Review of the book

103 Trigonometry Problems (From the Training of the USA IMO Team)

by Titu Andreescu and Zuming Feng,

Birkhäuser Boston, 2005, 214 p., Softcover, ISBN: 0-8176-4334-6

This book is not for professional mathematicians but rather it is aimed at students of mathematics, be they eager high school students or undergraduates, and those who teach them. It is an inspiring book that will give them an idea of how enchanting mathematics can be.

103 Trigonometry Problems (From the Training of the USA IMO Team) is written as a textbook to be used in advanced problem solving courses, or as a reference source for people interested in tackling challenging mathematical problems. The main authors' purpose is to stimulate young people to become interested in mathematics, to inspire and to challenge them, their parents and their teachers with the wonder, excitement, power, and relevance of mathematics. The book can serve as a text for problem solving courses or as a guide for individual study. It can be used by undergraduate students preparing for contests such as the William Lowell Putnam Mathematical Competition, and by advanced high school students studying for the American Mathematical Olympiad, regional competitions or the International Mathematical Olympiad. It can also be read without any course or contest in mind, just for the pleasure of working on some interesting problems.

This book is a very well written introduction to trigonometry and it contains a fine collection of excellent exercises ranging in difficulty from the fairly easy, to the more challenging. The problems, of the type that usually appear in the AIME, USAMO, IMO and the W. L. Putnam competition, and similar contests from other countries, are clustered in five chapters: *Trigonometric Fundamentals, Introductory Problems, Advanced Problems, Solutions to Introductory Problems, Solutions to Advanced Problems.* The first chapter is divided into 24 exciting sections, such as: *Area and Ptolemy's Theorem, Heron's Formula and Brahmagupta's Formula, Brocard points, The Cauchy-Schwarz Inequality, Constructing Sinusoidal Curves with a Straightedge, Traveling on Earth, Where Are You?*, etc. Each section is self-contained, independent of the others, and focuses on one main idea. All sections start with a short essay discussing basic facts, and one or more representative examples. Next, a number of carefully chosen problems are listed, to be solved by the reader. Many of them also illustrate significant mathematical ideas. The solutions to all problems are given in detail in the second part of the book. In the last section the authors have included a glossary of definitions and fundamental properties used in the book.

In this book, as is so often the case in mathematics, a little effort on the part of the reader will open a world of ideas. The book is so much more than an account of a few subjects on elementary mathematics. Anyone who has at least inkling that mathematics is important, interesting and beautiful will find the book inspiring, interesting and beautiful will find the book as very enjoyable.

A problem book review would be incomplete without the reviewer's favorite problems in the collection. I have chosen the following two problems.

1. *Let ABC be an acute triangle. Prove that*

$$(\sin 2B+\sin 2C)^2 \sin A+(\sin 2C+\sin 2A)^2 \sin B+(\sin 2A+\sin 2B)^2 \sin C \leq 12\, \sin A \sin B \sin C.$$

The authors give two different solutions to this problem. The first one relies on addition and subtraction formulas for trigonometric functions, while the second proof combines the extended law of sines with the following auxiliary result.

Lemma. *Let AD, BE, CF be the altitudes of an acute triangle ABC, with D, E, F on sides BC, CA, AB, respectively. Then*

$$|DE| + |DF| \leq |BC|.$$

Equality holds if and only if $|AB| = |BC|$.

2. *For any real number x and any positive integer n, prove that*

$$\left| \sum_{k=1}^{n} \frac{\sin kx}{k} \right| \leq 2\sqrt{\pi}.$$

The proof is based on Abel's inequality, an elementary identity, and the inequality

$$\left| \sum_{k=m+1}^{n} \frac{\sin kx}{k} \right| \leq \frac{1}{(m+1)|\sin \frac{x}{2}|},$$

where $x \in \mathbb{R} \setminus 2\pi\mathbb{Z}$ and m, n are positive integers with $m < n$.

This marvelous book is the fruit of the prodigious experience of the authors in preparing students for various mathematical competitions, which allowed them to present a nice collection of beautiful problems. I warmly recommend this book to all students curious about the force of the trigonometry, especially those who are bored at school and ready for a challenge. Teachers would find this book to be a welcome resource, as will contest organizers.

March 11, 2006

Prof. Vicentiu RADULESCU
Department of Mathematics
University of Craiova
200585 Craiova, Romania
E-mail: `radulescu@inf.ucv.ro, vicentiu.radulescu@math.cnrs.fr`
http://www.inf.ucv.ro/~radulescu