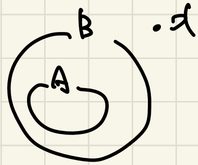
Ch 1.4

#1. We want to show that if $x \in B^c$, then $x \in A^c$.

+3

If $x \in B^c$, then $x \notin B$.



Since $A \subset B$, $x \notin A$.

That implies $x \in A^c$.

+1 : try something, but doesn't make sense.

#2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$x \in A \cap (B \cup C)$

+2: try to show if $x \in B^c$, then $x \in A^c$, but wrong steps.

$\Leftrightarrow$ $x \in A$ and $x \in B \cup C$

$\Leftrightarrow$ $x \in A$ and either $x \in B$ or $x \in C$.

$\Leftrightarrow$ either $x \in A \cap B$ or $x \in A \cap C$

$\Leftrightarrow$ $x \in (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$x \in A \cup (B \cap C)$

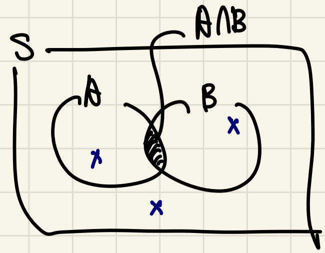
$\Leftrightarrow$ either $x \in A$ or $x \in B \cap C$

$\Leftrightarrow$ either $x \in A$ or $x \in B$ and $x \in C$

$\Leftrightarrow$ $x \in A \cup B$ and $x \in A \cup C$

$\Leftrightarrow$ $x \in (A \cup B) \cap (A \cup C)$

#3 $(A \cap B)^c = A^c \cup B^c$



$x \in (A \cap B)^c$

$\iff x \notin A \cap B$

$\iff x \notin A$ or $x \notin B$

$\iff x \in A^c$ or $x \in B^c$

$\iff x \in A^c \cup B^c$.

#8. A = blood reacts with anti-A

B = " anti-B.

Type A $= A \cap B^c$

Type B $= A^c \cap B$

Type AB $= A \cap B$

Type O $= A^c \cap B^c$

#11.

a. $S = \{ (x,y) : 0 \leq x \leq 5 \text{ and } 0 \leq y \leq 5 \}$.

b. $A = \{ (x,y) \in S : x+y \geq 6 \}$

$B = \{ (x,y) \in S : x=y \}$

$C = \{ (x,y) \in S : x > y \}$

$D = \{ (x,y) \in S : 5 < x+y < 6 \}$

c. $\{ (x,y) : x=y \text{ and } x+y \leq 5 \}$.

$= B \cap D^c \cap A^c$.

D. $\{ (x,y) : x+y < 6, \ y > x \}$

$= A^c \cap C^c \cap B^c$.

Ch 1.5

#4. Let A = student A fails

B = " B " .

$P(A) = .5$, $P(B) = .2$, $P(A \cap B) = .1$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ by general additivity for two events,

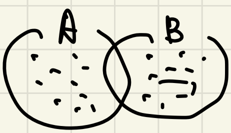
$= .5 + .2 - .1 = .6$.

#5 $P(A^c \cap B^c) = P((A \cup B)^c)$

$= 1 - P(A \cup B)$

$= 1 - .6 = .4$.

#6. $P((A \cap B^c) \cup (B \cap A^c)) = P(A \cup B) - P(A \cap B)$

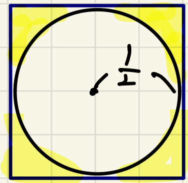


$= .6 - .1 = .5$.

#11.

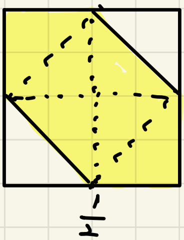
probability = area.

+1 (a)



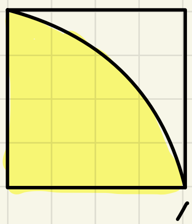
$$1 - \pi\left(\frac{1}{2}\right)^2 = 1 - \frac{\pi}{4}.$$

+1 (b)



$$\frac{6}{8} = \frac{3}{4}$$

+1 (c)

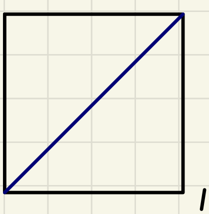


$$\int_0^1 1 - x^2 \, dx$$

$$= \left[x - \frac{1}{3}x^3 \right]_0^1$$

$$= \left(1 - \frac{1}{3}\right) = \frac{2}{3}.$$

+1 (d)



Since the area of a line is 0, the probability is 0.

Ch 1.6.

#2. $S = \{(1,1), (1,2), (1,3), \cdots, (6,6)\}$.

A = the sum is even

$= \{(1,1), (1,3), (1,5), \cdots, (6,6)\}$.

Since S is a simple sample space,

$$P(A) = \frac{|A|}{|S|} = \frac{18}{36} = \frac{1}{2}.$$

#3. B = difference is less than 3

+3

$= \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3),$
$(2,4), (3,1), (3,2), (3,3), (3,4), (3,5),$
$(4,2), (4,3), (4,4), (4,5), (4,6), (5,3),$
$(5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\}$.

Since S is a simple sample space,

$$P(B) = \frac{|B|}{|S|} = \frac{24}{36} = \frac{2}{3}.$$

#4. $S = \{G1, G2, G3, G4, G5, G6\}$

$$P(G2) = P(G3) = P(G4) = P(G5) = P(G6) = \frac{1}{2} P(G1)$$

Since S is a finite sample space,

$$\sum_{i=1}^{6} P(Gi) = 1.$$

Therefore, $P(G1) = \frac{2}{7}$, $P(G2) = \cdots = P(G6) = \frac{1}{7}$.

#5. $P(\{G1, G3, G5\}) = P(G1) + P(G3) + P(G5)$

$$= \frac{2}{7} + \frac{1}{7} + \frac{1}{7}$$

$$= \frac{4}{7}.$$