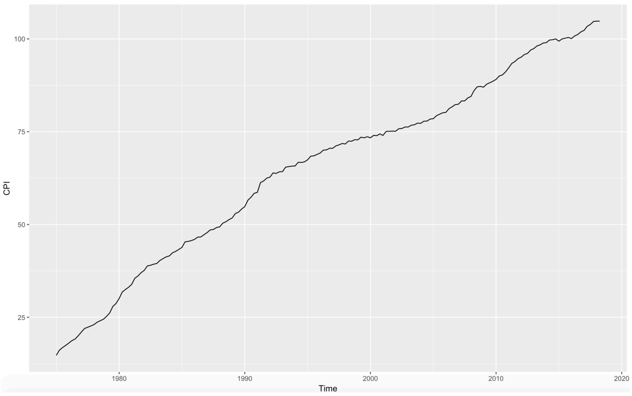
Time-Series Homework

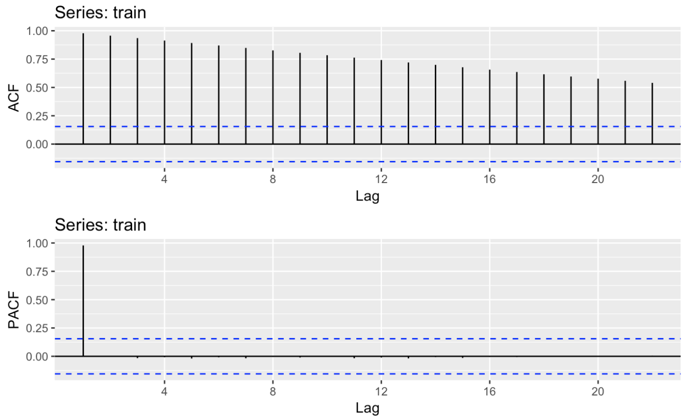
Anisimov Maksim, Vardanyan Garik

We have taken quarterly data of CPI, real GDP and government bond yield spread between 10-year and 3-month government bonds of the United Kingdom from 1975 to 2018 from investing.com and fred.stlouisfed.org. We begin with univariate forecasting, continue with multivariate forecasting and structural analysis.

Univariate Forecasting

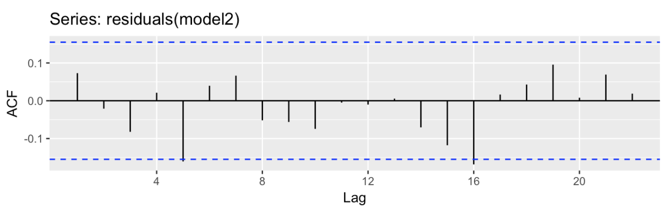
For univariate case we decided to forecast CPI. The train of data consists of observations from 1975 to 2015, while test consists of data from 2015 to second quarter of 2018. From the plot of the graph we see that the variance is stabilised during the considered time-frame and therefore we don't need to make Box-Cox transformations.

