NE 495 – Elements of Nuclear Engineering – Exam I – Solution

Concept Check. Provide brief responses for each problem. Where applicable, a small sketch can be used.

1. (6 points) Explain the first three terms of the semi-empirical mass formula, i.e., $a_v A, -a_s A^{2/3}$, and $-a_c Z(Z-1)/A^{1/3}$.

   The first term accounts for the fact the binding energy should be proportional to the liquid drop volume (and, hence, number of nucleons), and because $r \propto A^{1/3}$, $V \propto A$. The second, negative term accounts for the fact that nucleons near the surface of the drop are bound by fewer neigbors (so that the BE is reduced). The number of these surface nucleons is proportional to the surface area $S \propto r^2 \propto A^{2/3}$. The third term represents the Coulombic potential each of Z protons is subjected to from each of the other $Z-1$ protons. That potential goes as $1/r = 1/A^{1/3}$.

   1 point for mentioning volume term
   1 point for mentioning surface term
   1 point for mentioning coulombic term
   3 point for explaining why the SEMF has each term. Simply listing the terms by name is not an explanation!

2. (3 points) Suppose we know the speeds of an electron and of a proton moving somewhere in space to within some very small fraction of a [m/s]. Of the two particles, which can we locate more precisely?

   The uncertainty principle says $\Delta x \Delta p \geq \hbar/2$. For clarity, assume the classical momentum expression $p = mv$ applies; then, $\Delta p = m\Delta v$. Thus, $\Delta x \geq \frac{\hbar}{2m\Delta v}$. With its larger mass, the **proton** can be located more precisely.

   1 point for Heisenberg.
   1 point for relating velocity uncertainty to momentum uncertainty.
   1 points for correctly picking the proton.

3. (2 points) How was the nucleus discovered?

   The gold foil experiment showed that some alpha particles were scattered at large angles by the gold, which could only be explained by the presence of a very small, very dense nucleus.

   1 point for (Rutherford's) gold foil experiment. Simply stating the name of the experiment does not describe enough.
   1 point for connecting deflection to a hard nuclear core.

4. (3 points) Describe two observations that show how light exhibits particle properties.

   We discussed the (1) photoelectric effect and (2) Compton scattering. The former showed that light must exist in discrete quanta since the intensity of light (number of photons) did not change the energy of the emitted electron. The latter showed that the photon must have momentum.

   0.5 point for listing each correct phenomenon.
   1 point for explaining each of them; stating is not enough.

5. (2 points) What are *isobars*? Provide an example.

   Isobars are nuclides with the same mass number A. Examples include the pair H-3 and He-3.

   1 point for correct definition.
   1 point for correct example.

Worked Problems. Show all of your work to get full credit. Assume nothing is obvious to the grader, so, when in doubt, explain your work.

6. (6 points) ^{238}U decays by emission of an α particle with an energy of around 4.3 MeV. A simple theory suggests the α particle first forms within the nucleus and bounces around for a comparatively long time before escaping. Is this theory consistent with what you've learned, i.e., could the α particle be confined to the nucleus? Provide quantitative evidence to support your claim. For reference, an α particle is equivalent to a ^{4}He nucleus.

   Solution:

   1. The nuclear radius of the ^{238}U is about $1.1 \times 238^{1/3} \approx 6.8$ fm.
   2. One version of Heisenberg's uncertainty principle says that $\Delta x \Delta p \geq \frac{h}{4\pi}$. Here, take the α particle to be classical, so that $p = \sqrt{2Em} = \sqrt{2 \times 4.3\ \text{MeV} \times 4\text{u} \times 931\ \text{MeV/c}^2}$ or $p = 179$ MeV/c.
   3. Planck's constant is 4.135×10^{-21} MeVs.
   4. Hence, the minimum Δx is $4.135 \times 10^{-21}/4/\pi/179 \approx 0.5$ fm. So, it seems that the α could happily sit inside the original nucleus without violating Heisenberg's uncertainty principle.
   5. The flip of this version is to show that were an α confined to the nucleus then its minimum energy is smaller than the stated energy (again, meaning no inconsistency).
   6. For reference, with a +2 charge, the α particle must overcome a large coulomb barrier (imagine a couple protons being surround by a bunch of other protons that are trapping the first two). That said, as it bounces repeatedly, there is a small, quantum-mechanical effect called "tunneling" that let's the α through with a low probability.
   7. For reference, a second version of Heisenberg could be used to show that the energy of the α is consistent with the half-life of ^{238}U.

