AwesomeMath Admission Test Cover Sheet

Your name (please print) _____ _____
Last *First*

Admission Test A/B/C (circle one)

Contact Information _____ (phone number)

(please print) _____ (email address)

Number of pages (not including this cover sheet) _____

AwesomeMath
May 4 - May 14, 2007

Admission Test C

- Don't be discouraged if you can't solve many of the questions: the test isn't made to be easy. We want to see the solutions you do come up with no matter how many you solve.
- You should include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.
- In this document, you will find a cover sheet and an answer sheet. Print out each one, and make several copies of the blank answer sheet. Fill out the top of each answer sheet as you go, and then fill out the cover sheet when you're finished. **Start each problem on a new answer sheet.**
- **All the work you present must be your own**.
- **Don't be intimidated!** Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.
- **Be patient and persistent!** Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.
- **You must postmark or submit your solutions by e-mail by Monday, May 14, 2007**. Make sure that the cover sheet is the first page of your submission, and that it is completely filled out.

  Solutions are to be mailed to the following address:

  Dr. Titu Andreescu
  1721 Monaco Drive, Allen, TX 75002.

  If you e-mail your solutions, please send them to

  titu@awesomemath.org

  E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file, and send just the one file.

Please go the next page for the problems.

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1. Find all integers n for which $4n + 9$ and $9n + 1$ are both perfect squares.

2. Time is displayed on an electronic clock from $00 : 00 : 00$ to $23 : 59 : 59$. How many times in 24 hours the display shows exactly four 4's?

3. Let $s_1, s_2, ..., s_{25}$ be the squares of some 25 consecutive integers. Prove that

$$\frac{s_1 + s_2 + ... + s_{25}}{25} - 52$$

is also the square of an integer.

4. Consider a cyclic quadrilateral $ABCD$. Let P be the point of intersection of its diagonals. Denote by A_1, B_1, C_1, D_1 the projections of P onto quadrilateral's sides. Prove that quadrilateral $A_1B_1C_1D_1$ is circumscribed about a circle.

5. Find all triples (x, y, z) of integers, solutions to the system of equations

$$\begin{cases} xy + z = 100 \\ x + yz = 101. \end{cases}$$

6. Let $ABCD$ be a trapezoid such that $AB \parallel CD$. Let P be the point of intersection of diagonals AC and BD. If $\text{area}_{PAB} = 16$ and $\text{area}_{PCD} = 25$, find area_{ABCD}.

7. An electronic board initially displays number 36. Each minute the number shown is multiplied or divided by either 2 or 3 and the new number is displayed. Can the number shown after one hour be 12?

8. Consider an equilateral triangle ABC and a point P on the small arc BC of its circumcircle. Let A' be the point of intersection of PA and BC. Prove that

$$\frac{1}{PA'} = \frac{1}{PB} + \frac{1}{PC}.$$

9. What is maximum number of knights you can place on a chessboard such that none is attacking another?

10. What is the least number of congruent triangles that dissect an 8×8 square with one of its 1×1 corners removed?

AwesomeMath Admission Test Answer Sheet

Your name (please print) _____

Admission Test A/B/C (circle one) Problem Number ___ Page ___ of ___

Write neatly! Write all work inside the box. Do NOT write on the back of the page.