



# Contents

<b>1</b>	<b>Some Useful Substitutions</b>	<b>1</b>
1.1	Theory and examples . . . . .	3
1.2	Problems for training . . . . .	20
<b>2</b>	<b>Always Cauchy-Schwarz...</b>	<b>25</b>
2.1	Theory and examples . . . . .	27
2.2	Problems for training . . . . .	43
<b>3</b>	<b>Look at the Exponent</b>	<b>47</b>
3.1	Theory and examples . . . . .	49
3.2	Problems for training . . . . .	67
<b>4</b>	<b>Primes and Squares</b>	<b>73</b>
4.1	Theory and examples . . . . .	75
4.2	Problems for training . . . . .	89
<b>5</b>	<b>T2's Lemma</b>	<b>93</b>
5.1	Theory and examples . . . . .	95
5.2	Problems for training . . . . .	111

<b>6</b>	<b>Some Classical Problems in Extremal Graph Theory</b>	115
6.1	Theory and examples . . . . .	117
6.2	Problems for training . . . . .	128
<b>7</b>	<b>Complex Combinatorics</b>	131
7.1	Theory and examples . . . . .	133
7.2	Problems for training . . . . .	148
<b>8</b>	<b>Formal Series Revisited</b>	153
8.1	Theory and examples . . . . .	155
8.2	Problems for training . . . . .	173
<b>9</b>	<b>A Brief Introduction to Algebraic Number Theory</b>	179
9.1	Theory and examples . . . . .	181
9.2	Problems for training . . . . .	200
<b>10</b>	<b>Arithmetic Properties of Polynomials</b>	205
10.1	Theory and examples . . . . .	207
10.2	Problems for training . . . . .	227
<b>11</b>	<b>Lagrange Interpolation Formula</b>	233
11.1	Theory and examples . . . . .	235
11.2	Problems for training . . . . .	259
<b>12</b>	<b>Higher Algebra in Combinatorics</b>	263
12.1	Theory and examples . . . . .	265
12.2	Problems for training . . . . .	282
<b>13</b>	<b>Geometry and Numbers</b>	289
13.1	Theory and examples . . . . .	291
13.2	Problems for training . . . . .	309
<b>14</b>	<b>The Smaller, the Better</b>	313
14.1	Theory and examples . . . . .	315
14.2	Problems for training . . . . .	327

<b>15 Density and Regular Distribution</b>	333
15.1 Theory and examples . . . . .	335
15.2 Problems for training . . . . .	350
<b>16 The Digit Sum of a Positive Integer</b>	353
16.1 Theory and examples . . . . .	355
16.2 Problems for training . . . . .	369
<b>17 At the Border of Analysis and Number Theory</b>	375
17.1 Theory and examples . . . . .	377
17.2 Problems for training . . . . .	394
<b>18 Quadratic Reciprocity</b>	399
18.1 Theory and examples . . . . .	401
18.2 Problems for training . . . . .	419
<b>19 Solving Elementary Inequalities Using Integrals</b>	425
19.1 Theory and examples . . . . .	427
19.2 Problems for training . . . . .	445
<b>20 Pigeonhole Principle Revisited</b>	451
20.1 Theory and examples . . . . .	453
20.2 Problems for training . . . . .	473
<b>21 Some Useful Irreducibility Criteria</b>	479
21.1 Theory and examples . . . . .	481
21.2 Problems for training . . . . .	501
<b>22 Cycles, Paths, and Other Ways</b>	505
22.1 Theory and examples . . . . .	507
22.2 Problems for training . . . . .	519
<b>23 Some Special Applications of Polynomials</b>	523
23.1 Theory and examples . . . . .	525
23.2 Problems for training . . . . .	543