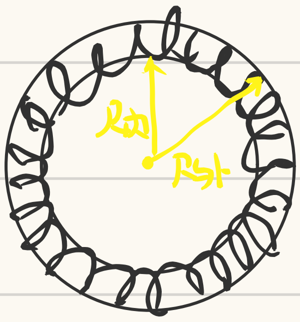
一 电磁学常见物理模型情景

螺线管

常涉及物理量 $\boxed{B\ \ H\ \ M}$ $\boxed{\Phi\ \ \Psi\ \ L\ \ M}$ $\boxed{\varepsilon_L\ \ \varepsilon_m}$

① 密绕螺线环（与长直螺线管情景不同，磁场不均匀）

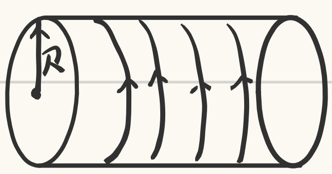


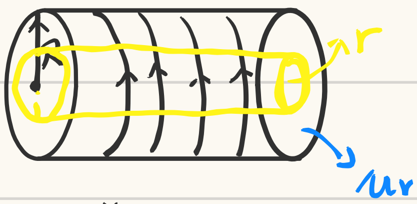
螺线管中 $\oint B\cdot dl = \mu_0 NI \Rightarrow B=\frac{\mu_0 NI}{2\pi r} \Rightarrow \begin{cases} H \\ M \end{cases}$

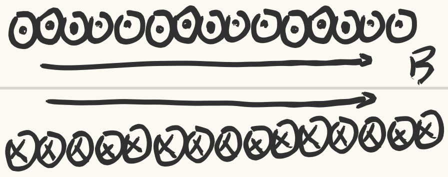
$\Phi=\int_{R_内}^{R_外} B(r)\cdot h\,dr \Rightarrow L=\frac{N\Phi}{I} \Rightarrow \varepsilon_L=-L\frac{dI}{dt}$

（螺线环场景下电磁感应通常考 $\varepsilon_L, \varepsilon$

② 长直螺线管（通常默认为 $L \gg R$）







螺线管内结论 $\begin{cases} B=\mu_0 nI \\ H=nI \end{cases}$ $n=\frac{N}{L}$（单位长度匝数）

在半无限长直螺线管中，在端点处磁场大小刚好是中心处一半

$\Phi=BS$ $L=\frac{N\Phi}{I}=\frac{nLBS}{I}=\mu_0 n^2 V$

对于电磁感应问题 $\frac{dB}{dt}$ $\begin{cases} E=\frac{r}{2}\left|\frac{dB}{dt}\right|, & r<R \\ E=\frac{R^2}{2r}\left|\frac{dB}{dt}\right|, & r>R \end{cases}$

$\oint E\,dl=\oiint\frac{dB}{dt}dS$

对于有内外表面的长直螺线管（通常就是内外 μ 不同）

表面磁化电流密度 $j_s=M=(\mu_r-1)\frac{NI}{L}$ $I_{R内}=j_s\cdot L$

与电荷有关问题

常涉及物理量 $\boxed{\substack{(线)\ (面)\\ \lambda\quad \sigma}}$ $\boxed{\omega\ \ T\ \ f}$ $\boxed{m\ \ M}$

① 带电圆盘

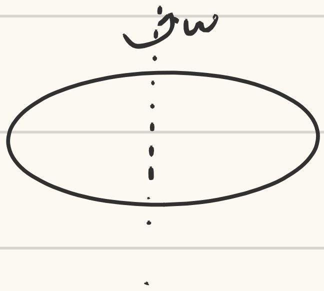
结论

$dq = \sigma ds = \sigma dr d\theta$ 每一个电荷元在中心 $dB = \frac{\mu_0}{4\pi}\frac{dq\,\vec{v}\times\vec{r}}{r^3}$ $B = \int_0^{2\pi}\int_0^{R} dB$

$I = \frac{Q}{T}$ $T=\frac{2\pi}{\omega}$ $dI = \frac{\sigma\cdot 2\pi r dr}{T} \Rightarrow dm = \pi r^2 dI \Rightarrow dM = |dm\times\vec{B}|$

