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Mixed Integer Linear Programming Model for Vehicle Routing Problem for Hazardous Materials Transportation ⋆

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Abstract: This paper presents a mathematical model to solve the Heterogeneous Vehicle Routing Problem (HVRP) in the context of hazardous materials (HazMat) transportation. To evaluate the model a linear approximation of the total routing risk is used as objective function. In the first stage a routing risk measure is proposed as a nonlinear function of the truck load. This function is approximated by means of two different piecewise linear functions (PLF). A genetic algorithm is employed to estimate the interval limits of PLF. These two functions are utilized to approximate the total routing risk for the best known solution for the benchmark instances of HVRP with fixed costs and unlimited fleet, both approaches are compared with the nonlinear risk function value. In the second stage the best piecewise linear approximation of the routing risk is integrated to a mixed integer linear programming (MILP) model for solving the risk optimization problem. The final model is tested on HVRP instances with 20 nodes. Results show that total cost minimization and total risk minimization appear to be conflicting objectives.

Keywords: Transportation science, Operations Research, Risk minimization

1. INTRODUCTION

Estimation and analysis of risks is an important aspect of hazardous materials (HazMat) transportation management. Selection of the safest routes is a main focus research in this area, Erkut et al. (2007). Routing in HazMat transportation are grouped in two categories: shortest path and vehicle routing, and scheduling problems. While the first type of routing problems has been substantially studied, scarce work has been found on the second type, see Androutsopoulos and Zografos (2012) and Pradhananga et al. (2014). The selection of a routing solution in HazMat transportation depends critically on the model adopted for the quantification of risk, Bell (2006). Therefore, a combined model considering both HazMat transportation risk and routing problem should be developed.

Tarantilis and Kiranoudis (2001) made one of the first studies that has explicitly considered HazMat transportation risk in vehicle routing problems. In this work the risk is defined as the number of people exposed to a HazMat transportation accident. The authors utilize an aggregated measure for computing the risk on an arc of the routing network. The traditional risk model is used in most of posterior studies, see Zografos and Androutsopoulos (2004), Zografos and Androutsopoulos (2008) and Pradhananga et al. (2010). The risk is computed as the product of the hazardous materials accident probability on an arc by the population exposed to the impacts of an accident on that arc. In the work of Androutsopoulos and Zografos (2010) both, the probability of accident and expected consequences arising in case of an accident occurring on an arc, are considered time dependent. In a later work, Androutsopoulos and Zografos (2012) considers that consequences expected radius of a hazardous materials accidents depends on the truck load, but the assessment of the computational performance of the solving algorithm was based on load-invariant risk values.

The volume of transported HazMat plays an important role in determining the likelihood of occurrence of an incident. In HazMat material distribution, the vehicle load is reduced by a quantity equals to the customer demand each time a client is visited, affecting the optimal path in risk minimization problems. Other important aspects to consider in HazMat transportation risk estimation are the probability of a truck tank accident, and the probability of a material release. Truck tank accident probability depends on the type of truck and the road conditions; meanwhile release probability depends on the nature of

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the accident, the type of HazMat, and the container conditions. In this context the probability of a HazMat incident depends on the load and type of the truck.

In this paper we focus on expected population exposure risk mitigation via selection of routes by solving a variant of heterogeneous vehicle routing problem (HVRP). The remainder of this paper is organized as follows. The next section provides a problem definition and presents the mixed integer linear programming (MILP) formulation of the HVRP in HazMat transportation. Section 3 presents the routing risk measure and it is formulated as a piecewise linear approximation. Section 4 presents the computational results of risk approximation and MIPL model application to the well-known benchmark instances of a HVRP variant. The final section concludes the paper and discusses future research issues.

2. PROBLEM DEFINITION

The vehicle routing problem for HazMat transportation using a heterogeneous fleet can be defined as the determination of the safest routes assigned to a fleet of different vehicles transporting a specific HazMat from a depot to a set of clients. This problem is characterized as the HVRP proposed by Golden et al. (1984) but with the introduction of the transportation risk objective function, that is expressed as the expected consequences of a hazardous materials accident.

To model the problem, a Mixed Integer Linear Programming (MILP) based on the proposal of Gheysens et al. (1984) and Baldacci et al. (2008) is proposed.

