Autocracy v Democracy

Problem Set - Game Theory

GV482*

In this problem set, we will consider how autocratic and democratic institutions may differ. We will recover the fundamental trade-off between efficiency and (re)distributional arrangements. Let me briefly reiterate this trade-off. It is very much possible, though you will investigate the issue further in next week's empirical problem set, that autocracies are more efficient than democracies. Autocracies are able to fulfil their objectives, their plans better than democracies. But who is to benefit from this? Is all this efficiency trickling down to the citizenry or is it all to this advantage of the elite? This problem set will serve to develop the logic of how this efficiency may very well be detrimental to the public. In the last lecture, we will see evidence that relates to this model. Both the model and the empirical analysis are taken from Martinez-Bravo, Padró i Miquel, Qian, and Yao's paper: "The Rise and Fall of Local Elections in China: Theory and Empirical Evidence on the Autocrat's Trade-off" (available here). As always, you can refer to the paper if you are stuck, but be aware that the models in the article and in this problem set may differ.

Before describing the set-up, note that the level of difficulty is close to what you should expect for your final examination. The length, on the other hand, is longer than what awaits you.

We consider the administration of a local village. There is a unit mass of villagers v (the idea of a unit mass, you will recall, means that there are very many villagers, it is to simplify the analysis, and plays little role in what follows). Among these villagers, one of them will be selected to serve as village official, denoted o. The method of selection will depend on the institutions in place. We will consider three forms of institution in turn: democracy, appointment, democracy with oversight. All described in details in what follows. For now, we give the elements common to all three institutions.

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The village official o makes two decisions. She decides between two policies $x \in \{0, 1\}$ and how much effort to exert to make a success of the policy she has picked: $e \geq 0$. For the policy to be successful, effort is not enough, however. Success requires the official to be competent. That is, the probability of a success, denoted $S = 1$, is:

$$Pr(S = 1) = \begin{cases} e & \text{if } o \text{ is competent} \\ 0 & \text{if } o \text{ is incompetent} \end{cases}$$

Notice that the policy is certain to fail with an incompetent official in power.

The competence of each villager is common knowledge **within the village**. That is, all villagers know if he himself is competent and who in the village is competent. In what follows, we assume that the probability a villager is competent is π. Since we have a mass of villagers, this means that there is a proportion π of competent villagers.

The payoff of the villagers depends on the success of the policy, the policy chosen, and an underlying state of the world, which determines which policy is the best for the villagers and is know by everyone in the village. We denote this state of the world (or, in other words, the best policy for villagers) by $\omega \in \{0, 1\}$. The payoff of a villager (when not an official) is:

$$U_v = \begin{cases} S & \text{if } x = \omega \\ -S & \text{if } x \neq \omega \end{cases}$$

Notice that our parameter S in the utility function. It means that each villager gets zero when the policy fails ($S = 0$). Remark, further, that the villagers suffer a disutility if the wrong policy is implemented.

The official, on the other hand, cares mostly about the rents from being in office, denoted R. Indeed, to make our life easy, we will assume that the official only cares about them. She enjoys these rents when in office and some further rents if she is retained. The retention process, like the selection process of the official, will depend on the institution in place (democracy, appointment, or oversight) described below. The official also suffers a disutility from effort (in other words, effort requires time and possibly resources that could be used for other purposes) so her utility is:

$$U_o(e) = R + \begin{cases} R & \text{if retained} \\ 0 & \text{if not retained} \end{cases} - C\frac{e^2}{2}$$

The term $C\frac{e^2}{2}$ captures the disutility from effort, which is paid whether the official is retained or not. Throughout, we assume that $C > R > 1$.

Part A - Democracy

We start with democracy. In democracy, the villagers have full control over the (s)election of the official and her retention. The game then proceeds as follows.

0. Nature draws the competence of each villager and the state $\omega \in \{0, 1\}$.
1. Each villager observes his own competence, the competence of all other inhabitants of the village.
2. Villagers decide whether to run at no cost. One villager is elected to be the village official. We assume that, in case of indifference, all villagers converge to the same (randomly picked) candidate.
3. The official and the villagers observe ω. The official picks a policy and exerts a level of effort $e \geq 0$ to bring it to fruition.
4. The outcome (success—$S = 1$—or not—$S = 0$) is realized.
5. The villagers observe the policy choice x and the outcome of the policy S. They then decide whether to retain or remove the official.
6. The game ends and payoffs are realized.

