AwesomeMath Admission Test Cover Sheet

Your Name Last Name _____ First Name _____

Admission Test ☐ A ☐ B ☐ C Check one

Contact Information Phone Number _____

Please Print Email _____

Number of pages (not including this cover sheet) _____

AwesomeMath Test C
May 15-30, 2009

• Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.

• Include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.

• In this document you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer heet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.

• All the work you present must be your own.

• Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.

• Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.

• Postmark or submit your solutions by e-mail by Saturday, May 30, 2009.

• Make sure that the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be mailed to the following address:

Solutions are to be mailed to the following address:

Dr. Titu Andreescu
1721 Monaco Drive, Allen, TX 75002.

If you e-mail your solutions, please send them to

titu@awesomemath.org

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Admission Test C

1. Find the least odd positive number whose sum of digits is 2009.

2. What is the greatest integer n for which $n^2+2009n$ is the square of an integer?

3. Find the least positive integer having exactly 2009 divisors.

4. Prove that from the product $1!2!\cdots 120!$ we can delete one of the factors $k!$ such that the remaining product is a perfect square.

5. Find the number of ordered pairs (m,n) of positive integers for which
$$\frac{1}{m}+\frac{1}{n}=\frac{1}{2009}.$$

6. Let ABC be a triangle with $\angle A = 120^\circ$. Point P lies on the angle bisector of $\angle A$ such that $PA = AB + AC$. Prove that triangle PBC is equilateral.

7. Let n be a positive integer. Prove that $\underbrace{44\ldots4}_{2n}-\underbrace{88\ldots8}_{n}$ is a perfect square.

8. Let a,b,c be positive real numbers. Prove that
$$\frac{a}{a+2b}+\frac{b}{b+2c}+\frac{c}{c+2a}\geq 1.$$

9. Write 2009^{2010} as sum of six distinct perfect squares.

10. Let $ABCD$ be a quadrilateral inscribed in a semicircle of diameter $AD = x$. If $AB = a, BC = b, CD = c$, prove that
$$x^3-(a^2+b^2+c^2)x-2abc=0.$$

AwesomeMath Answer Sheet

Your Name

Problem Number Page of

Write neatly! Write all work inside the box. Do NOT write on the back of the page