AwesomeMath
making x,y,z as easy as a,b,c

AwesomeMath Admission Test Cover Sheet

Your Name Last Name _____ First Name _____

Admission Test ☐ A ☐ B ☐ C Check one

Contact Information Phone Number _____

Please Print Email _____

Number of pages (not including this cover sheet) _____

• Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.

• Include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.

• In this document , you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer heet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.

• All the work you present must be your own.

• Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.

• Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.

• Postmark or submit your solutions by e-mail by Saturday, April 18, 2009.

• Make sure that the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be mailed to the following address:

Dr. Titu Andreescu
1721 Monaco Drive, Allen, TX 75002

If you e-mail your solutions, please send them to

titu@awesomemath.org

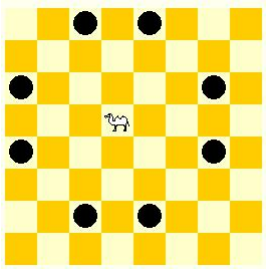
E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

Admission Test B

April 3 - 18, 2009

1. What is the angle formed by the hour and minute hands of an analog clock at 3:54 PM?
2. Find all positive integers n for which $\sqrt{\sqrt{n}+\sqrt{n+2009}}$ is an integer.
3. How many perfect cubes divide 2009^9?
4. Find the least positive integer whose square ends in 2009.
5. A simple magic square of order n is an arrangement of numbers $1, 2, \ldots, n^2$ in a square such that the n numbers in all rows, all columns, and both diagonals sum to the same constant. Find this constant.
6. Find all integers n for which there are distinct odd integers a, b, c such that $n + 2009 = a + b + c$ and $n + abc = ab + bc + ca$.
7. Let $ABCD$ be a parallelogram and let X and Y be the points outside $ABCD$ that make triangles BCX and CDY equilateral. Prove that triangle AXY is also equilateral.
8. In several variants of chess, a *camel* jumps in the opposite corner of a 2×4 rectangle:



   Prove that on an arbitrarily large chessboard a camel can return to its original position only after an even number of jumps.
9. Let $a_n = 2 - \frac{1}{n^2+\sqrt{n^4+\frac{1}{4}}}$, $n = 1, 2, \ldots$ Prove that $\sqrt{a_1} + \sqrt{a_2} + \cdots + \sqrt{a_{119}}$ is an integer.
10. There is a pile of 2009 chips on a table. You are allowed to perform repeatedly the following operation: choose a pile containing more than two chips, throw away a chip from the pile, then divide it into two smaller (not necessarily equal) piles. Is it possible that eventually all the piles on the table consist of exactly three chips?

AwesomeMath Answer Sheet

Your Name

Problem Number Page of

Write neatly! Write all work inside the box. Do NOT write on the back of the page