   2 points for Heisenberg and correct interpretation.
   2 points for radius
   2 points for Δx

7. (6 points) 303 stainless steel is a non-magnetic, austenitic stainless steel that is not hardenable by heat treatment. Although the composition of 303 (and most other alloys) is defined only to within certain tolerances, assume that its composition is 17% Cr, 9% Ni, 0.15% C, 1% Si, 2% Mn, 0.1% P, 0.1% S, and 0.75% Mn (all percents by mass), with the remainder being Fe.

   Assume the density of this steel is 8030 kg/m^3. How many ^{32}S atoms are in 1 g of the alloy?

   Solution:

1. Compute the number of S atoms:

$$\begin{aligned} n_{\mathrm{S}} &= \frac{m_{\mathrm{S}} N_a}{M_{\mathrm{S}}} \\ &= \frac{0.001 \text{ g S} \times 6.022 \cdot 10^{23} \text{ atoms/mol}}{(32 \cdot 0.949309 + 33 \cdot 0.00780 + 34 \cdot 0.04290) \text{ g S/mol}} \\ &= 1.876 \cdot 10^{19} \text{ atoms S/cm}^3 . \end{aligned}$$

2. Not all S is S-32:

$$n_{\text{Ni-32}} = 0.94930 \text{ atom S-32/atom S} 1.876 \cdot 10^{19} \text{ atoms S}$$

$$\boxed{n_{\text{S-32}} = 1.781 \cdot 10^{19} \text{ atoms S}}$$

1 point for correct use of steel mass density (or mass).
2 points for correct number (density) equation.
2 points for correct Si number (density).
1 points for correct Si-32 number (density).

8. (6 points) Compute the kinetic energy of a muon ($m_0 \approx 105.7$ MeV$/c^2$) traveling at a speed equal to $0.95c$.

Solution:

1. The total relativistic energy is given by

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-(v/c)^2}} = T + m_0 c^2$$

2. The relativistic kinetic energy is, therefore,

$$T = \frac{m_0 c^2}{\sqrt{1-(v/c)^2}} - m_0 c^2$$

3. The kinetic energy of the muon is

$$T = \frac{105.7 \text{ MeV}}{\sqrt{1-0.95^2}} - 105.7 \text{ MeV} \approx 232.8 \text{ MeV} (\approx 3.72 \cdot 10^{-11} \text{ J})$$

4. For reference, the classical value gives

$$T = \frac{1}{2} m v^2 = \frac{1}{2} 105.7 \text{ MeV} - (0.95)^2 \approx 42.8 \text{ MeV} (\approx 6.78 \cdot 10^{-12} \text{ J})$$

3 point for relativistic kinetic energy definition.
3 points for correct answer (given energy used).

9. (6 points) It has been proposed to extract uranium from sea water to produce nuclear reactor fuel. Assume that the total volume of the oceans is 1.3×10^9 km^3, the uranium concentration in the ocean is 3 parts per billion water molecules, and the total annual electricity consumption in the U.S. is 4,095 Billion kWh (1 kWh = 3.6×10^6 J). Assume further that every single ^{235}U nucleus fissions according to the reaction $n + {}^{235}U \rightarrow {}^{139}Ba + {}^{95}Kr + 2\,n$ and that 30% of all of the energy can be recovered for electrical production.

   How many years could the oceanic uranium power the U.S.?

   Solution:

   1. Assume water has a density of $1\ \text{g/cm}^3$. (A symbolic density is fine, as long as it is carried through. Water density is a good number to know as a mechanical engineer.)
   2. Mass of oceanic water is approximately $m_w = 1.3 \cdot 10^9 \times 100000^3 \text{cm} \times 1 \text{g/cm}^3 = 1.3 \times 10^{24}$g.
   3. The number of oceanic water molecules is $N_w = \frac{m_w N_a}{18} \approx 4.349 \cdot 10^{46}$.
   4. The number of oceanic uranium nuclei is $N_U = 4.35 \cdot 10^{46} \times 3/10^9 = 1.305 \cdot 10^{38}$.
   5. The number of oceanic U-235 is $N_{235} = 0.0072 N_U = 9.394 \cdot 10^{35}$.
   6. The Q value of the fission is $931.5(M_{235} + m_n - M_{139} - M_{95} - 2m_n) \approx 170$ MeV.
   7. The total energy from fission to be recovered is $E_U = Q\text{MeV} \times 1.6 \cdot 10^{-13}\text{J/MeV} \times N_2 35 \times 0.3 \approx 7.666 \cdot 10^{24}$J.
   8. The total annual US consumption is $E = 4095 \times 3.6 \cdot 10^6$J.
   9. The number of years is therefore $E_U / E \approx 520000$.

   2 points for correct volume and number of water molecules
   1 points for correct number of U-235
   2 point for correct Q value (should be 150–200 MeV, very loosely)
   1 points for correct number of years given the number of fissions and Q