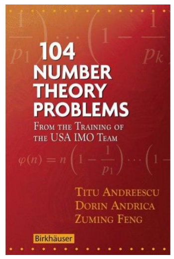
Review of the book

104 Number Theory Problems from the Training of the USA IMO Team

by Titu Andreescu, Dorin Andrica, and Zuming Feng,
Birkhäuser, 2007, 204 p., Softcover, ISBN: 978-0-8176-4527-4



Regarding to number theory, the famous British mathematician Godfrey Harold Hardy (1877-1947) wrote in his essay *A Mathematician's Apology*: "...there is one science [number theory] whose very remoteness from ordinary human activities should keep it gentle and clean". I would say that, indeed, the book under review is organized around this phrase, even if, due to its content, it is not entirely inspired from "ordinary human activities". At the same time, it is addressed to the most "gentle and clean" human spirit because it is written for advanced high school students, undergraduates, mathematics majors, instructors, mathematics coaches, but also for researchers in pure mathematics.

This book contains problems developed for various mathematical contests and it is written as a textbook to be used either in advanced problem solving courses or as a reference source for people interested in tackling challenging mathematical problems. *104 Number Theory Problems From the Training of the USA IMO Team* is also intended for undergraduate students preparing for difficult contests, such as the International Mathematical Olympiad, the William Lowell Putnam Mathematical Competition, the American Mathematical Olympiad, as well as various (difficult!) regional or national mathematical competitions for high-school students.

The book is a nice introduction to elementary number theory and it contains a fine collection of excellent exercises ranging in difficulty from the fairly easy, to very challenging. The problems are clustered in three chapters: *Number Theory Foundations, Introductory Problems, and Advanced Problems*. The next two contain complete solutions to *Introductory Problems*, and *Advanced Problems*, respectively. Most of the problems are of an elementary and combinatorial nature. Many seem to be understood without too much effort, but solving them is a different matter; indeed, some of the problems are hopelessly difficult, including a few notorious ones, such as the Erdös-Suranyi representation theorem (Problem 39, Chapter 3). A complete solution to an original problem often generates yet more related problems. The first chapter is divided into 23 sections, such as: *The Fundamental Theorem of Arithmetic, Bezout's Identity, Modular Arithmetic, Euler's Totient Function, Linear Diophantine Equations, Legendre Function, Fermat's Numbers, Mersenne's Numbers*, etc. Challenging introductory and advanced problems are listed in Chapters 2 and 3. Many of these problems illustrate significant mathematical ideas. The solutions to all problems are given in detail in the last two chapters. The book also contains a glossary of definitions and fundamental properties used in the text, as well as suggestions for further reading.

The review would be incomplete without a sample problem. I choose, quite subjectively, the following one.

(Problem 29, Chapter 2.) Knowing that 2^{29} is a nine-digit number all whose digits are distinct, without computing the actual number determine which of the ten digits is missing.

The authors give the following solution to this problem. It is observed that $2^3 \equiv -1 \pmod 9$, hence $2^{29} \equiv (2^3)^9 \cdot 2^2 \equiv -4 \equiv 5 \pmod 9$. The ten digit number containing all digits 0 through 9 is a multiple of 9, because the sum of its digits has this property. So, in our nine-digit number, 4 is missing. (Indeed, a straightforward computation shows that $2^{29} = 536870912$)

This marvelous work is the fruit of the long experience of the authors in preparing students for various mathematical competitions, which allowed them to present such a nice collection of beautiful problems. The reviewer is convinced that every adept of number theory who works through the problems of this book will gain large benefits. Last but not least, let us recall the deep words of the celebrated mathematician André Weil (1906-1998), a founding member of the Bourbaki group: "In order to become proficient in mathematics, or in any subject, the student must realize that most topics involve only a small number of basic ideas."

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