The equilibrium concept, here and throughout this problem set, is Subgame Perfect Nash Equilibrium. Notice that at the time of the retention decision, the villagers are always indifferent between retaining the official or replacing her (the game ends after, so no matter what villagers do, it has no impact on their payoffs). Villagers are, thus, free to pick the best retention rule for themselves (as if they could commit to it). We will see that this retention rule takes a very simple form. For the moment, we will denote $\rho(x, S)$ the probability that the official is retained after picking policy

x and outcome S.[1]

Before turning to the analysis, Table 1 provides a point of reference by describing the model's main parameters and variables.

Variables	Definition
$x \in \{0,1\}$	Policy choice
$e \geq 0$	Effort to successfully implement policy
$S \in \{0,1\}$	Outcome: success ($S = 1$) or failure ($S = 0$)
$\omega \in \{0,1\}$	Ideal policy of villagers (state of the world)
π	Proportion of competent villagers
R	Rents from being retained for an official
$C\frac{e^2}{2}$	Cost of effort
$\rho(x, S)$	Probability o is retained as a function of policy choice x and outcome S

Table 1: Main variables and parameters

Q1 Explain briefly why if $\omega = 1$, then the best retention rule for the villagers is: $\rho(1,1) = 1$ and $\rho(x,S) = 0$ in all other cases (for all others combinations of x and S); if $\omega = 0$, then the best retention rule for the villagers is: $\rho(0,1) = 1$ and $\rho(x,S) = 0$ in all other cases.

Comment: You are just asked to give a quick intuition for this. You can do so by referring to the villagers' objective in their relationship with their official.

Q2 From now on, we will use the retention rule defined in **Q1**. In this question, we determine the action of the official.

(a) Show that a competent official picks $x = 1$ if $\omega = 1$ and $x = 0$ if $\omega = 0$. Show that her equilibrium level of effort is $e^D(c) = \frac{R}{C}$ in both states, where D stands for democracy and c for competent.

(b) Explain briefly why an incompetent official is indifferent between $x = 1$ and $x = 0$ (whatever the state) and always chooses effort $e^D(nc) = 0$.

[1] A retention rule is, thus, the list of all retention decisions under different values of x and S: $\rho(1,1)$, $\rho(1,0)$, $\rho(0,1)$, $\rho(0,0)$ for each possible ω.

Q3 We can then conclude our analysis of democracy by looking at the electoral choice of the villagers.

(a) Explain briefly why the villagers always choose competent candidates over incompetent ones.

(b) Show that all competent villagers run (remember that with many candidates and villagers randomly coordinate to one of them when indifferent).

Part B - Appointment

In this section, we consider the case when the official is appointed by a centralised autocratic government G and her fate is also determined by G. To do so, we need to describe the objective of and information available to G.

Unlike villagers, G always wants policy $x = 1$ to be successfully implemented by the village official. In the case of China, one can think of the sale of agricultural land, industrialisation, one child policy, etc., which are the objectives of the central government, and may not always be aligned with local demands. Indeed, we are introducing here the possibility of divergence. The payoff of the central government is:

$$U_G = \begin{cases} S & \text{if } x = 1 \\ -S & \text{if } x = 0 \end{cases}$$

The government G is imperfectly informed. First, it does not know the competency of each villager. It can only rely on the prior knowledge that $\pi\%$ of villagers are competent. Second, G does not always observes the policy picked by official. There is a probability λ that the government observes $\omega \in \{0, 1\}$, $x \in \{0, 1\}$ and $S \in \{0, 1\}$. Otherwise, the central government observes *nothing*. We also assume that there is a probability α that the state ω is 1. This parameter will play an important role in Part D.

The game in the case of appointment proceeds as follows.

0. Nature draws the competence of each villager and the state $\omega \in \{0, 1\}$.
1. Each villager observes his own competence, the competence of all other inhabitants of the village.

2. G randomly selects one villager to become the village official.
3. The official and the villagers observe ω. The official picks a policy and exerts a level of effort $e \geq 0$ to bring it to fruition.
4. The outcome (success—$S = 1$—or not—$S = 0$) is realized.
5. The government G observes ω, x and S with probability λ. With probability $1-\lambda$, G observes nothing. G decides whether to retain the official.
6. The game ends and payoffs are realized.

Notice that at the time of the retention decision, the government is indifferent between retaining the official or replacing her. It can pick the optimal retention rule. For the moment, we will denote $\kappa(x, S, d)$ the probability that the official is retained by G after picking policy x and outcome S as a function of the government's information, which we denote by d with $d = 1$ if the government observes ω, x, and S and $d = 0$ if not.