($m = NIS\vec{e}_n$, 方向垂直线圈平面)

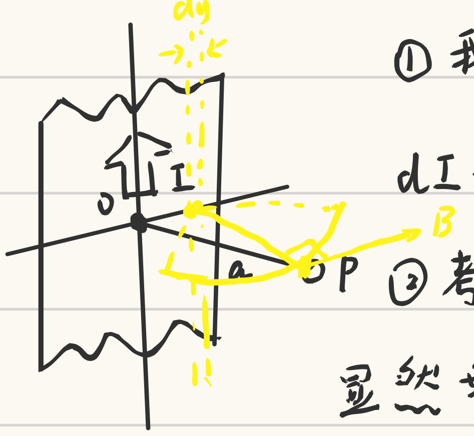
② 带电圆环



$I = \frac{\lambda\cdot 2\pi r}{T}$ $B = \frac{\mu_0 I}{2r}$ $T = \frac{2\pi}{\omega}$ $\Rightarrow I = \lambda\omega r$

$m = IS = \lambda\omega\pi r^3$ $M = m\times\vec{B} = \lambda\omega\pi r^3 B$

薄板积分法求磁场 => 主要是要考虑清楚 dI 的分布



① 我们要考虑清楚 dI如何表达

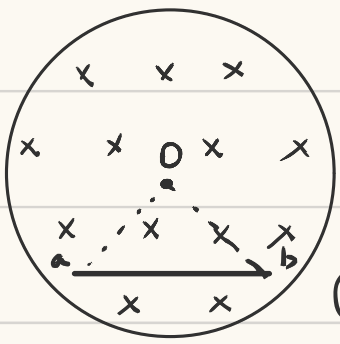
$dI = \frac{I}{b}\cdot dy$ $dB = \frac{\mu_0}{2\pi r}dI$ $r = \sqrt{a^2+y^2}$

② 考虑清楚方向以及如何积分

显然垂直板的B被抵消 $\Rightarrow dB' = dB\cos\theta \Rightarrow B = \int_{-\frac{b}{2}}^{\frac{b}{2}}\cdot dB$

电磁感应相关

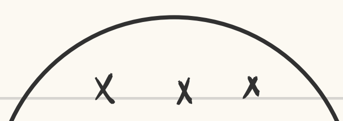
① 感生电动势相关 求 ε_{ab}



当 l_{ab} 在内部时

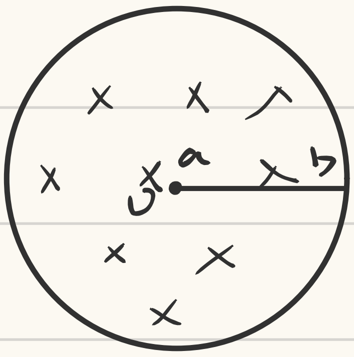
① $\oint E_i dl = \iint \frac{dB}{dt} ds = \frac{r}{2}\frac{dB}{dt}$ $E_i = \frac{r}{2}\frac{dB}{dt}$ $d\varepsilon = E_i\cdot dx$ $\varepsilon_{ab} = \int d\varepsilon$

② $\varepsilon_{ab} = \frac{d\Phi}{dt} = S_{\triangle OAB}\frac{dB}{dt}$



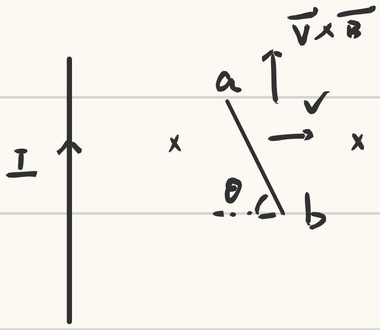
① $\mathcal{E}_{ab}=\frac{d\Phi}{dt}=S_{oa'b'}\frac{dB}{dt}=\frac{R^2\theta}{2}\frac{dB}{dt}$

② $E_i=\frac{R^2}{2r}\frac{dB}{dt}$ $\quad d\mathcal{E}=E_i\cdot dx$ $\quad \mathcal{E}_{ab}=\int d\mathcal{E}$



