AwesomeMath
making x,y,z as easy as a,b,c

AwesomeMath Admission Test Cover Sheet

Your Name Last Name _____ First Name _____

Admission Test ☐ A ☐ B ☐ C Check one

Contact Information Phone Number _____

Please Print Email _____

Number of pages (not including this cover sheet) _____

• Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.

• Include all significant steps in you r reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.

• In this document , you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.

• All the work you present must be your own.

• Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.

• Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.

• Postmark or submit your solutions by e-mail by Monday, May 12, 2008.

• Make sure that the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be mailed to the following address:

Dr. Titu Andreescu
1721 Monaco Drive, Allen, TX 75002

If you e-mail your solutions, please send them to

titu@awesomemath.org

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

Admission Test C

May 2 - May 12, 2008

1. Write $1,000,000$ as a sum of a prime number and a perfect square.

2. Can the discriminant of a quadratic polynomial with integer coefficients be a) 2007? b) 2008?

3. Let n be an integer divisible by 4. Prove that n^2+2 can be written as $a^4+b^4+c^4+d^4-4abcd$ for some nonnegative integers a, b, c, and d.

4. Let ABC be a triangle with $AB = AC = 20$ and $BC = 24$. Point D lies on the small arc AB of its circumcircle such that $AD = 15$ and $BD = 7$. Prove that CD is a diameter of this circle.

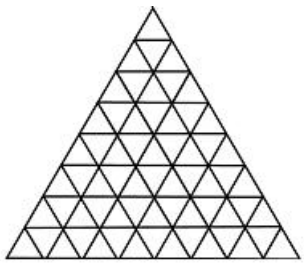
5. The odd positive integers are grouped as follows:

$$\{1\}, \{3,5\}, \{7,9,11\}, \{13,15,17,19\}, \ldots$$

Prove that the sum of the numbers in the n^{th} group is n^3.

6. Points M and N lie on a semicircle of diameter AB such that $AM - BM = 3$ and $AN - BN = 7$. Let P be the point of intersection of AN and BM. Evaluate area AMP − area BNP.

7. How many regular hexagons appear in the figure below?



8. Let r be a positive real number such that $\sqrt[4]{r} - \frac{1}{\sqrt[4]{r}} = 14$. Prove that

$$\sqrt[6]{r} + \frac{1}{\sqrt[6]{r}} = 6.$$

9. Find the greatest n for which there are points $P_1, P_2, \ldots, P_n$ in the plane such that each triangle whose vertices are among $P_1, P_2, \ldots, P_n$ has a side less than 1 and a side greater than 1.

10. Find the least real number r such that for each triangle with sidelength a, b, c,

$$\frac{\max(a,b,c)}{\sqrt[3]{a^3+b^3+c^3+3abc}} < r.$$

AwesomeMath Answer Sheet

Your Name

Problem Number Page of

Write neatly! Write all work inside the box. Do NOT write on the back of the page