Before turning to the analysis, Table 2 summarizes the additional parameters and variables.

Variables	Definition
α	Common knowledge probability that $\omega = 1$
λ	Probability that the government learns x and S
$\kappa(x, S, d)$	Probability o is retained by G as a function of policy choice x, outcome S, and G's information d

Table 2: Main variables and parameters

Q4 Explain briefly why if G's retention rule satisfies $\kappa(1, 1, 1) = 1$ (the official is always retained after successfully implementing $x = 1$ and G learning it) and $\kappa(x, S, 1) = 0$ for all other x and S.
Comment: As for **Q1**, you are just asked to give a quick intuition for this result.

Q5 In the retention rule above, we have not determined what G should do after learning nothing ($d = 0$). We know that in this case, G cannot condition her retention decision on x or S (for the simple fact that it does not learn it). Hence, the retention rule must satisfy $\kappa(1, 1, 0) = \kappa(1, 0, 0) = \kappa(0, 1, 0) = \kappa(0, 0, 0) = \kappa_0$. We will solve for κ_0 below. In this question, we consider the official's choices.

(a) Given the (partial) retention rule defined in **Q4**, explain why a competent official always picks $x = 1$.

(b) Explain briefly why a competent official's maximization problem is:

$$\max_{e \geq 0} \lambda \times e \times R + (1-\lambda)(e \times \kappa_0 + (1-e) \times \kappa_0) \times R - C\frac{e^2}{2}$$

(c) Show that a competent official's equilibrium level of effort is $e^A(c) = \lambda \frac{R}{C}$ (where A stands for appointment).

Q6 We can then conclude our analysis of appointment by considering whether G retains the official conditional on learning nothing.

Explain briefly why the government is indifferent between all $\kappa_0 \in [0, 1]$

Part C - Democracy with oversight

We now consider an hybrid model. In this case, villagers elect the official. When it comes to the retention, villagers first decide whether to retain the official and, then, the government may decide whether to confirm or revert the decision (no matter what the villagers did). The game in this case then proceeds as follows.

0. Nature draws the competence of each villager and the state $\omega \in \{0, 1\}$.
1. Each villager observes his own competence, the competence of all other inhabitants of the village.
2. Villagers elect one of them to become the village official.
3. The official and the villagers observe ω. The official picks a policy and exerts a level of effort $e \geq 0$ to bring it to fruition.
4. The outcome (success—$S = 1$—or not—$S = 0$) is realized.
5. Villagers observe S and x with probability 1. The government G observes ω, x, and S with probability λ. With probability $1 - \lambda$, G observes nothing. Villagers decide whether to retain

the official. The government then also decides whether to retain the official based on its information.

6. The game ends and payoffs are realized.

In this case, we have a potential conflict of interest between villagers and the central government. When the state is $\omega = 0$, which occurs with probability $1 - \alpha$, the villagers want the official to exert effort on $x = 0$, whereas the central government wants the official to exert effort on $x = 1$. We address the issue of the official's policy choice in **Q7**.

We will make some assumptions to make our life easier. We will assume that when the official picks the preferred policy of the villagers, they always retain the official when she is successful and always removes her when she is unsuccessful. That is, $\rho(\omega, 1) = 1$ and $\rho(\omega, 0) = 0$. In state $\omega = 1$, when there is no disagreement between the central government and the villagers, the villagers will never retain an official who picks $x = 0$ (that is, no matter the outcome of the project). In state $\omega = 0$, when the official picks $x = 1$, the policy the villagers do not like, then the villagers retain if the official *fails* to implement the policy and removes the official if she is successful. That is, $\rho(1, 1) = 0$ and $\rho(1, 0) = 1$ if $\omega = 0$.

When it comes to the government, we assume that if it observes nothing it rubber stamps the decision of the villagers (i.e., always retains the official if the villagers have decided to retain her, always confirm the removal otherwise). When it observes ω, x and S $(d = 1)$, the government retains the official if she is successful in implementing $x = 1$: $\kappa(1, 1, 1) = 1$ and always removes the official if she fails to implement policy $x = 1$: $\kappa(1, 0, 1) = 0$. When the state is $\omega = 1$, then G would remove an official who implements $x = 0$ no matter the outcome of the project. If $\omega = 0$, then the government removes the official if she successfully implements policy 0: $\kappa(0, 1, 1) = 0$. G, however, keeps the official after failure to implement $x = 0$ in $\omega = 0$: $\kappa(0, 0, 1) = 1$.

