



**AwesomeMath**  
making x,y,z as easy as a,b,c

## AwesomeMath Admission Test Cover Sheet

Your Name

Last Name

First Name

Admission Test

☐

A

☐

B

☐

C

Check one

Contact Information

Phone Number

Please Print

Email

Number of pages (not including this cover sheet)

- Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.
- Include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.
- In this document, you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.
- All the work you present must be your own.
- Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.
- Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.
- Postmark or submit your solutions by e-mail (preferred) by Friday, April 8, 2011.
- Make sure that the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be mailed to the following address:

Dr. Titu Andreescu  
3425 Neiman Road, Plano TX 75025

If you e-mail your solutions, please send them to

tandreescu@gmail.com

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

## Admission Test B

March 18 - April 8, 2011

1. Find the greatest positive integer  $n$  with the following properties: none of its digits is zero and reading from left to right each three consecutive digits of  $n$  form a perfect square.
2. Find all integers that can be written as  $432 \times 0.ab5ab5ab5\dots$ , where  $a$  and  $b$  are different digits.
3. Find all primes  $p$  such that  $2011p = 2 + 3 + 4 + \dots + n$ , for some positive integer  $n$ .
4. A positive integer is called *descending* if in its decimal representation there are at least two digits and each digit is less than any digit to its left. How many descending positive integers are there?
5. A parallelogram has integer side-lengths and diagonals 40 and 42. What is its area?
6. Find all primes  $q_1, q_2, q_3, q_4, q_5$  such that  $q_1^4 + q_2^4 + q_3^4 + q_4^4 + q_5^4$  is the product of two consecutive even integers.
7. In triangle  $ABC$ ,  $\angle A = 30^\circ$  and  $\angle BMC = 45^\circ$ , where  $M$  is the midpoint of  $AB$ . Find  $\angle C$ .
8. Find all positive integers  $n$  for which the equation  $x^3 + y^3 = n! + 4$  has solutions in integers.
9. Our designer placed a geometric figure on the back page of the AwesomeMath brochure. My sixth grade son, an AMSP participant himself, asked me what was the problem having that picture as support. He did not like my "I don't know" answer and challenged me to write a problem based on the figure. After a few minutes, I came up with: Let  $A$  and  $B$  be points on a semicircle of center  $C$  and diameter  $MN$  such that  $AC \perp BC$ . The circumcircle of triangle  $ABC$  intersects  $MN$  for the second time at  $P$ . Prove that  $(AP - BP)^2 = 2CP^2$ .  
a) Justify this conclusion and b) (optional) Can you come up with a better problem inspired by that figure?
10. Let  $n$  be a positive integer such that  $2^n$  and  $5^n$  start with the same three digits. What are these digits?

Dr. Titu Andreescu

# AwesomeMath Answer Sheet

Your Name

Problem Number

Page

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Write neatly! All work should be inside the box. Do NOT write on the back of the page!