



AwesomeMath
making x,y,z as easy as a,b,c

AwesomeMath Admission Test Cover Sheet

Your Name

Last Name

First Name

Admission Test

☐

A

☐

B

☐

C

Check one

Contact Information

Phone Number

Please Print

Email

Number of pages (not including this cover sheet)

- Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.
- Include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.
- In this document, you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.
- All the work you present must be your own.
- Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.
- Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.
- Postmark or submit your solutions by e-mail by Monday, Feb 9, 2009.
- Make sure that the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be mailed to the following address:

Dr. Titu Andreescu
1721 Monaco Drive, Allen, TX 75002

If you e-mail your solutions, please send them to

titu@awesomemath.org

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

Admission Test A

Jan.30 - Feb.9, 2009

1. When $10!$ seconds have elapsed, how many weeks have gone by?
2. All digits of a number N are equal to 1. Prove that if 7 divides N , then so does 13.
3. From a 29×29 sheet can we cut a 39×2 rectangle?
4. What is the least 22-digit number divisible by 22 whose sum of digits is 22?
5. For a real number a , let $\lfloor a \rfloor$ and $\{a\}$ denote its integer part and fractional part, respectively. Find all x such that

$$\lfloor x \rfloor \cdot \{x\} = \left(\frac{2}{5}x\right)^2.$$

6. In triangle ABC , M is the midpoint of side BC . There is a point D on side AB for which CD intersects AM at E such that $DE = AD$. Prove that $CE = AB$.
7. Let $T_k = \frac{k(k+1)}{2}$, $k = 1, 2, 3, \dots$. Prove that there are infinitely many n for which T_n is divisible by the sum of its digits.
8. Let a and b be distinct real numbers. Prove that for each positive real number x ,

$$\frac{8x^2}{|a-b|} + \frac{a^2 + b^2}{x} \geq 6x.$$

9. Numbers 1 through 10 are written on a board. At any time, numbers a, b, c may be replaced by

$$\frac{2(b+c)-a}{3}, \frac{2(c+a)-b}{3}, \frac{2(a+b)-c}{3}.$$

Can a number greater than 20 eventually appear on the board?

10. Let $P(x) = 2009x^9 + a_1x^8 + \dots + a_9$ such that $P\left(\frac{1}{n}\right) = \frac{1}{n^3}$, $n = 1, 2, \dots, 9$.
Find $P\left(\frac{1}{10}\right)$.

AwesomeMath Answer Sheet

Your Name

Problem Number

Page

of

Write neatly! Write all work inside the box. Do NOT write on the back of the page