In this model the HVRP is defined on a complete directed graph $\mathcal{G}(\mathcal{N},\mathcal{L})$. The node set $\mathcal{N}=\{0,1,2,\ldots,n\}$ includes the depot node 0, and a set of customer nodes (service stations), $\mathcal{C}$. Each client $i\in\mathcal{C}$ has a demand d_i and it is connected with other node $j\in\mathcal{N}$ by an arc $(i,j)\in\mathcal{L}$. Each arc is characterized by a length al_{ij}, a cost c_{ij}, and a number of persons exposed to the consequences of a HazMat release PD_{ij}. To satisfy the demands there is a set of $\mathcal{K}$ different types of trucks. A truck type $k\in\mathcal{K}$ is characterized by a maximal capacity Q_k, a fixed cost f_k, and an accident rate $TTAR_k$.

A solution is composed of a set of routes $\mathcal{SR}$ satisfying all customers demands once. Each route $r\in\mathcal{SR}$ starts and ends at the depot, and respects the vehicle capacity Q_k. Split deliveries are not allowed.

Two types of decision variables are defined:

$$y_{ij}^k:\ \begin{array}{l}\text{flow of goods from node } i \text{ to node } j\\ \text{in a vehicle of type } k\end{array}$$

$$x_{ij}^k:\begin{cases}1 \text{ if a vehicle of type } k \text{ travels the arc } (i,j)\\ 0 \text{ otherwise}\end{cases}$$

The HVRP for HazMat transportation is formulated as follows:

$$z=\sum_{r\in\mathcal{SR}}R(r) \tag{1}$$

subject to:

$$\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{N}}x_{ij}^k=1,\ \forall j\in\mathcal{N}\setminus\{0\} \tag{2}$$

$$\sum_{i\in\mathcal{N}}x_{ij}^k-\sum_{i\in\mathcal{N}}x_{ji}^k=0,\ \forall k\in\mathcal{K},\forall j\in\mathcal{C} \tag{3}$$

$$\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{N}}y_{ij}^k-\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{N}}y_{ji}^k=d_j,\ \forall j\in\mathcal{C} \tag{4}$$

$$d_j\sum_{k\in\mathcal{K}}x_{ij}^k\le\sum_{k\in\mathcal{K}}y_{ij}^k\ \forall i,j\in\mathcal{N},\ i\neq j \tag{5}$$

$$y_{ij}^k\le x_{ij}^k(Q_k-d_i)\ \forall i,j\in\mathcal{N}\ \ i\neq j,\ \forall k\in\mathcal{K} \tag{6}$$

$$y_{ij}^k\ge 0,\ \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{L} \tag{7}$$

$$x_{ij}^k\in\{0,1\},\ \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{L} \tag{8}$$

Equation (1) expresses objective function for minimizing the total risk, being $R(r)$ the risk associated to route r. The set of constraints (2) ensures that each customer is visited exactly once, and the set (3) and (4) represents the conservation flux constraints. Additionally, (4) guarantees demands satisfaction. Constraints (5) and (6) state that no goods are transported from i to j if no vehicle is serving the arc (i,j), and (6) define the load of the vehicle k when traversing arc (i,j).

3. TRANSPORTATION RISK ASSESSMENT

In order to include the risk in the MILP proposed on section 2, a model for the route risk must be defined and after it must be linearized.

3.1 Route Risk Model

The proposed route risk model is based on the traditional one that uses the expected consequence as a measure of risk, Erkut et al. (2007). In this model population exposure is the consequence measure but other exposed receptors can be considered as the environment, or the properties in vicinity of the HazMat transportation incident.

Let r be a route composed of $(r_1,r_2),(r_2,r_3),\cdots,(r_{n-1},r_n)$ arcs, where r_i represents the client visited on $i-th$ position on the route. To evaluate the risk associated to a route r we use the total expected consequences expressed as:

$$R(r)=\sum_{u=1}^{n-1}\left[\prod_{v=1}^{u-1}(1-PI_{r_vr_{v+1}}^k)\right]PI_{r_ur_{u+1}}^kPD_{r_ur_{u+1}} \tag{9}$$

Where $PI_{r_ur_{u+1}}^k$ is the probability of occurrence of an incident using a type of vehicle k, and PDr_ur_{u+1} is the exposed population within a buffer distance surrounding the arc (r_ur_{u+1}), see Erkut et al. (2007). To estimate $R(r)$ we considered that $PI_{r_ur_{u+1}}^kPI_{r_vr_{v+1}}^k\cong 0$ for all pair of consecutive arcs $(r_ur_{u+1}),(r_vr_{v+1})$, given the fact that HazMat incident probability takes small values, Erkut et al. (2007). In consequence, (9) is reduced to:

$$R(r) = \sum_{u=1}^{n-1} PI^k_{r_u r_{u+1}} PD_{r_u r_{u+1}} = \sum_{(i,j)\in r} PI^k_{ij} PD_{ij} \quad (10)$$

To evaluate the incident probability PI^k_{ij} and consequence PD^k_{ij} it is necessary to consider that an accident occurs, generating a material release that could have several outcomes (jet-fire, pool fire, toxic cloud, explosion, etc.) affecting a population, as is presented on Fig. 1. All the events involved in a HazMat incident have an associated probability that is used to estimate the risk.