With this in mind, we turn to the analysis of democracy with oversight.

Q7 In this question, we consider the official's best responses (plural, since it consists of effort and policy choice) given the retention rules above.

(a) Show that a competent official maximizes the following expected payoff with respect to e in state $\omega = 1$ after choosing $x = 1$:

$$e \times R - C\frac{e^2}{2}$$

(b) Show that a competent official exerts $e^O(c;1,1) = \frac{R}{C}$ after picking $x = 1$ in state $\omega = 1$.

Comment: the notation $e^O(c;1,1)$ denotes the effort of a competent official after choosing policy 1 in state 1 under oversight (O). More generally, we will use the following notation $e^O(c;x,\omega)$ for an effort after policy x in state ω.

(c) Briefly explain why a competent official never picks $x = 0$ in state $\omega = 1$.

(c) Show that a competent official maximizes the following expected payoff with respect to e after choosing policy $x = 0$ in state $\omega = 0$ is:

$$(1-\lambda)e \times R + \lambda(1-e) \times R - C\frac{e^2}{2}$$

(d) Show that a competent official exerts $e^O(c;0,0) = \max\{(1-2\lambda)\frac{R}{C}, 0\}$ after picking $x = 0$ in state $\omega = 0$.

(e) Show that the official's expected utility from choosing $x = 0$ in state $\omega = 0$ (taking into account her effort) is:

$$\begin{aligned} V^o(0,0) =& R + \lambda R + (1-2\lambda)e^O(c;0,0)R - C\frac{(e^O(c;0,0))^2}{2} \\ =& R + \lambda R + \big(\max\{(1-2\lambda), 0\}\big)^2 \frac{R^2}{2C} \end{aligned}$$

(f) Repeating **(c)** and **(d)**, show that a competent official exerts $e^O(c;1,0) = \max\{(2\lambda-1)\frac{R}{C}, 0\}$ after picking $x = 1$ in state $\omega = 0$.

(g) Show that the official's expected utility from choosing $x = 1$ in state $\omega = 0$ (taking into account her effort) is:

$$V^o(1,0) = R + (1-\lambda)R + \big(\max\{(2\lambda-1), 0\}\big)^2 \frac{R^2}{2C}$$

(h) Demonstrate that, in state $\omega = 0$, a competent official chooses $x = 1$ if and only if $\lambda \leq 1/2$ and $x = 0$ if and only if $\lambda > 1/2$. What is her level of effort then? Provide some intuition for this result.

Comment: Note that we assume the official follows the "orders" of the central government when indifferent (this assumption plays no role in the results).

Q8 Explain briefly why all competent villagers run and the villagers (randomly) pick one of these competent candidates to be elected.

Part D - Comparison

In this part, we will compare the villagers' and the central government's welfare across different systems. We denote by $\mathcal{W}_G(K)$ the expected welfare of the central government under system $K \in \{A, D, O\}$ (with A standing for appointment, D for democracy, and O for democracy with oversight). Similarly, we denote $\mathcal{W}_v(K)$ the welfare of a villager when non in office under system $K \in \{A, D, O\}$.

Like in your assessed work, this last part does not provide guidance for the algebra. Some of these questions can be answered without any algebra, just by understanding the logic of the model.

Q9 Show that all villagers (again, those not in office) strictly prefer democracy to democracy with oversight and strictly prefer democracy with oversight to appointment: $\mathcal{W}_v(D) > \mathcal{W}_v(O) > \mathcal{W}_v(A)$.

Q10 Show that the central government strictly prefers democracy with oversight to pure democracy: $\mathcal{W}_G(D) < \mathcal{W}_G(O)$.

Q11 Show that the central government strictly prefers the central government prefers appointment over democracy with oversight whenever $\alpha < \pi\lambda$. Provide some intuition for this result.

Q12 Suppose that efficiency is measured by the likelihood that the central government's preferred policy is successfully implemented. Consider a researcher who compares efficiency in a system of democracy with oversight and a system of appointment within the same country (e.g., performing a difference-in-differences research design).[2] The researcher finds that appointment is more efficient than oversight. Explain why the finding of the researcher is likely to be upwardly biased (i.e., the positive effect of appointment the researcher finds is too high).

Q13 Explain why efficiency (as defined above) is not everything.

[2] If we are in autocracy, these would be the only two systems ever observed by **Q10**.