Review of the book
Complex Numbers from A to...Z
by Titu Andreescu and Dorin Andrica,
Birkhäuser, 2005, 321 p., Softcover, ISBN: 0-8176-4326-5

Mathematics is amazing not only in its power and beauty, but also in the way that it has applications in so many areas. The aim of this book is to stimulate young people to become interested in mathematics, to enthuse, inspire, and challenge them, their parents and their teachers with the wonder, excitement, power, and relevance of mathematics. This book is a very well written introduction to the fascinating theory of complex numbers and it contains a fine collection of excellent exercises ranging in difficulty from the fairly easy, if calculational, to the more challenging. The book is mainly devoted to complex numbers and to their wide applications in various fields, such as geometry, trigonometry or algebraic operations. An important feature of this marvelous book is that it presents a wide range of problems of all degrees of difficulties, but also that it includes easy proofs and natural generalizations of many theorems in elementary geometry. The authors show how to approach the solution of such problems, emphasizing the use of methods rather than the mere use of formulas. Of course, the more sophisticated the problems become, the more specific this approach has to be chosen.

The book is self-contained; no background in complex numbers is assumed and complete solutions to routine problems and to olympiad-caliber problems are presented in the last chapter of the book. The aim of the core part of each chapter is to develop key mathematical ideas and to place them in the context of novel, interesting, and unexpected applications to real-world problems. The first chapter deals with complex numbers in algebraic form and leads up to the geometric interpretations of the modulus and of the algebraic operations. The second chapter deals with various applications to trigonometry, starting with elementary facts on the polar representation of complex numbers and going up to more sophisticated properties related to nth roots of unity and their applications in solving binomial equations. Chapter 3 is devoted to the applications of complex numbers in solving problems in Plane and Analytic Geometry. This chapter includes a lot of interesting properties related to collinearity, orthogonality, concyclicity, similar triangles, as well as very useful analytic formulas for the geometry of a triangle and of a circle in the complex plane. Chapter 4 contains much more powerful results such as: the nine-point circle of Euler, some important distances in a triangle, barycentric coordinates, orthopolar triangles, Lagrange's theorem, geometric transformations in the complex plane. This chapter also includes a marvelous theorem known in the mathematical folklore under the name of "Morley's Miracle" and which simply states that *the three points of intersection of the adjacent trisectors of any triangle form an equilateral triangle*. As stated in the book, this theorem was mistakenly attributed to Napoleon Bonaparte. The proof of this theorem follows directly from Theorem 3 on page 155, a deep result which was obtained by the celebrated French mathematician Alain Connes (Fields Medal in 1982 and Clay Research Award in 2000), in connection with his revolutionary results in Noncommutative Geometry. Chapter 5 illustrates the force of the method of complex numbers in solving several Olympiad-caliber problems where this technique works very efficiently.

A problem book review would be incomplete without the reviewer's favorite problem in the collection. I have chosen a problem that is due to the Romanian mathematician Tzitzeica and which is known as "the five–coin problem":

Chapter 5, Problem 3, p. 192. *Three equal circles* $\mathcal{C}_1(O_1, r)$, $\mathcal{C}_2(O_2, r)$ *and* $\mathcal{C}_3(O_3, r)$ *have a common point* O. *Circles* $\mathcal{C}_1$ *and* $\mathcal{C}_2$, $\mathcal{C}_2$ *and* $\mathcal{C}_3$, $\mathcal{C}_3$ *and* $\mathcal{C}_1$, *meet again at points* A, B, C *respectively. Prove that the circumradius of triangle* ABC *is equal to* r.

This very successful book is the fruit of the prodigious activity of two well-known creators of mathematics problems in various mathematical journals. The vast experience of the authors in preparing students for various mathematical competitions allowed them to present a big collection of beautiful problems. This book continues the tradition making national and international mathematical competition problems available to a wider audience and is bound to appeal to anyone interested in mathematical problem solving. I very strongly recommend this book to all students curious about elementary mathematics, especially those who are bored at school and ready for a challenge. Teachers would find this book to be a welcome resource, as will contest organizers. This book is meant both to be read and to be used. All in all, an excellent book for its intended audience!

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