a minima

7) Which of the following statements are true about the matrix $A = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$

- ○ A is positive definite.
- ○ A is positive semidefinite.
- ● A is neither positive definite nor positive semi-definite.
- ○ Can not be determined.

8) The matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is positive definite

$\det \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = -3$ — ve means not positive definitive

- ○ True
- ● False

9) Which of the following statement is true about the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$?

for diagonal matrices $\lambda_1, \lambda_2, \lambda_3$ = eigen values since all are positive

- ● A is positive definite.
- ○ A is positive semi-definite.
- ○ A is neither positive definite nor positive semi-definite.
- ○ Can not be determined

10) The non-zero singular values of the matrix $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ are

$A^TA = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

$A^TA = \begin{bmatrix} 2 & 2 & 0 & 1 \\ 2 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}$

eigen values $= 3+\sqrt{3},\ 3-\sqrt{3}$

then $\sigma = \sqrt{3+\sqrt{3}},\ \sqrt{3-\sqrt{3}}$

- ○ $3+\sqrt{3}, 3-\sqrt{3}$
- ● $\sqrt{3+\sqrt{3}}, \sqrt{3-\sqrt{3}}$
- ○ $2+\sqrt{2}, 2-\sqrt{2}$
- ○ $\sqrt{2+\sqrt{2}}, \sqrt{2-\sqrt{2}}$

11) The SVD of the matrix $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ is

- ○ $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 & 0 \\ 0 & \sqrt{3} & 0 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 \end{bmatrix}$
- ○ $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \end{bmatrix} \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$
- ● $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$
- ○ $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 & 0 \\ 0 & \sqrt{3} & 0 & 0 \end{bmatrix} \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$A^TA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} =$

eigen value 2 & 2