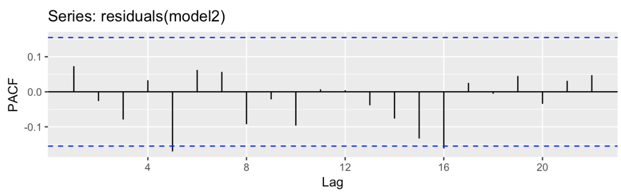




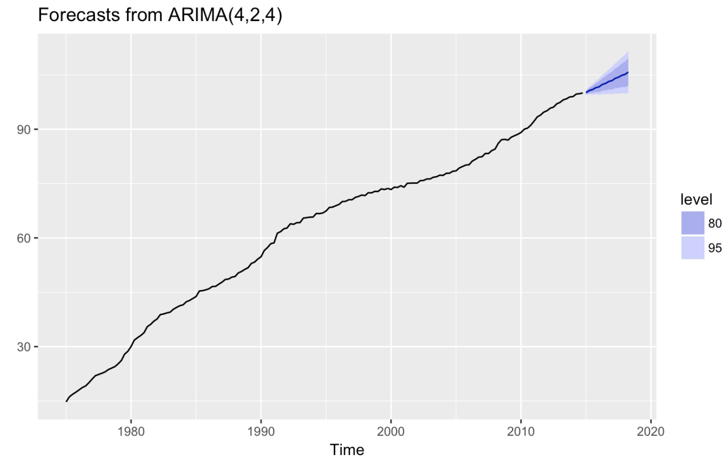
The plot of data clearly indicates that data is non-stationary. ACF of data is slowly decreasing, so we can see that there is a sign of unit root in our series. The PACF also indicates unit root. Augmented Dickey-Fuller test suggests that for our initial time series null hypothesis of non stationarity can not be rejected. We take first difference and we get that for the new series the null hypothesis of non stationarity is rejected towards alternative hypothesis of stationarity at any viable level. We build our model using default function of auto.arima. Auto ARIMA suggests that our data is second order difference stationary. It computes that ARIMA(0,2,1) is the optimal choice for our data. ACF and PACF of residuals suggest that residuals are not White Noise. Box-Ljung test indicates that null hypothesis is rejected and there are no White Noise residuals in our model. In order to check other models, we look at different set of parameters choosing the ones with lowest AIC.

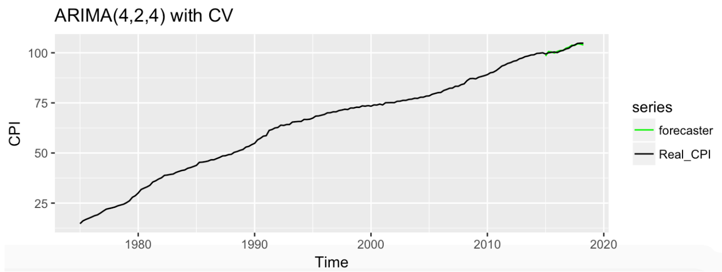
We get that best model is ARIMA(4,2,4), and use it for our further analysis. We also try models ARIMA(4,1,4) and ARIMA(3,2,4) that have AIC close to the best, maybe in test data this models will be better. For ARIMA(4,2,4) we plot ACF and PACF of residuals.





Box-Ljung test indicates that null hypothesis of White Noise residuals can't be rejected and we can move further in our analysis. We build forecast from 2015 and draw confidence intervals for them.





The intervals are big, so we check residuals for normality with tests of Jarque-Bera and Shapiro and get that our residuals are not normal. We also build forecasts with other optimal models and for their case get that residuals are not normal as well. RMSE on test equals to 1 and is higher than on train (0.3). Our model is overfitted, but overall it works good, because of the range of our target variable.

In order to make our model even better we perform cross validation with window 1. Obviously, the results of cross validation are much better because for every period we have the value of the recurrent period. RMSE of CV forecasts is equal to 0.53, so our original opinion was correct. Our final best model is ARIMA(4,2,4) with cross validation.

Multivariate Forecasting

Our Vector Autoregression Model is built to forecast UK GDP, CPI and bond yield spread. Our approach is justified by some theoretical and empirical works that study relationships between GDP and bond yield spreads and GDP and inflation. There are studies reveal that in well-developed financial markets there is a positive statistical significance between the economic growth and the yield spread (Moffatt and Zang, 2012). In addition, it is necessary to consider an effect of CPI on both GDP since the UK implemented inflation targeting in 1992. Finally, GDP can influence inflation (when demand increases, prices grow) and bond yields (through rising expectations about future rates).

We split data to training and test in order to check the quality of our forecasts. We assume that predicting for more that 2 years is unreasonable since too many outside shocks can happen during this period. Thus, our training sample covers the period 1975-2016 Q1, and our test sample begins in 2016 Q2 and finishes in 2018 Q1.

