AwesomeMath Admission Test Cover Sheet

Your Name Last Name _____ First Name _____

Admission Test ☐ A ☐ B ☐ C Check one

Contact Information Phone Number _____

Please Print Email _____

Number of pages (not including this cover sheet) _____

Awesome Math Test B
March 26 - April 16, 2010

• Do not be discouraged if you cannot solve all of the questions: the test is not made to be easy. We want to see the solutions you come up with no matter how many problems you solve.

• Include all significant steps in your reasoning and computation. We are interested in your ability to present your work, so unsupported answers will receive much less credit than well-reasoned progress towards a solution without a correct answer.

• In this document , you will find a cover sheet and an answer sheet. Print out each one and make several copies of the blank answer sheet. Fill out the top of each answer heet as you go, and then fill out the cover sheet when you are finished. Start each problem on a new answer sheet.

• All the work you present must be your own.

• Do not be intimidated! Some of the problems involve complex mathematical ideas, but all can be solved using only elementary techniques, admittedly combined in clever ways.

• Be patient and persistent. Learning comes more from struggling with problems than from solving them. Problem-solving becomes easier with experience. Success is not a function of cleverness alone.

• Postmark or submit your solutions by e-mail by April 16, 2010.

• Make sure that the cover sheet is the first page of your submission, and that it is completely filled out. Solutions are to be mailed to the following address:

Dr. Titu Andreescu
3425 Neiman Rd., Plano, TX 75025

If you e-mail your solutions, please send them to

tandreescu@gmail.com

E-mailed solutions may be written and scanned or typed in TeX. They should be sent as an attachment in either .doc or .pdf format. If you write and scan your solutions, insert the scans into a .doc or .pdf file and send just the one file.

Please go to the next page for the problems

Admission Test B

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1. If the numbers $4a-3$ and $4b-3$ add up to 2010, find the sum of the numbers $\frac{a}{3}-4$ and $\frac{b}{3}-4$.

2. Each of the arrays 1,2,3,4,5,6,7,8,9,10 and 11,12,13,14,15,16,17,18,19,20 contains four prime numbers. Such and array is called a *hard decade*. Find the sum of the numbers in the next hard decade.

3. (a) Find the largest prime p such that p^2 divides $2009!+2010!+2011!$
   (b) Find the second largest prime with this property.

4. Let $a \geq b \geq c \geq 0$. Prove that
$$(a-b+c)\left(\frac{1}{a}-\frac{1}{b}+\frac{1}{c}\right) \geq 1.$$

5. Find all integers n such that $n^2+2010n$ is a perfect square.

6. Find all n for which there are n consecutive integers whose sum of squares is a prime.

7. Find all pairs (x,y) of positive integers such that
$$x^2+y^2+33^2 = 2010\sqrt{x-y}.$$

8. In quadrilateral $ABCD$, diagonals AC and BD intersect at O. Denote by P,Q,R,S the orthogonal projections of O onto AB, BC, CD, DA, respectively. Prove that
$$PA\cdot AB + PC\cdot CD = \frac{1}{2}(AD^2+BC^2)$$
if and only if
$$PB\cdot BC + PD\cdot DA = \frac{1}{2}(AB^2+CD^2).$$

9. Find all triples (x,y,z) of real numbers such that
$$x^2+y^2+z^2+1 = xy+yz+zx+|x-2y+z|.$$

10. Consider a triangle ABC and a point P in its interior. Lines PA, PB, PC intersect BC, CA, AB at A', B', C', respectively. Prove that
$$\frac{BA'}{BC}+\frac{CB'}{CA}+\frac{AC'}{AB}=\frac{3}{2}$$
if and only if at least two of the triangles PAB, PBC, PCA have the same area.

AwesomeMath Answer Sheet

Your Name

Problem Number Page of

Write neatly! Write all work inside the box. Do NOT write on the back of the page