

AwesomeMath Admission Test Cover Sheet

Your name (please print) _____
Last *First*

Admission Test A/B/C (circle one)

Contact Information _____ (phone number)

(please print) _____ (email address)

Number of pages (not including this cover sheet) _____

Admission Test A

- Don't be discouraged if you can't solve many of the questions: the test isn't made to be easy. We want to see the solutions you do come up with no matter how many you solve.
- You should include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.
- In this document, you will find a cover sheet and an answer sheet. Print out each one, and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you're finished. **Start each problem on a new answer sheet.**
- **All the work you present must be your own.**
- **Don't be intimidated!** Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.
- **Be patient and persistent!** Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.
- **You must postmark or submit your solutions by e-mail by Monday, February 12, 2007.** Make sure that the cover sheet is the first page of your submission, and that it is completely filled out.

Solutions are to be mailed to the following address:

Dr. Titu Andreescu
1721 Monaco Drive, Allen, TX 75002.

If you e-mail your solutions, please send them to

titu@awesomemath.org

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file, and send just the one file.

Please go the next page for the problems.

Admission Test A

1. In a *magic square*, the sum of the three entries in each row, column, or diagonal has the same value. The figure shows four of the entries of a *magic square*. What is x ?

		3
x	4	5

2. Find the least positive integer whose product of digits is $10!$.

3. The numbers d_1, d_2, \dots, d_6 are distinct digits of the decimal system and are different from 6. Prove that $d_1 + d_2 + \dots + d_6 = 36$ if and only if $(d_1 - 6)(d_2 - 6) \cdot \dots \cdot (d_6 - 6) = -36$.

4. In the standard 8×8 chessboard (with squares colored alternatively black and white) there are 64 1×1 squares, 49 2×2 squares, etc. How many of those squares have more than half of their areas colored black?

5. In a chess (round-robin) tournament, five participants withdrew after having played two games each. If 100 games were played in all, what was the initial number of participants?

6. Find all quadruples (x, y, z, w) of positive integers such that

$$x^2 + y^2 + z^2 + w^2 = 3(x + y + z + w).$$

7. The www.awesomemath.org homepage displays the triangle "Forum", the equilateral triangles "Summer Program", "Year-round" and "Math Reflections", as well as three "white" lines that appear to pass through the same point. Prove that these lines are indeed concurrent.

8. The average age of the 2006 AwesomeMath Summer Program participants (students and assistants) would have increased by one month either if three additional 18 year olds had enrolled or if three 12 year olds had not participated. What was the number of participants?

9. Find the least positive integer n such that each n -element subset of the set $\{1, 2, \dots, 2007\}$ contains two elements, not necessarily distinct, such that their sum is a power of 2.

10. Let I be the incenter of triangle ABC and let the line passing through I and perpendicular to AI intersect BC at A' . Points B' and C' are defined similarly. Prove that A', B', C' lie on a line perpendicular to OI , where O is the circumcenter of triangle ABC .

Suggestions for Writing Proofs

(By Tiankai Liu)

- All proofs should be written neatly and coherently in paragraphs of standard American English. Mathematical symbols like \equiv and \leq should be used only in equations, not as verbs or prepositions in a sentence. Do not write things like “all of the \triangle ’s angles are $\leq 90^\circ$ ”—this should be “ $\angle A, \angle B, \angle C \leq 90^\circ$ ” or “all of the triangle’s angles are at most 90° .” Avoid the symbols $\wedge \vee \therefore \because \forall \exists$; instead, write out “and,” “or,” “because,” “therefore,” “for all,” “there exists.” Similarly, do not use \implies or \Rightarrow except as part of a sequence of equations.
- Write a statement using words rather than symbols unless this would be unnecessarily awkward. Do not invent more notation than is necessary to explain your solution.
- Write true statements. Do not write something that is only partially true, and then say how to fix it later. If you assume something in one of your statements, say clearly what you are assuming. Define all terms you make up. If you use figures, graphs, tables, etc., explain thoroughly what they represent.
- Use the following formats for common proof patterns:
 - *Proof by induction:* (1) state the claim, (2) check the base case, (3) prove the induction step, and (4) conclude with the words “induction is complete.”
 - *Proof by contradiction:* (1) state the claim, (2) state that you are assuming the opposite, (3) derive a contradiction (and say why it is a contradiction), and (4) conclude that the claim follows.
 - *Proof by case analysis:* (1) state the claim, (2) state the various cases, (3) say why they exhaust all possibilities, (4) analyze the cases one by one, and (5) conclude that the claim follows.
- Draw accurate diagrams with compass and straightedge for geometry problems. This is for your own good as well as the graders’.

AwesomeMath Admission Test Answer Sheet

Your name (please print) _____

Admission Test A/B/C (circle one) Problem Number ____ Page ____ of ____

Write neatly! Write all work inside the box. Do NOT write on the back of the page.