

The Democratic Critiques and Populism

GV482 Problem Set - Game Theory

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Part III - The Populist's entry decision (part of this will be solved on Thursday 2 February, the rest on Thursday 9 February)

In the previous part, we have computed the electoral decision of a voter from group C as a function of her information. We use our previous answers to compute P 's electoral chances and entry decision under different scenarios.

Q7

In this question, we suppose that mainstream candidates find it optimal to the commoners' preferred policy.

- (a) Using your answers to **Q3**, explain briefly why P never wins the election if he enters in this case.

As we have shown in **Q3**, the probability of P winning the election is 0 if he enters.

If P proposes the same platform (commoners' preferred policy) as A and B , P will get no vote as both the commoners and elite will vote for the mainstream candidate with a positive valence shock. This corresponds to **Q3** (a).

If P proposes a different platform, and

- the commoners and the elite's preferred policy is the same, P will get no vote as both the commoners and elite will vote for the mainstream candidate with a positive valence shock. This corresponds to **Q3** (b).
- the commoners and the elite's preferred policy is different, P will only get the elite's votes. This accounts for σ of the vote. Since this is less than 50%, P will not win the election. This corresponds to **Q3** (c).

- (b) Explain why the populist never enters when $\sigma + (1 + \sigma)(1 - p) < \frac{1}{2}$.

As we have shown in **Q2** (e), $\sigma + (1 + \sigma)(1 - p) < \frac{1}{2}$ implies that the mainstream candidates converge on the commoners preferred policy. As we discussed in Part (a), P has no chance of winning in this case. As there is a cost to entry, P will not enter:

$$U_P(\text{enter}) = -c < U_P(\text{not enter}) = 0$$

Q8

In question **Q8** and **Q9**, we turn to the case when mainstream candidates converge to the elite citizens' preferred policy ($x_A = x_B = \omega_E$ and this is anticipated by commoners). In this question, we assume that $\alpha > 1/2$ and look at P 's entry decision then.

- (a) Assume that mainstream candidates propose $x_A = x_B = 0$ and the populist offers $x_P = 1$. Assume further than the state is $\omega_C = 0$ (i.e., that is the optimal policy happens to be zero for the common voters). Using your answer to **Q5**(i), show that the vote share of candidate P is: $(1 - \sigma)(1 - p)$. Explain why P does not win the election then.

The elite know their preference well. They will vote for one of the mainstream candidates and not populist P .

As for the commoners,

- some receive signal of 1 ($s_{i,C} = 1$). Their posterior belief (from **Q5** (j)) is:

$$\mu(1, 0, 0, 1) = \frac{(1 - \alpha)p^2}{(1 - \alpha)p^2 + \alpha(1 - p)^2} > \frac{1 + |\delta|}{2}.$$

They will vote for P .

- Some receive signal of 0 ($s_{i,C} = 0$). Their posterior belief is:

$$\mu(0, 0, 0, 1) = 1 - \alpha$$

Since we assume $\alpha > 1/2$, $\mu(0, 0, 0, 1) = 1 - \alpha < \frac{1}{2} < \frac{1 + |\delta|}{2}$. They will not vote for P .

Therefore, only the commoners who receive signal of 1 will vote for P . The probability of receiving this wrong signal $Pr(s_{i,C} = 1 \mid \omega_C = 0) = 1 - p$. They therefore account for $(1 - \sigma)(1 - p)$ of total voters. Since $p > 0.5$, this is smaller than 50%. Other voters will unite around the mainstream candidate with a positive valence shock, who will receive more than 50% of the vote. Therefore, P will not win the election.

- (b) Assume that mainstream candidates propose $x_A = x_B = 0$ and the populist offers $x_P = 1$. Assume further than the state is $\omega_C = 1$ (i.e., that is the optimal policy happens to be one for the common voters). Show that the vote share of candidate P is: $(1 - \sigma)p$. Explain why P loses the election then.

The elite know their preference well. They will vote for one of the mainstream candidates and not populist P .

As for the commoners,

- some receive signal of 1 ($s_{i,C} = 1$). Their posterior belief (from **Q5** (j)) is:

$$\mu(1, 0, 0, 1) = \frac{(1 - \alpha)p^2}{(1 - \alpha)p^2 + \alpha(1 - p)^2} > \frac{1 + |\delta|}{2}.$$

They will vote for P .

- Some receive signal of 0 ($s_{i,C} = 0$). Their posterior belief is:

$$\mu(0, 0, 0, 1) = 1 - \alpha$$

Since we assume $\alpha > 1/2$, $\mu(0, 0, 0, 1) = 1 - \alpha < \frac{1}{2} < \frac{1 + |\delta|}{2}$. They will not vote for P .

