AwesomeMath
making x,y,z as easy as a,b,c

AwesomeMath Admission Test Cover Sheet

Your Name Last Name _____ First Name _____

Admission Test ☐ A ☐ B ☐ C Check one

Contact Information Phone Number _____

Please Print Email _____

Number of pages (not including this cover sheet) _____

Awesome Math Test C
April 29 - May 20, 2011

• Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.

• Include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.

• In this document, you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.

• All the work you present must be your own.

• Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.

• Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.

• Postmark or submit your solutions by e-mail (preferred) by Friday, May 20, 2011.

• Make sure that the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be mailed to the following address:

Dr. Titu Andreescu
3425 Neiman Road, Plano TX 75025

If you e-mail your solutions, please send them to

tandreescu@gmail.com

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

Admission Test C

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1. Find the greatest positive integer n with the following properties: none of its digits is 0 and, reading from left to right, each three consecutive digits of n form either a perfect square or a perfect cube.

2. What is the least number of the form $2011 + 2007 + 2003 + 1999 + \cdots$ that has exactly three divisors greater than 1 ?

3. Find all primes p and q such that both $pq - 2p$ and $pq + 2q$ are perfect squares.

4. Solve in positive integers the system of equations $\begin{cases} x - yz = 2 \\ xy - z = 23. \end{cases}$

5. What is the product of the real zeros of the polynomial $p(x) = x^4 + 4x^3 + 6x^2 + 4x - 2011$?

6. Find all pairs (x, y) for which $x! + y! + 3$ is a perfect cube.

7. If $x^2 + x\sqrt{5} + 1 = 0$, find the real number a such that $x^{10} + ax^5 + 1 = 0$.

8. Find all integers n for which both $n + 27$ and $8n + 27$ are perfect cubes.

9. Evaluate the sum
$$\sum_{n \geq 2} \frac{3n^2 - 1}{(n^3 - n)^2}.$$

10. Quadrilateral $ABCD$ is inscribed in a semicircle of diameter $AD = 2$. Prove that
$$AB^2 + BC^2 + CD^2 + AB \cdot BC \cdot CD = 4.$$

Dr. Titu Andreescu

AwesomeMath Answer Sheet

Your Name

Problem Number Page Of

Write neatly! All work should be inside the box. Do NOT write on the back of the page!