$\mathcal{E}_{ab}=0$

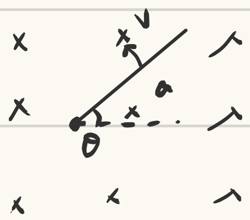
② 动生电动势 相关 (一般用于单导线切割的求法) $d\mathcal{E}_i$



$d\mathcal{E}_i=(\vec v\times\vec B)\cdot d\vec l$ $\quad d\mathcal{E}_i=vB\,dl\cdot\cos(\frac{\pi}{2}+\theta)=-vB\sin\theta\,dl$

$\mathcal{E}_{ab}=-\int_a^b vB\sin\theta\,dl=-\int_a^b vB\tan\theta\,dr$

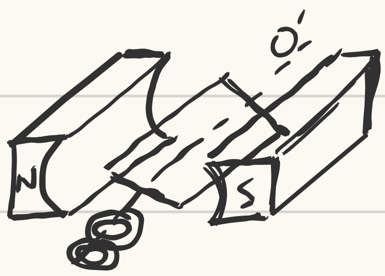
$B=\frac{\mu_0 I}{2\pi r}\Rightarrow\mathcal{E}_{ab}=-\frac{\mu_0 I v}{2\pi}\tan\theta\ln(1+\frac{L}{d}\cos\theta)$



$d\mathcal{E}=(\vec v\times\vec B)\cdot dl=-\omega lB\,dl$

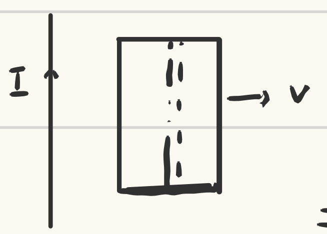
$\mathcal{E}=\int d\mathcal{E}=\int_0^L-\omega lB\,dl=-\frac{1}{2}\omega BL^2$ $\quad a\to O$ $\quad$ O点电势更高

③ 常用定义法求解



$\theta=\omega t$ $\quad \Phi=BS=BS\cos\omega t$

$\mathcal{E}_i=-N\frac{d\Phi}{dt}=NBS\omega\sin\omega t=\mathcal{E}_m\sin\omega t$



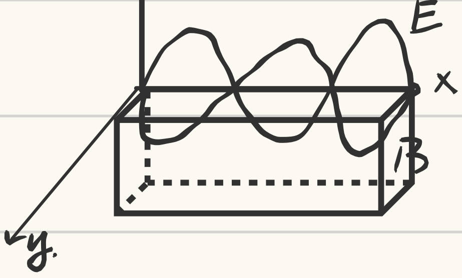
$B=\frac{\mu_0 I}{2\pi x}\Rightarrow d\Phi=B\,dS=\frac{\mu_0 I}{2\pi x}l\,dx$

$\Rightarrow\Phi=\int d\Phi\Rightarrow\mathcal{E}=-N\frac{d\Phi}{dt}$

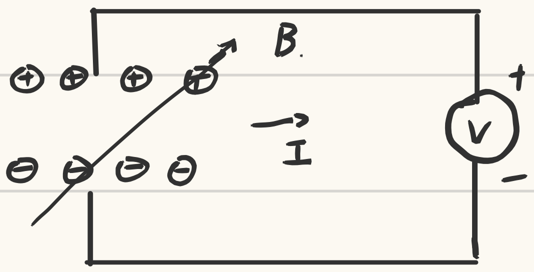
④ E与B

传播 (垂直) $\quad$ 大小 $\frac{E}{B}=v$(介质中速度)

$=\frac{1}{\sqrt{\mu\varepsilon}}$



霍尔元件



① $F_电 = F_洛$ $qE_H = qvB$ $E_H = vB$

② $I = nqSv$ $U_H = E_H h = \frac{IB}{nqd} = R_H \frac{IB}{d}$

③ 对于 R_H $\begin{cases} \text{正负 载流子电性} \\ \text{大小 载流子浓度} \end{cases}$ $R_H = \frac{1}{ne}$

$E_H = R_H \cdot J \cdot B$, $J = nqv$ / $\frac{E}{\rho}$

(欧姆定律微分式)