Therefore, only the commoners who receive signal of 1 will vote for P . The probability of receiving this right signal $Pr(s_{i,C} = 1 \mid \omega_C = 1) = p$. They therefore account for $(1 - \sigma)p$ of total voters. For the mainstream candidates to converge on the elite's preference, it requires:

$$\begin{aligned} \sigma + (1 - \sigma)(1 - p) &\geq \frac{1}{2} \\ \sigma + (1 - \sigma) - (1 - \sigma)p &\geq \frac{1}{2} \\ (1 - \sigma)p &\leq \frac{1}{2} \end{aligned}$$

Therefore, P only receives less than half of the votes, whereas the mainstream candidate with a positive valence shock gathers more than half of the votes. P therefore loses.

- (c) Assume that mainstream candidates propose $x_A = x_B = 1$ and the populist offers $x_P = 0$. Explain why P never wins the election.

The elite know their preference well. They will vote for one of the mainstream candidates and not populist P .

As for the commoners,

- some receive signal of 1 ($s_{i,C} = 1$). Their posterior belief (from **Q6** (a)) is:

$$\mu(1, 1, 1, 0) = \alpha.$$

As we have shown in **Q6** (d), commoners with signal 1 in this case only vote for P if and only if $\alpha < 1/2$. As we assume $\alpha > 1/2$ in this question, they will not vote for P .

- Some receive signal of 0 ($s_{i,C} = 0$). Their posterior belief is:

$$\mu(0, 1, 1, 0) = \frac{(1-p)^2\alpha}{(1-p)^2\alpha + p^2(1-\alpha)}$$

As we have shown in **Q6** (d), commoners with signal 0 in this case always vote for P .

Therefore, only commoners with signal 0 will vote for P . The probability of receiving this signal is either p or $1-p$. They therefore account for $(1-\sigma)p$ or $(1-\sigma)(1-p)$ of total voters. As we explored in Part (c), neither is greater than 50% and one of the mainstream candidates gathers the majority of the vote. Therefore, P will not win the election.

- (d) Explain briefly why P never enters.

We have shown that, when $\alpha > 1/2$, the populist candidate P never wins the election. No matter which policy the mainstream candidates converge on and what policy the populist candidate P proposes, she has no chance of winning. Since there is a cost of running for election, P will never enter the election.

Q9

We now look at P 's electoral chances and entry decision when $\alpha < 1/2$.

- (a) Assume that mainstream candidates propose $x_A = x_B = 0$ and the populist offers $x_P = 1$. Using your answer to **Q5**(i), show that the vote share of candidate P is: $(1-\sigma)$. Explain why P wins the election then.

The elite will also vote for the mainstream candidates as their platform aligns with the elite's preference.

As for the commoners,

- some receive signal of 1 ($s_{i,C} = 1$). Their posterior belief (from **Q5** (j)) is:

$$\mu(1, 0, 0, 1) = \frac{(1 - \alpha)p^2}{(1 - \alpha)p^2 + \alpha(1 - p)^2} > \frac{1 + |\delta|}{2}.$$

They will vote for P .

- Some receive signal of 0 ($s_{i,C} = 0$). Their posterior belief is:

$$\mu(0, 0, 0, 1) = 1 - \alpha$$

Since we assume $\alpha < 1/2$, $\mu(0, 0, 0, 1) = 1 - \alpha > \frac{1 + |\delta|}{2}$. This is shown in **Q5** (j). They will also vote for P .

Therefore, the P will enjoy all the votes, and only the votes of the commoners, who account for $(1 - \sigma)$ of all voters. Since there are more commoners than elite $1 - \sigma > 1/2$, P will win the election.

- (b) Assume that mainstream candidates propose $x_A = x_B = 1$ and the populist offers $x_P = 0$. Explain why P always wins the election then.

The elite will also vote for the mainstream candidates as their platform aligns with the elite's preference.

As for the commoners,

- some receive signal of 1 ($s_{i,C} = 1$). Their posterior belief (from **Q6** (a)) is:

$$\mu(1, 1, 1, 0) = \alpha.$$

As we have shown in **Q6** (d), $\mu(1, 1, 1, 0) = \alpha < \frac{1 - |\delta|}{2}$ when $\alpha < 1/2$ and these commoners with signal of 1 will vote for P .

- some receive signal of 0 ($s_{i,C} = 0$). Their posterior belief is:

$$\mu(0, 1, 1, 0) = \frac{(1 - p)^2 \alpha}{(1 - p)^2 \alpha + p^2(1 - \alpha)}$$

We have shown in **Q6** (d) it is true that $\mu(0, 1, 1, 0) < \frac{1 - |\delta|}{2}$ for all values of α . Therefore these commoners with signal of 1 will also vote for P .

Therefore, the P will enjoy all the votes, and only the votes of the commoners, who account for $(1 - \sigma)$ of all voters. Since there are more commoners than elite $1 - \sigma > 1/2$, P will win the election.

We now need to compute P 's expected payoff if P enters. We consider the populist's entry decision so we no longer assume that $x_P = s_{P,C}$ (this has to be a choice of the populist). However, we still assume below that citizens anticipate that $x_P = s_{P,C}$ if the populist enters (we will see that this anticipation is correct below). In turn, both the citizens and the populist anticipate that the mainstream candidates propose $x_A = x_B = \omega_E$ (again, we will see that this anticipation is correct in Part IV).