We implement a majority view that all data in VAR must be stationary (Enders, 2015) and check various transformed data for stationarity. As a result, we use in our model difference between yield spreads, difference between logarithms of GDPs and, surprisingly, logarithm of CPI (according to ADF test). We use logarithm of CPI, not difference as in univariate case, just for purpose of better interpretation.

AIC(n)	HQ(n)	SC(n)	FPE(n)
6	6	1	6

Lag selection procedure gives the following results shown in the table.

Since AIC usually overestimates the appropriate number of lags, we start with building VAR(1). To carry out a test for serially correlated errors we use Breusch-Godfrey statistic test and asymptotic Portmanteau statistic since our sample is large enough (165 observations). We build VARs only with drift as there has to be no trend in our stationary series but we expect a vector of constants to be significant in our model.

ADF test results (p-value)	**Initial time-series**	**First difference**	**Second difference**	**Natural logarithm**	**Difference of logarithms**
GDP	Non-stationary (0.45)	Stationary (0.01)	Stationary (0.01)	Non-stationary (0.6504)	Stationary (0.01)

ADF test results (p-value)	Initial time-series	First difference	Second difference	Natural logarithm	Difference of logarithms
CPI	Non-stationary (0.47)	Stationary at 6% (0.054)	Stationary (0.01)	Stationary (0.01)	-
Bond yield spread	Stationary at 5% (0.04)	Stationary (0.01)	Stationary (0.01)	-	-

Then, we stop building models at lag 3 as in all VARs with 1,2 and 3 lags there is no error autocorrelation and these models are parsimonious.

	Asymptotic Portmanteau p-value	Breusch-Godfrey p-value	Jarque-Bera test result
VAR(1), VAR(2), VAR(3)	<0.01	<0.01	Errors are not normal

Errors in these three models are not normal according to Jarque-Bera test, thus confidence intervals in both models will not be reliable. To choose the best model we want to look at forecasting ability of each VAR.

We build forecasts for both training and test sets using VAR(1), VAR(2) and VAR(3). We seek choosing the model that has better predictive ability and is as parsimonious as possible. We use mean absolute scaled error (MASE) as a quality measure to eliminate scale problems when comparing forecasts for variables of different scale. As a benchmark forecast we use naive forecast for difference between yield spreads and seasonal naive forecast for difference between logarithms of GDPs and logarithm of CPI since they are influenced by some seasonal effects during the year.

MASE	Log GDP difference		Log CPI		Yield Spread difference		
	Training set	Test set	Training set	Test set	Training set	Test set	Cumulative test quality
VAR(1)	0.64	0.225	0.145	0.423	0.629	0.2797	0.9277
VAR(2)	0.63	0.205	0.1447	0.353	0.624	0.281	0.839
VAR(3)	0.614	0.257	0.133	0.313	0.614	0.2772	0.8472
Naive	-	-	-	-	0.858	0.2856	1.1406
Seasonal naive	1	0.298	1	0.557	-	-	

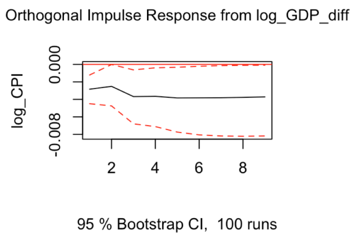
We use sum of MASE (because this metric is indifferent to scale) to help us choose the best model. As we want a parsimonious and accurate model simultaneously, we select VAR(2) as the best for forecasting this system and use it in structural analysis.

Multivariate structural analysis

We assume that the following chain of effects in the given quarter exists:
Bond yield spread difference responds to changes in differences between logarithms of GDP (call it GDP acceleration) and changes in CPI, logarithm of CPI responds to changes in GDP acceleration, GDP acceleration does not respond to CPI and yield spread difference changes. This is a rather common assumption in many economic researches, but in our model bond yield spread mimics a role of an interest rate. We consider prices in the UK sticky, thus the CPI is not affected by changes in yield spread difference in the same quarter, but it is surely dependent on current economic activity. Bond yield spread difference is easily affected by current shocks as it is determined by well-developed UK financial market, and GDP acceleration is causally prior to other variables in the system.

