



AwesomeMath
making x,y,z as easy as a,b,c

AwesomeMath Admission Test Cover Sheet

Your Name

Last Name

First Name

Admission Test

☐

A

☐

B

☐

C

Check one

Contact Information

Phone Number

Please Print

Email

Number of pages (not including this cover sheet)

- Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.
- Include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.
- In this document, you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.
- All the work you present must be your own.
- Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.
- Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.
- Postmark or submit your solutions by e-mail (preferred) by Friday, Feb. 4, 2011.
- Make sure that the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be mailed to the following address:

Dr. Titu Andreescu
3425 Neiman Road, Plano TX 75025

If you e-mail your solutions, please send them to

tandreescu@gmail.com

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

Admission Test A

Jan. 14 - Feb. 4, 2011

1. Using each of the 10 digits exactly once, form three numbers such that their sum S is as small as possible. Find S .

2. Evaluate

$$\left(1 - \frac{2011}{2}\right) \left(1 - \frac{2011}{3}\right) \cdots \left(1 - \frac{2011}{2010}\right).$$

3. Find all primes p such that $2011p + 8$ is the product of two consecutive odd numbers.
4. How many positive integers less than 2011 are divisible by 5 and 6 but not divisible by 7 or 8?
5. Prove that among any five perfect squares there are two whose difference is divisible by 12.
6. Let ABC be a triangle with $\angle A = 90^\circ$ and let P be a point on the hypotenuse BC . Prove that

$$\frac{AB^2}{PC} + \frac{AC^2}{PB} \geq \frac{BC^3}{PA^2 + PB \cdot PC}.$$

7. Find all positive integers n for which $(n-2)! + (n+2)!$ is a perfect square.
8. Prove that any parallelogram can be dissected in 2011 cyclic quadrilaterals.
9. Are there distinct prime numbers p, q, r such that qr divides $p^2 + 11$, rp divides $q^2 + 11$, and pq divides $r^2 + 11$? What if 11 is replaced by 10?
10. Find all integers $n \geq 2$ for which $\sqrt[n]{3^n + 4^n + 5^n + 8^n + 10^n}$ is an integer.

AwesomeMath Answer Sheet

Your Name

Problem Number

Page

Of

Write neatly! All work should be inside the box. Do NOT write on the back of the page!