Graded Assignment 4

Due date for this assignment: 2024-10-20, 23:59 IST.

You may submit any number of times before the due date. The final submission will be considered for grading.

Note: This assignment will be evaluated after the deadline passes. You will get your score 48 hrs after the deadline. Until then the score will be shown as Zero.

1) If P is a projection matrix, then the eigenvalue corresponding to every nonzero vector orthogonal to the column space of P is

- (●) 0
- ( ) 1
- ( ) -1

2) Consider the following statements regarding a real symmetric matrix A

The eigenvalues of A are always real.
The eigenvalues of A may be imaginary.
The eigenvectors corresponding to different eigenvalues of A are linearly independent.
The eigenvectors corresponding to different eigenvalues of A are not linearly independent.
A is orthogonally diagonalizable.
A is not diagonalizable.

Which of the above statements are true?

- ( ) 2, 3 and 5
- ( ) 1, 4 and 5
- ( ) 2, 4 and 6
- (●) 1, 3 and 5

3) The eigenvectors corresponding to distinct eigenvalues of a matrix

- [x] are linearly independent
- [ ] are linearly dependent
- [ ] have no relation

4) The determinant of a 3×3 matrix having eigenvalues 1, -2 and 3 is

- ○ 2
- ○ 0
- ○ 6
- ● -6
- ○ -2

5) The trace of a 2×2 matrix is -1 and its determinant is -6. Its eigenvalues will be

$x_1 + x_2 = -1 \longrightarrow -\frac{6}{x_2} + x_2 = -1$

$x_1 \cdot x_2 = -6 \qquad -6 + x_2^2 = -x_2$

$x_1 = -6/x_2 \qquad x^2 + x - 6 = 0$

$x - 2x + 3x - 6 = 0$

$x(x-2) + 3(x-2) = 0$

$(x-2)(x+3) = 0$

$\underline{2, -3}$

- ○ -1, 3
- ○ 2, 3
- ● 2, -3
- ○ -2, 3

6) If the eigenvalues of a matrix are -1, 0 and 4 are

Find the Trace?

3

7) Find the Determinant?

0

8) The characteristic polynomial for the matrix $A=\begin{bmatrix}1 & 1\\ 1 & 3\end{bmatrix}$ is

$|A-\lambda I| \longrightarrow \begin{vmatrix}1-\lambda & 1\\ 1 & 3-\lambda\end{vmatrix} \longrightarrow (3-\lambda-3\lambda+\lambda^2)-1$

$= \lambda^2-4\lambda+3-1$

$= \lambda^2-4\lambda+2$

- ○ $\lambda^2-4\lambda+1$
- ○ $\lambda^2-4\lambda$
- ○ $\lambda^2-4\lambda-2$
- ○ $\lambda^2+4\lambda+2$
- ● $\lambda^2-4\lambda+2$

9) The eigenvalues of matrix $A=\begin{bmatrix}1 & 1\\ 2 & 3\end{bmatrix}$ are

$|A-\lambda I| = \left|\begin{bmatrix}1-\lambda & 1\\ 2 & 3-\lambda\end{bmatrix}\right| = 3-\lambda-3\lambda+\lambda^2-2$

$= \lambda^2-4\lambda+1$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{4\pm\sqrt{(-4)^2-4(1)(1)}}{2a}$

$= \frac{4\pm\sqrt{12}}{2} = 2\pm\sqrt{3}$

- ● $2+\sqrt{3}, 2-\sqrt{3}$
- ○ $\sqrt{3}, -\sqrt{3}$
- ○ 0,1
- ○ $\sqrt{5}, -\sqrt{5}$

10) If the eigenvalues of a matrix A are 0 -1 and 5, then the eigenvalues of A^3 are

$S^{-1}A^kS = \lambda^k$

- ○ 0, -1 and 5
- ● 0, -1 and 125
- ○ 0, 1 and -125
- ○ 0, 1 and -5

11) The 110th term of Fibonacci sequence is approximately given by

$$F_K \simeq \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^K$$

- [x] $\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^{110}$
- [ ] $\frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^{110}$
- [ ] $\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^{-110}$
- [ ] $\frac{-1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^{110}$

12) Let A be an $n \times n$ matrix. Which of the following statements is/are false?

- [ ] If A has r non-zero eigenvalues, then rank of A is at least r.
- [x] If one of the eigenvalues of A are zero, then $|A| \neq 0$.
- [ ] If x is an eigenvector of A, then so are all the multiples of x.
- [ ] If 0 is an eigenvalue of A, then A cannot be inverted.

13) The eigenvalues of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ are

$$\begin{matrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{matrix}$$

$$\cancel{4}-\lambda-4\lambda+\lambda^2-\cancel{4}=0$$

$$\lambda^2-5\lambda=0$$

$$\lambda(\lambda-5)=0$$

$$\lambda_1=0 \quad \lambda_2=5$$

- [x] 0
- [ ] 2
- [ ] 3
- [x] 5

14) For the matrix given in the previous question, which of the following vectors is/are its eigenvector(s)?

- [x] $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- [x] $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\lambda_1 = 0$

$(A - \lambda I)x = 0$

$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} x = 0$

$x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$\lambda_2 = 5$

$(A - 5I)x = 0$

$\begin{bmatrix} 1-5 & 2 \\ 2 & 4-5 \end{bmatrix} x = 0$

$\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} x = 0$ $\quad x \doteq \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

15) Suppose that A, P are 3 × 3 matrices, and P is an invertible matrix.

If $P^{-1}AP = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 3 & 8 \\ 0 & 0 & 4 \end{bmatrix}$, then the eigenvalues of the matrix A^2 are

- [x] 1
- [ ] 12
- [x] 9
- [x] 16

$S^{-1}A^kS = \lambda^k$

Ans : 1, 9, 16

16) The best second degree polynomial that fits the data set

x	y
0	0
1.3	1.5
4	1.2

is

- ○ $1.35x^2 + 0.3x$
- ○ $1.25x^2 + 0.45x$
- ● $-0.316x^2 + 1.56x$
- ○ $-0.25x^2 + 0.5$

$y = ax^2 + bx + c$

1) For 0, 0 : $a(0)^2 + b(0) + c = 0$

$c = 0$

2) For 1.3, 1.5: $a(1.3)^2 + b(1.3) + c = 1.5$

$1.69a + 1.3b + c = 1.5$ — ①

3) For 4, 1.2 $\quad a(4)^2 + b(4) + c = 1.2$

$16a + 4b + c = 1.2$ — ②

Solving ① & ②

$a = -0.316$

$b = 1.56$