IRF

It is rather difficult to interpret effects of differences on differences but we try to make this as clear as possible. Also we admit that we could have captured the true data-generating process, which was impossible without differencing and logarithm transformation that we used.



Each shock of difference has to be interpreted in the following way: "If the variable X grew by z units in the previous period, then in the following quarter in will increase by z+1 units". In case of difference between logarithms we just use not absolute change but percentage change.

1. Influence of shocks in GDP acceleration

IRF demonstrates that shock in GDP acceleration increases GDP acceleration significantly, and the cumulative effect is roughly 0.012 during 8 quarters. An increase in GDP acceleration can be an indicator of good economic conditions, thus it improves GDP in some following quarters.

One unit shock in GDP acceleration leads to almost insignificant decrease in CPI during 8 quarters. This can be a result of inflation targeting policy of the UK central bank that would try to cool down price growth during the economic boom. Cumulative response of spread difference is significant. Both long-term and short-term rates react to economic boom due to higher demand in the debt market, but short-term rates are usually more sensitive.

2. Influence of shocks in logarithm of CPI.

Sharp growth in CPI leads to a rise in CPI in the next quarter. However, after the 5th quarter influence decreases which can be a sign of efficient inflation targeting policy.

Effect of CPI shock on spread movement is insignificant as both short-term and long-term rates, evidently, react to price level shock quite similarly. Effect on GDP acceleration is significant and exists because a considerable rise in prices reduces economic activity.

3. Influence of shocks in government bond yield spread difference.

In this case spread difference increases in response considerably, perhaps, because of growing expectations about long-term rates implied in yield curve. The cumulative effect converges to roughly 0.8, so such shock increases spread by 0.8 p.p. in general.

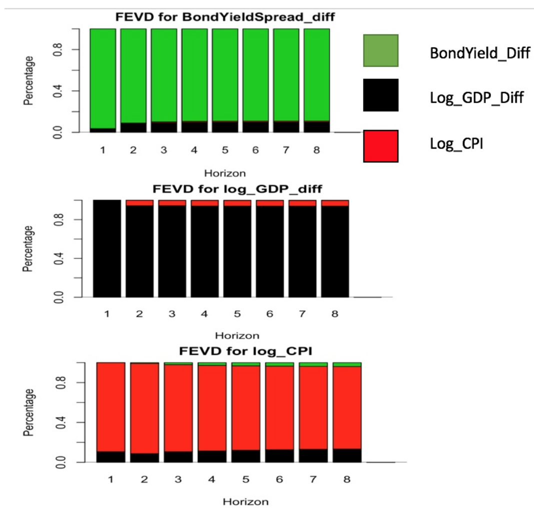
This shock does not affect significantly CPI as the confidence interval intersects 0. Again, this result can be explained by a strong inflation control in the UK. This shock has an insignificant effect on GDP acceleration according to our SVAR. Increase in yield spread is often a sign of recession, but it is not obvious that it can be a reason of slowing down GDP acceleration.

FEVD

Forecast error of difference between quarterly yield spreads in long run (8 quarters) is almost entirely due to its shocks. However, there is a marginal effect of GDP acceleration shock as it surely somehow affects debt market.

In long-run forecast error of quarterly GDP acceleration is explained in general by itself as well. But CPI shock has some influence because, perhaps, considerable deviation of prices from some targeted level is highly unexpected because the economy believes in inflation targeting.

Forecast of CPI is barely affected by bond market since inflation targeting policy is used. CPI is affected by GDP shocks, but this effect accelerates only in short-term when inflation targeting is difficult.



At h = 8 effect of	Forecasted Value at h = 8		
	GDP acceleration	Log CPI	Spread difference
GDP acceleration	0.94	0.134	0.03
Log CPI	0.058	0.828	0.007
Spread difference	0.002	0.038	0.963