

To specified the point n of max magnitude (for rise-time)

$$\begin{aligned}
 h(n) &= \sum_{k=0}^n r_1^k r_2^{n-k} \\
 &= r_1^n r_2^0 + r_1^{n-1} r_2^1 + r_1^{n-2} r_2^2 + \dots + r_1^0 r_2^n \\
 &= r_1^n r_2^0 + r_1^{n-1} r_2^1 + \dots + r_1^0 r_2^n = h(n) \\
 \frac{r_2}{r_1} h(n) &= r_2 + \frac{r_2}{r_1} r_1^n - \frac{r_2}{r_1} r_1^{n-1} \\
 \left(\frac{r_2}{r_1} - 1 \right) h(n) &= \frac{r_2^{n+1}}{r_1} - r_1^n \\
 h(n) &= \frac{\frac{r_2^{n+1}}{r_1} - r_1^n}{\frac{r_2}{r_1} - 1} = \frac{\frac{r_2^{n+1} - r_1^{n+1}}{r_1}}{\frac{r_2 - r_1}{r_1}} \\
 &= \frac{r_2^{n+1} - r_1^{n+1}}{r_2 - r_1} \\
 h(n) - h(n-1) &= \frac{r_2^{n+1} - r_1^{n+1}}{r_2 - r_1} - \frac{r_2^n - r_1^n}{r_2 - r_1} = 0 \\
 \frac{r_2^{n+1} - r_1^{n+1} - r_2^n + r_1^n}{r_2 - r_1} &= \frac{r_2^n (r_2 - 1) + r_1^n (1 - r_1)}{r_2 - r_1} = 0 \\
 r_2^n (1 - r_2) &= r_1^n (1 - r_1) \\
 \left(\frac{r_2}{r_1} \right)^n &= \left(\frac{1 - r_1}{1 - r_2} \right) \Rightarrow n \log \frac{r_2}{r_1} = \log \frac{1 - r_1}{1 - r_2} \\
 n &= \frac{\log \frac{1 - r_1}{1 - r_2}}{\log \frac{r_2}{r_1}}
 \end{aligned}$$

For the rising time:

I use the below equation to calculate the rising time:

rise time $tr = n/Fs$

set $Ta = 0.5$, $Tb = 0.8$, $Fs = 8000$

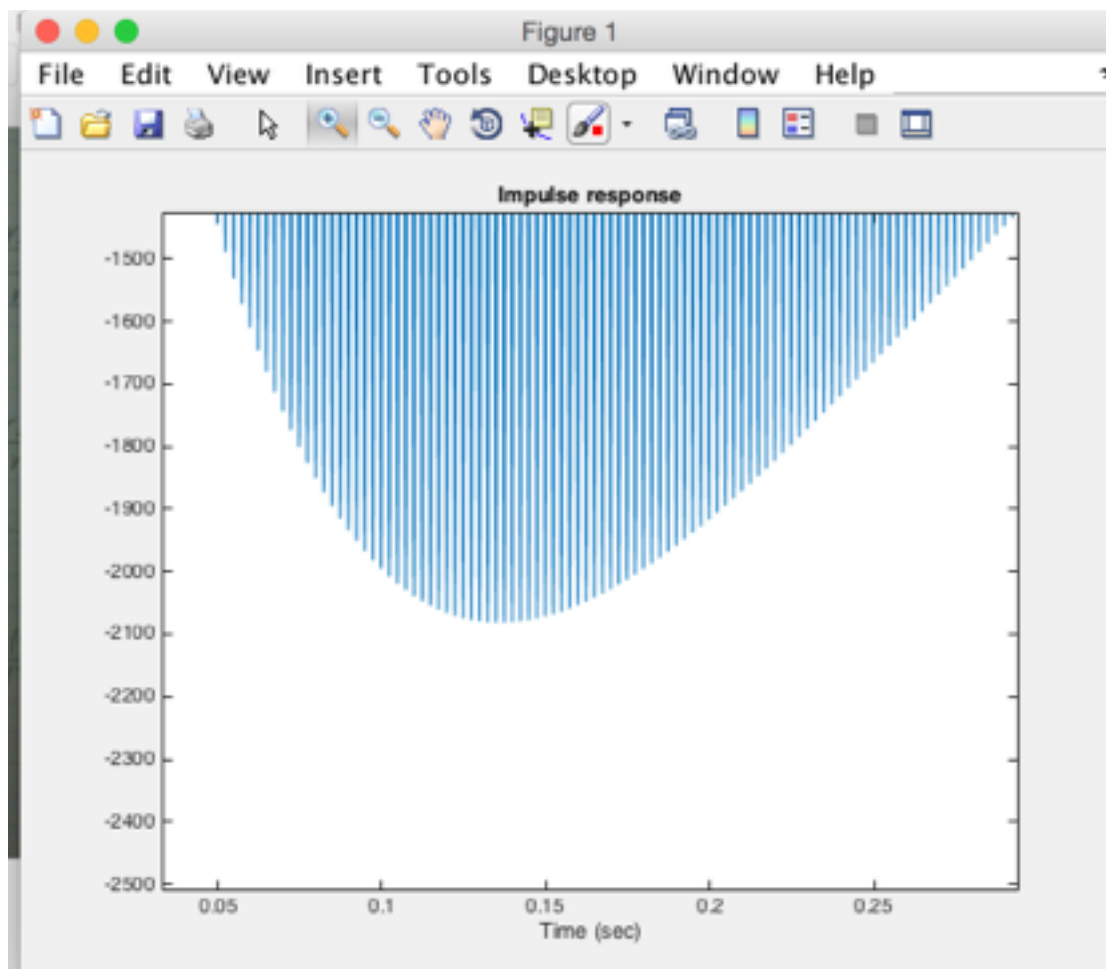
$r1 = 0.01^{(1/(Ta*Fs))} = 0.9988$

$r2 = 0.01^{(1/(Tb*Fs))} = 0.9993$

$n = 1076.096$

$\rightarrow tr = 0.1345$

$$peak = \log((1-r1)/(1-r2))/\log(r2/r1)/8000$$



for the falling time:

I just modified the value of the T_a , and then the falling time will increase when I add the value of the T_a .

Python implementation:

See the file Lab_2_4_10_phc307: