Triangle Bordered With Squares

Cătălin Barbu

Abstract

The triangle determined by the centers of the squares built outside on the sides of an acute triangle is called the exterior Vecten triangle. Similarly we define the interior Vecten triangle if the squares are built to the interior of the triangle. In this article we give properties for squares with sides equal to half of triangles's sides that are built with respect to the midpoints of the sides of the triangle.

Denote by $O_a^{'}, O_b^{'}, O_c^{'}$ the centers of the squares $A_bA_b^{'}A_c^{'}A_c$, $B_cB_c^{'}B_a^{'}B_a$, $C_aC_a^{'}C_b^{'}C_b$, $A_b, A_c \in [BC]$, $B_c, B_a \in [CA]$, $C_a, C_b \in [AB]$ constructed externally on the sides BC, CA, AB; by $O_a^{''}, O_b^{''}, O_c^{''}$ the centers of the squares $A_bA_b^{''}A_c^{''}A_c$, $B_cB_c^{''}B_a^{''}B_a$, $C_aC_a^{''}C_b^{''}C_b$ constructed internally; by M_a, M_b, M_c, $A_a^{'}$, $B_b^{'}$, $C_c^{'}$ the midpoints of the segments BC, CA, AB, $B_a^{'}C_a^{'}$, $C_b^{'}A_b^{'}$, $A_c^{'}B_c^{'}$; by $G_a^{'}$, $G_b^{'}$, $G_c^{'}$, $G_a^{''}$, $G_b^{''}$, $G_c^{''}$ the centroids of triangles $O_b^{'}M_aO_c^{'}$, $O_c^{'}M_bO_a^{'}$, $O_a^{'}M_cO_b^{'}$, $O_b^{''}M_aO_c^{''}$, $O_c^{''}M_bO_a^{''}$, $O_a^{''}M_cO_b^{''}$; by $G_1^{'}, G_2^{'}, G_3^{'}, G_1^{''}, G_2^{''}, G_3^{''}$ the centroids of triangles $O_a^{'}O_b^{''}O_c^{''}$, $O_b^{'}O_c^{''}O_a^{''}$, $O_c^{'}O_a^{''}O_b^{''}$, $O_a^{''}O_b^{'}O_c^{'}$, $O_b^{''}O_c^{'}O_a^{'}$, $O_c^{''}O_a^{'}O_b^{'}$, and $\{A^{'}\} = B_c^{'}B_a^{'} \cap C_a^{'}C_b^{'}$, $\{B^{'}\} = C_a^{'}C_b^{'} \cap A_b^{'}A_c^{'}$, $\{C^{'}\} = A_b^{'}A_c^{'} \cap B_c^{'}B_a^{'}$.

Let O be the circumcenter of triangle ABC. Consider a complex plane with origin at O. Denote by the corresponding lowercase letter the coordinate of a point denoted by an uppercase letter. Then

$$m_a = \frac{b+c}{2}, a_b = \frac{3b+c}{4}, a_c = \frac{3c+b}{4}.$$

Point $O_a^{'}$ is obtained from point A_b by a rotation of center M_a and angle $90°$. Hence

$$o_a^{'} = m_a + i(a_b - m_a) = \frac{b+c}{2} + i \cdot \frac{b-c}{4}.$$

Likewise,

$$o_b^{'} = \frac{c+a}{2} + i \cdot \frac{c-a}{4}, o_c^{'} = \frac{a+b}{2} i \cdot \frac{a-b}{4}, \tag{1}$$

$$o_a^{''} = \frac{b+c}{2} - i \cdot \frac{b-c}{4}, o_b^{''} = \frac{c+a}{2} - i \cdot \frac{c-a}{4}, o_c^{''} = \frac{a+b}{2} - i \cdot \frac{a-b}{4}, \tag{2}$$

$$a_b^{'} = \frac{3b+c}{4} + i \cdot \frac{b-c}{2}, a_c^{'} = \frac{3c+b}{4} + i \cdot \frac{b-c}{2} b_c^{'} = \frac{3c+a}{4} + i \cdot \frac{c-a}{2},$$

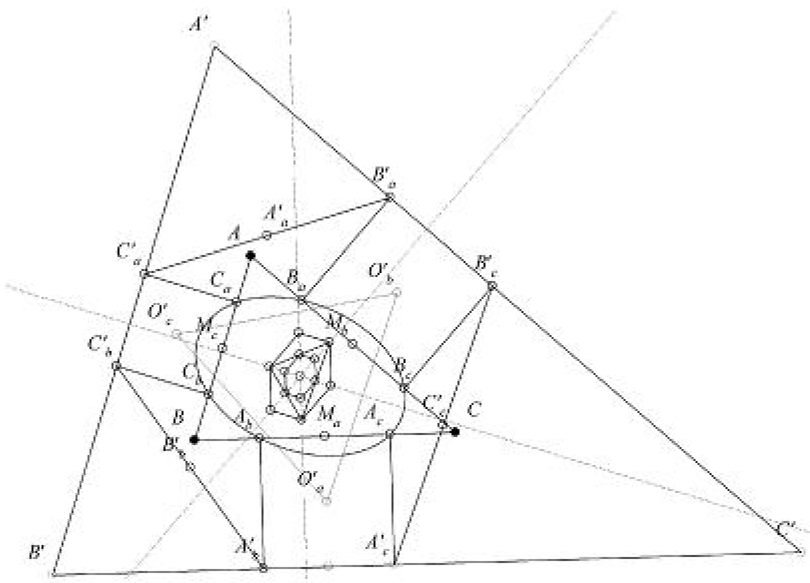
$$b_a^{'} = \frac{3a+c}{4} + i \cdot \frac{c-a}{2}, c_a^{'} = \frac{3a+b}{4} + i \cdot \frac{a-b}{2}, c_b^{'} = \frac{3b+a}{4} + i \cdot \frac{a-b}{2} \tag{3}$$

$$a_b^{''} = \frac{3b+c}{4} - i \cdot \frac{b-c}{2}, a_c^{''} = \frac{3c+b}{4} - i \cdot \frac{b-c}{2}, b_c^{''} = \frac{3c+a}{4} - i \cdot \frac{c-a}{2},$$

$$b_a^{''} = \frac{3a+c}{4} - i \cdot \frac{c-a}{2}, c_a^{''} = \frac{3a+b}{4} - i \cdot \frac{a-b}{2}, c_b^{''} = \frac{3b+a}{4} - i \cdot \frac{a-b}{2} \tag{4}$$

The coordinates of the midpoints $A_a^{'}, B_b^{'}, C_c^{'}$ of the segments $B_a^{'}C_a^{'}, C_b^{'}A_b^{'}, A_c^{'}B_c^{'}$ are

$$a_a^{'} = \frac{6a+b+c}{8} + i \cdot \frac{c-b}{4}, b_b^{'} = \frac{6b+c+a}{8} + i \cdot \frac{a-c}{4}, c_c^{'} = \frac{6c+a+b}{8} + i \cdot \frac{b-a}{4}. \qquad (5)$$



Theorem 1. *Lines $AO_a^{'}, BO_b^{'}$ and $CO_c^{'}$ are concurrent.*

Proof. Because triangles $BO_a^{'}C, CO_b^{'}A$ and $AO_c^{'}B$ are similar isosceles triangles, the concurrence follows from Kiepert's Theorem [1]. □

Theorem 2. *Lines $AO_a^{"}, BO_b^{"}$ and $CO_c^{"}$ are concurrent.*

Proof. The proof is similar to the previous one. □

Theorem 3. *Lines AA', BB' and CC' are concurrent.*

Proof. The proof follows from the fact that triangles ABC and $A'B'C'$ have parallel sides. □

Remark 1. *If the squares have the sides equal with the sides of the triangle, then lines $AO_a^{'}, BO_b^{'}$ and $CO_c^{'}$ are concurrent in the external Vecten point, lines $AO_a^{"}, BO_b^{"}$ and $CO_c^{"}$ are concurrent in the internal Vecten point, AA', BB' and CC' are concurrent in the point of Lemoine, the $A'B'C'$ triangle beeing Grebe's triangle of triangle ABC [1].*

Theorem 4. *Triangles $ABC, O_a^{'}O_b^{'}O_c^{'}, O_a^{"}O_b^{"}O_c^{"}, A_b^{'}B_c^{'}C_a^{'}, A_c^{'}B_a^{'}C_b^{'}, A_b^{"}B_c^{"}C_a^{"}, A_c^{"}B_a^{"}C_b^{"}$ have the same centroid G.*

Proof. Triangles $ABC, O_a^{'}O_b^{'}O_c^{'}, O_a^{"}O_b^{"}O_c^{"}, A_b^{'}B_c^{'}C_a^{'}, A_c^{'}B_a^{'}C_b^{'}, A_b^{"}B_c^{"}C_a^{"}, A_c^{"}B_a^{"}C_b^{"}$ have the same centroid because

$$\begin{aligned} \frac{o_a^{'}+o_b^{'}+o_c^{'}}{3} &= \frac{o_a^{"}+o_b^{"}+o_c^{"}}{3} = \frac{a_b^{'}+b_c^{'}+c_a^{'}}{3} = \frac{a_c^{'}+b_a^{'}+c_b^{'}}{3} = \\ \frac{a_b^{"}+b_c^{"}+c_a^{"}}{3} &= \frac{a_c^{"}+b_a^{"}+c_b^{"}}{3} = \frac{a+b+c}{3} = g. \end{aligned}$$

□

Theorem 5. *Lines $A_a^{'}O_a^{'}, B_b^{'}O_b^{'}$ and $C_c^{'}O_c^{'}$ are concurrent.*

Proof. The equations of lines $A_a^{'}O_a^{'}, B_b^{'}O_b^{'}$ and $C_c^{'}O_c^{'}$ are:

$$z\left(\overline{o_a^{'}}-\overline{a_a^{'}}\right)-\overline{z}\left(o_a^{'}-a_a^{'}\right)+o_a^{'}\overline{a_a^{'}}-a_a^{'}\overline{o_a^{'}}=0,$$

$$z\left(\overline{o_b^{'}}-\overline{b_b^{'}}\right)-\overline{z}\left(o_b^{'}-b_b^{'}\right)+o_b^{'}\overline{b_b^{'}}-b_b^{'}\overline{o_b^{'}}=0,$$

and

$$z\left(\overline{o_c^{'}}-\overline{c_c^{'}}\right)-\overline{z}\left(o_c^{'}-c_c^{'}\right)+o_c^{'}\overline{c_c^{'}}-c_c^{'}\overline{o_c^{'}}=0.$$

By adding up the previous equations we obtain

$$o_a^{'}\overline{a_a^{'}}-a_a^{'}\overline{o_a^{'}}+o_b^{'}\overline{b_b^{'}}-b_b^{'}\overline{o_b^{'}}+o_c^{'}\overline{c_c^{'}}-c_c^{'}\overline{o_c^{'}}=0.$$

The last relation is equivalent to

$$[\overline{a}(b+c)-a(\overline{b}+\overline{c})]+[\overline{b}(c+a)-b(\overline{c}+\overline{a})]+[\overline{c}(b+a)-c(\overline{b}+\overline{a})]=0,$$

which is clear. □

Theorem 6. *Lines $A_a^{'}O_a^{"}, B_b^{'}O_b^{"}$ and $C_c^{'}O_c^{"}$ are concurrent.*

Proof. The proof is similar with Theorem's 6 proof. □

Theorem 7. *Lines $A_a^{'}M_a, B_b^{'}M_b$ and $C_c^{'}M_c$ are concurrent.*

Proof. The proof is similar with Theorem's 6 proof. □

Theorem 8. *Lines $A^{'}O_a^{'}, B^{'}O_b^{'}$ and $C^{'}O_c^{'}$ are concurrent.*

Proof. The proof is similar with Theorem's 6 proof. □

Theorem 9. *Lines $A^{'}O_a^{"}, B^{'}O_b^{"}$ and $C^{'}O_c^{"}$ are concurrent.*

Proof. The proof is similar with Theorem's 6 proof. □

Theorem 10. *The following relations are true: $AO_a^{'}\perp B_a^{'}C_a^{'}$, $BO_b^{'}\perp A_b^{'}C_b^{'}$, $CO_c^{'}\perp A_c^{'}B_c^{'}$ and $AO_a^{'}=B_a^{'}C_a^{'}$, $BO_b^{'}=A_b^{'}C_b^{'}$, $CO_c^{'}=A_c^{'}B_c^{'}$.*

Proof. Because $b_a^{'} - c_a^{'} = i\left(\frac{b+c-2a}{2} + i\frac{b-c}{4}\right) = i(o_a^{'} - a)$, we have $AO_a^{'} \perp B_a^{'}C_a^{'}$ and $\left|b_a^{'} - c_a^{'}\right| = \left|i(o_a^{'} - a)\right| = \left|(o_a^{'} - a)\right|$, hence $AO_a^{'} = B_a^{'}C_a^{'}$. In the same way we demonstrate the other relations. $\square$

Theorem 11. *If by $A_{[XYZ]}$ we denote the area of triangle XYZ, then*

$$A_{[O_a'O_b'O_c']} = \frac{7}{16}A_{[ABC]} + \frac{1}{16}\left(3R^2 + \frac{BC^2 + CA^2 + AB^2}{2}\right).$$

Proof. Because

$$A_{[ABC]} = \frac{i}{4}\begin{vmatrix} a & \overline{a} & 1 \\ b & \overline{b} & 1 \\ c & \overline{b} & 1 \end{vmatrix} = \frac{i}{4}[b\overline{c} + c\overline{a} + a\overline{b} - a\overline{c} - b\overline{a} - c\overline{b}]$$

and

$$BC^2 + CA^2 + AB^2 = |c-b|^2 + |a-c|^2 + |b-a|^2 =$$
$$(c-b)(\overline{c} - \overline{b}) + (a-c)(\overline{a} - \overline{c}) + (b-a)(\overline{b} - \overline{a}) =$$
$$2[3R^2 - (b\overline{c} + c\overline{a} + a\overline{b} + a\overline{c} + b\overline{a} + c\overline{b})],$$

we have

$$b\overline{c} + c\overline{a} + a\overline{b} + a\overline{c} + b\overline{a} + c\overline{b} = 3R^2 - \frac{BC^2 + CA^2 + AB^2}{2}$$

(where $|a| = |b| = |c| = R$ is circumradius of triangle ABC). We obtain

$$A_{[O_a'O_b'O_c']} = \frac{i}{4}\begin{vmatrix} o_a^{'} & \overline{o_a^{'}} & 1 \\ o_b^{'} & \overline{o_b^{'}} & 1 \\ o_c^{'} & \overline{o_c^{'}} & 1 \end{vmatrix}$$

$$= \frac{i}{4} \cdot \frac{1}{16} \cdot [7(b\overline{c} + c\overline{a} + a\overline{b} - a\overline{c} - b\overline{a} - c\overline{b}) + 4i(b\overline{c} + c\overline{a} + a\overline{b} + a\overline{c} + b\overline{a} + c\overline{b}) - 24iR^2],$$

hence

$$A_{[O_a'O_b'O_c']} = \frac{1}{16}\left[7A_{[ABC]} - \left(3R^2 - \frac{BC^2 + CA^2 + AB^2}{2}\right) + 6R^2\right]$$
$$= \frac{1}{16}\left(7A_{[ABC]} + 3R^2 + \frac{BC^2 + CA^2 + AB^2}{2}\right).$$

$\square$

Theorem 12. *If by $A_{[XYZ]}$ we denote XYZ triangle's area, then*

$$A_{[O_a''O_b''O_c'']} = \frac{7}{16}A_{[ABC]} - \frac{1}{16}\left(3R^2 + \frac{BC^2 + CA^2 + AB^2}{2}\right).$$

Proof. The proof is similar to the previous one. $\square$

Theorem 13. *If $\{P_1\} = O_a^{'}O_c^{'} \cap BC$, $\{P_1^{'}\} = O_a^{'}O_b^{'} \cap BC$, $\{Q_1\} = O_b^{'}O_a^{'} \cap CA$, $\{Q_1^{'}\} = O_b^{'}O_c^{'} \cap CA$, $\{R_1\} = O_c^{'}O_b^{'} \cap AB$, $\{R_1^{'}\} = O_c^{'}O_a^{'} \cap AB$, then points $P_1, P_1^{'}, Q_1, Q_1^{'}, R_1$ and $R_1^{'}$ are on the same conic.*

Proof. Because ABC and $O_a^{'}O_b^{'}O_c^{'}$ are homological triangles (See Theorem 1), then by using Salmon's theorem [2] the conclusion follows. $\square$

Theorem 14. *If* $P_2\} = O_a^{"}O_c^{"} \cap BC$, $\{P_2^{'}\} = O_a^{"}O_b^{"} \cap BC$, $\{Q_2\} = O_b^{"}O_a^{"} \cap CA$, $\{Q_2^{'}\} = O_b^{"}O_c^{"} \cap CA$, $\{R_2\} = O_c^{"}O_b^{"} \cap AB$, $\{R_2^{'}\} = O_c^{"}O_a^{"} \cap AB$, *then points* $P_1, P_1^{'}, Q_1, Q_1^{'}, R_1$ *and* $R_2^{'}$ *are on the same conic.*

Proof. The proof is similar to the previous one. $\square$

Theorem 15. *The following relations are true:* $G_a^{'}G \perp BC, G_b^{'}G \perp CA, G_c^{'}G \perp AB$ *and* $BC = 12G_a^{'}G$, $CA = 12G_b^{'}G$, $AB = 12G_c^{'}G$.

Proof. We have $g_a^{'} = \frac{m_a + o_b^{'} + o_c^{'}}{3} = g + i \cdot \frac{c-b}{12}$, hence $\frac{g_a^{'} - g}{c-b} = i \cdot \frac{1}{12}$. Since $G_a^{'}G \perp BC$ and $\left|\frac{g_a^{'} - g}{c-b}\right| = \left|i \cdot \frac{1}{12}\right| = \frac{1}{12}$, we obtain $12G_a^{'}G = BC$. $\square$

Theorem 16. *The following relations are true:* $G_a^{"}G \perp BC, G_b^{"}G \perp CA, G_c^{"}G \perp AB$ *and* $BC = 12G_a^{"}G$, $CA = 12G_b^{"}G$, $AB = 12G_c^{"}G$.

Proof. We have $g_a^{"} = \frac{m_a + o_b^{"} + o_c^{"}}{3} = g - i \cdot \frac{c-b}{12}$, then $G_a^{"}G \perp BC$ and $12G_a^{"}G = BC$. $\square$

Corollary 1. *The centroid of the triangle* ABC *is the midpoint of the segments* $G_a^{'}G_a^{"}, G_b^{'}G_b^{"}$ *and* $G_c^{'}G_c^{"}$.

Theorem 17. *Triangles* $G_a^{'}G_b^{'}G_c^{'}, G_a^{"}G_b^{"}G_c^{"}$ *and* ABC *have the same centroid* G..

Proof. We have $\frac{g_a^{'} + g_b^{'} + g_c^{'}}{3} = \frac{g_a^{"} + g_b^{"} + g_c^{"}}{3} = \frac{a+b+c}{3}$, as desired. $\square$

Theorem 18. *The following relations are true:* $G_a^{'}G_b^{'} \perp AM_a, G_c^{'}G_a^{'} \perp BM_b, G_a^{'}G_b^{'} \perp CM_c$ *and* $AM_a = 6G_b^{'}G_c^{'}$, $BM_b = 6G_c^{'}G_a^{'}$, $CM_c = 6G_a^{'}G_b^{'}$.

Proof. We have $g_b^{'} - g_c^{'} = \frac{i}{6}(a - m_a)$, then $G_b^{'}G_c^{'} \perp AM_a$ and $AM_a = 6 \cdot G_b^{'}G_c^{'}$. $\square$

Theorem 19. *The following relations are true:* $G_b^{"}G_c^{"} \perp AM_a, G_c^{"}G_a^{"} \perp BM_b, G_a^{"}G_b^{"} \perp CM_c$ *and* $AM_a = 6G_b^{"}G_c^{"}$, $BM_b = 6G_c^{"}G_a^{"}$, $CM_c = 6G_a^{"}G_b^{"}$.

Proof. We have $g_b^{"} - g_c^{"} = -\frac{i}{6}(a - m_a)$, then $G_b^{"}G_c^{"} \perp AM_a$ and $AM_a = 6 \cdot G_b^{"}G_c^{"}$. $\square$

Remark 2. *The sides of triangles* $G_a^{'}G_b^{'}G_c^{'}$ *and* $G_a^{"}G_b^{"}G_c^{"}$ *have lenghts equal to one-sixth of the lengths if the medians of triangle* ABC*.The existence of triangles* $G_a^{'}G_b^{'}G_c^{'}$ *and* $G_a^{"}G_b^{"}G_c^{"}$ *implies, also, the existence of a medians triangle(a triangle that has sides of equal lenght with medians the lengths of the triangle* ABC*) [1].*

Corollary 2. *Quadrilaterals* $G_b^{'}G_c^{'}G_b^{"}G_c^{"}, G_c^{'}G_a^{'}G_c^{"}G_a^{"}, G_a^{'}G_b^{'}G_a^{"}G_b^{"}$ *are parallelograms.*

Proof. By Theorems 20 and 21 we get

$$G_b^{'}G_c^{'} \parallel G_b^{"}G_c^{"}, G_c^{'}G_a^{'} \parallel G_c^{"}G_a^{"}, G_a^{'}G_b^{'} \parallel G_a^{"}G_b^{"}$$

and

$$G_b^{'}G_c^{'} = G_b^{"}G_c^{"}, G_c^{'}G_a^{'} = G_c^{"}G_a^{"}, G_a^{'}G_b^{'} = G_a^{"}G_b^{"}.$$

$\square$

Corollary 3. *Triangles $G_a^{'}G_b^{'}G_c^{'}$ and $G_a^{"}G_b^{"}G_c^{"}$ are congruent.*

Theorem 20. *The pairs of triangles $G_a^{'}G_b^{'}G_c^{'}$ and ABC, $G_a^{"}G_b^{"}G_c^{"}$ and ABC are bilogic.*

Proof. The solutions results from Theorems 16-21, the common center of orthology being centroid of the triangle ABC. □

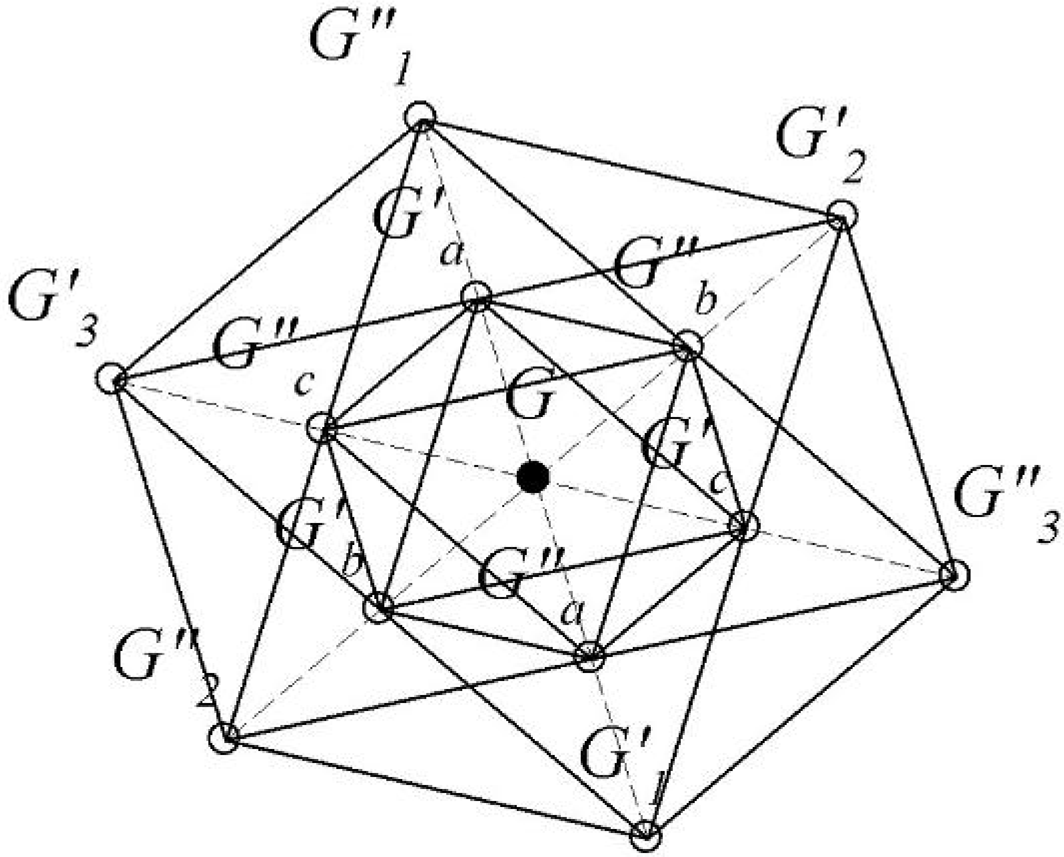
Theorem 21. *If by $A_{[XYZ]}$ we denote XYZ triangle's area, then $A_{[G_a^{'}G_b^{'}G_c^{'}]} = A_{[G_a^{"}G_b^{"}G_c^{"}]} = \frac{1}{48}A_{[ABC]}$.*

Proof. Triangle $G_a^{'}G_b^{'}G_c^{'}$ is similar with the medians triangle $M_1M_2M_3$, so $m(\widehat{G_a^{'}G_b^{'}G_c^{'}}) = m(\widehat{M_1M_2M_3}) = 180^\circ - m(\widehat{BGC})$. We have

$$A_{[G_a^{'}G_b^{'}G_c^{'}]} = \frac{G_a^{'}G_b^{'} \cdot G_a^{'}G_c^{'} \cdot \sin \widehat{G_a^{'}G_b^{'}G_c^{'}}}{2} = \frac{\frac{CM_c}{6} \cdot \frac{BM_b}{6} \cdot \sin(\widehat{BGC})}{2} =$$

$$\frac{1}{16}A_{[BGC]} = \frac{1}{16} \cdot \frac{1}{3}A_{[ABC]} = \frac{1}{48}A_{[ABC]}.$$

Triangles $G_a^{'}G_b^{'}G_c^{'}$ and $G_a^{"}G_b^{"}G_c^{"}$ are congruent, so $A_{[G_a^{'}G_b^{'}G_c^{'}]} = A_{[G_a^{"}G_b^{"}G_c^{"}]} = \frac{1}{48}A_{[ABC]}$. □



Theorem 22. *The following relations are true:* $G_1'G \perp BC, G_2'G \perp CA, G_3'G \perp AB$ *and* $BC = 6G_1'G$, $CA = 6G_2'G,\ AB = 6G_3'G$.

Proof. We have $g_1' = \frac{o_a' + o_b'' + o_c''}{3} = g + i \cdot \frac{b-c}{6}$, so $\frac{g_1' - g}{b-c} = \frac{i}{6}$. Because $G_1'G \perp BC$ and $\left| \frac{g_1' - g}{b-c} \right| = \left| \frac{i}{6} \right|$, we obtain $BC = 6G_1'G$. □

Theorem 23. *The following relations are true:* $G_1''G \perp BC, G_2''G \perp CA, G_3''G \perp AB$ *and* $BC = 6G_1''G$, $CA = 6G_2''G,\ AB = 6G_3''G$.

Proof. The proof is similar to the one of the previous theorem. □

Theorem 24. *Triangles* $G_1'G_2'G_3'$, $G_1''G_2''G_3''$ *and* ABC *have the same centroid.*

Proof. We have $\frac{g_1' + g_2' + g_3'}{3} = \frac{g_1'' + g_2'' + g_3''}{3} = \frac{a+b+c}{3} = g$, as desired □

Corollary 4. *The centroid of the triangle* ABC *is the midpoint of the segments* $G_a'G_a''$, $G_b'G_b''$, $G_c'G_c''$, $G_1'G_1''$, $G_2'G_2''$ *and* $G_3'G_3''$.

Corollary 5. *Points* $G_1'', G_a', G, G_a'', G_1'$ *are collinear and* $G_1''G_a' = G_a'G = GG_a'' = G_a''G_1'$.

Theorem 25. *Quadrilaterals* $G_1'G_3''G_2'G, G_1''G_3'GG_2'$ *and* $G_3'G_2''G_1'G$ *are parallelograms.*

Proof. Because $g + g_3'' = g_1' + g_2' = 2g_c'$, quadrilateral $G_1'G_3''G_2'G$ is a parallelogram. Similarly, quadrilaterals $G_1''G_3'GG_2'$ and $G_3'G_2''G_1'G$ are parallelograms. □

Corollary 6. *Hexagons* $G_a''G_c'G_b''G_a'G_c''G_b'$ *and* $G_1'G_3''G_2'G_1''G_3'G_2''$ *are homotetic, the center of homotethy being the centroid of triangle* ABC.

Corollary 7. *The points* $G_a', G_b', G_c', G_a'', G_b''$ *and* G_c'' *are same conic*

Proof. We have $G_a'G_c'' \parallel G_a''G_c', G_b'G_a'' \parallel G_b''G_a', G_c'G_b'' \parallel G_b''G_c'$. As two parallel lines have the same infinity it follows that the lines are intersected in the same point of the infinity. Then, the intersection points between the pair of parallel lines enumerated above belong to the infinite line and from Pascal's reverse theorem the conclusion follows. □

Corollary 8. *Points* $G_1', G_2', G_3', G_1'', G_2''$ *and* G_3'' *are on the same conic.*

References

[1] C. Barbu, Teoreme fundamentale din geometria triunghiului, Ed. Unique, Bacău, 2008.
[2] G. Salmon, Traité de géométrie analytique, Ed. Gauthier –Villars, Paris, 1903.
[3] T. Andreescu, D. Andrica, Complex Numbers from A to...Z, Birkhäuser, Boston, 2006.
[4] D. Andrica, K. Nguyen, A note on the Nagel and Gergonne points, Creative Math. & Inf., **17** (2008), 127-136.
[5] R. Musselman, The triangle bordered with squares, *American Mathematical Monthly*, **43** (1936), 539-548.
[6] J. Neuberg, Bibliographie du Triangle et du Tétračdre, *Mathesis*, **37** (1923), 289-293.

Cătălin Barbu
"Vasile Alecsandri" College,
Bacău, Romania
E-mail address: kafka_mate@yahoo.com