AwesomeMath
making x,y,z as easy as a,b,c

AwesomeMath Admission Test Cover Sheet

Your Name Last Name _____ First Name _____

Admission Test ☐ A ☐ B ☐ C Check one

Contact Information Phone Number _____

Please Print Email _____

Number of pages (not including this cover sheet) _____

Awesome Math Test A
January 15 - February 5, 2010

• Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.

• Include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.

• In this document, you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.

• All the work you present must be your own.

• Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.

• Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.

• Postmark or submit your solutions by e-mail by February 5, 2010.

• Make sure that the cover sheet is the first page of your submission and that it is completely filled out. Solutions are to be mailed to the following address:

Dr. Titu Andreescu
1721 Monaco Drive, Allen, TX 75002

If you e-mail your solutions, please send them to

titu@awesomemath.org

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

Admission Test A

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1. In the table

$$\begin{array}{ccccccc} 0 & & & & & & \\ 1 & 2 & 3 & & & & \\ 4 & 5 & 6 & 7 & 8 & & \\ 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ \multicolumn{7}{c}{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots} \end{array}$$

what is the number directly below 2010?

2. Consider n distinct positive integers whose arithmetic mean is less than n. Prove that there are at least two consecutive integers among the n considered.

3. Is the 2010-digit number 100...09 a prime?

4. (a) How many positive integers less than 1,000 are divisible by at least one of the numbers 3,4,5?
   (b) How many positive integers less then 1,000 contain at least one of the digits 3,4,5?

5. Alice notices that for her Social Security number ABC-DE-$FGHI$ the addition $ABC+DE=FGHI$ is correct, with $F\neq 0$. Find all such Social Security numbers not containing the digit 7 whose digits are all different.

6. Is there a perfect square whose last 10 digits are all different?

7. Let $a_0=1$ and $a_{n+1}=a_0\cdot\ldots\cdot a_n+3$ for $n\geq 0$. Prove that
$$a_n+\sqrt[3]{1-a_na_{n+1}}=1$$
for all $n\geq 1$.

8. The squares of a regular chessboard are randomly labeled 1 through 64. Prove that there are two adjacent squares such that the positive difference of their labels is at least 5.

9. Let a,b,c be the side lengths of a triangle. Prove that
$$0\leq\frac{a-b}{b+c}+\frac{b-c}{c+a}+\frac{c-a}{a+b}<1.$$

10. Let m and n be positive integers, $m<n$. Evaluate
$$\sum_{k=m+1}^{n}k(k^2-1^2)(k^2-2^2)\cdots(k^2-m^2).$$

AwesomeMath Answer Sheet

Your Name

Problem Number Page of

Write neatly! Write all work inside the box. Do NOT write on the back of the page