AwesomeMath
making x,y,z as easy as a,b,c

AwesomeMath Admission Test Cover Sheet

Your Name Last Name _____ First Name _____

Admission Test ☐ A ☐ B ☐ C Check one

Contact Information Phone Number _____

Please Print Email _____

Number of pages (not including this cover sheet) _____

• Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want t o see the solutions you come up with no matter how many problems you solve.

• Include al l significant steps in you r reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.

• In this document , you wil l find a cove r sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill o ut t he top of each answer heet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.

• All the work you present must be your own.

• Do not be intimidated! Some of the problems involve complex mathematical ideas, but all ca n b e solved using only elementary techniques, admittedly combined in clever ways.

• Be patient and persisntent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.

• Postmark or submit your solutions by e-mail by Monday, Apr 7, 2008.

• Make sure tha t the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be mailed to the following address:

Dr. Titu Andreescu
1721 Monaco Drive, Allen, TX 75002

If you e-mail your solutions, please send them to

titu@awesomemath.org

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

Admission Test B

Mar.28 - Apr.7, 2008

1. Construct a square using tiles of the form

2. Find all primes p such that $47p^2 + 1$ is a perfect square.

3. The numbers 1 through 10 are written on a board. By deleting some of them, the remaining numbers can be divided into two groups such that the product of the numbers in each group is the same. What is the least number of numbers that need to be deleted?

4. Find the least positive integer n such that for each prime p, $p^2 + n$ is not a prime.

5. Find the minimum of $x^4 + y^4 + z^4 - 4xyz$ over all real numbers x, y, z.

6. Al and Bo play the following game: there are 22 cards labeled 1 through 22. Al chooses one of them and places it on a table. Bo then places one of the remaining cards at the right of the one placed by Al such that the sum of the two numbers on the cards is a perfect square. Al then places one of the remaining cards such that the sum of the numbers on the last two cards played is a perfect square, and so on. The game ends when all the cards are played or no more cards can be placed on the table. The winner is the one who played the last card. Does Al have a winning strategy?

7. Find all triples (x, y, z) of positive integers such that $x^3 + y^3 + z^3 = 2008$.

8. In quadrilateral $ABCD$ the point of intersection of the perpendicular bisectors of AD and BC lies on AB. Prove that $AC = BD$ if and only if $\angle A = \angle B$.

9. In each square of an 8×8 board Michael writes a positive integer not exceeding 10, such that each two numbers that appear in adjacent or diagonally adjacent squares are relatively prime. Prove that some number appears at least 11 times.

10. Let $A_1A_2 \ldots A_{10}$ be a regular decagon. Find the number of obtuse triangles whose vertices are among $A_1, A_2, \ldots, A_{10}$.

AwesomeMath Answer Sheet

Your Name

Problem Number Page of

Write neatly! Write all work inside the box. Do NOT write on the back of the page