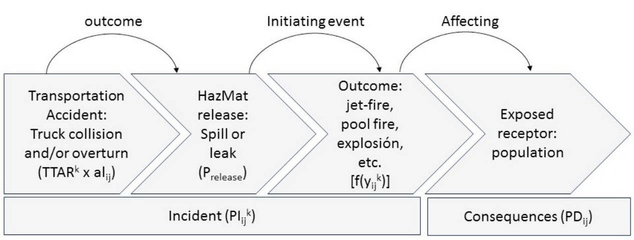

Fig. 1. Risk assessment in HazMat Transportation

PI^k_{ij} is computed by using (11) where, δ^k_{ij} is the incident rate for a truck type k, and al_{ij} is the length of arc (i, j).

$$PI^k_{ij} = \delta^k_{ij} \times al_{ij} \quad (11)$$

The incident rate per kilometer for a truck type k (δ^k_{ij}) is function of the incident probability given a truck accident on an arc (i, j) (IP^k_{ij}), and the truck tank accident rate ($TTAR^k$), that depends on the vehicle type, Button and Reilly (2000):

$$\delta^k_{ij} = TTAR^k \times IP^k_{ij} \quad (12)$$

Equation (13) defines IP^k_{ij} as the product of the release probability of HazMat in a truck accident ($P_{release}$) and the probability of a certain outcome arising as a consequence of the initiating event, Ronza et al. (2007).

$$IP_{ij} = P_{release} \times (\beta \times (y^k_{ij})^\alpha) \quad (13)$$

As mentioned above y^k_{ij} is the load of a truck type k traversing the arc (ij), while α and β are constant values that depend on the type of material. The release probability in a truck accident, $P_{release}$, is obtained from the combination of probabilities of accident outcomes with the probabilities of different types of accidents for a truck carrying a HazMat, Saccomanno et al. (1993), Button and Reilly (2000) and Kazantzi et al. (2011b). The probability of a certain consequence of the initiating event as fire or explosion is estimated using the model proposed by Ronza et al. (2007). This model is based on empirical approaches to predict ignition and explosion probabilities for land transportation spills as a function of the substance, the load, and the transportation mode.

The population exposure in a radius (buffer size) is defined depending on the type of HazMat transported and the maximum volume that can be carried on a truck.

Combining (10)-(13), the route risk for a given route r is estimated as:

$$\begin{aligned} R(r) = TTAR^k \times P_{release} \times \beta \times \\ \sum_{(i,j)\in r} (y^k_{ij})^\alpha \times (al_{ij} \times PD_{ij}) \end{aligned} \quad (14)$$

The objective risk function for the mathematical model becomes:

$$\begin{aligned} z = P_{release} \times \beta \times \\ \sum_{(i,j)\in\mathcal{L}} \sum_{k\in\mathcal{K}} TTAR^k \times (y^k_{ij})^\alpha \times al_{ij} \times PD_{ij} \end{aligned} \quad (15)$$

As this is a nonlinear function on, y^k_{ij}, a piecewise linear approximation is used.

3.2 Linear Approximation

Let $[q_0, q_M]$ be a bounded interval for y^k_{ij}, this interval is divided into an increasing sequence of M breakpoints $\{l_0, \dots, l_M\}$. The value of $(y^k_{ij})^\alpha$ is then approximated by using linear interpolations over the M segments according to (16).

$$\begin{aligned} (y^k_{ij})^\alpha := \{a_m + b_m y^k_{ij}, \\ y^k_{ij} \in [l_{m-1}, l_m]\ \forall m \in \{1, \dots, M\} \end{aligned} \quad (16)$$

where $a_m \in R$, $b_m \in R$ are the intercepts and the slopes of the linear functions, respectively, and $l_0 < l_1 < \cdots < l_M$. Two different kinds of piecewise linear approximations are performed: first looks for define an straight line joining two points on the function ($[l_{m-1}, l_m]$) and the second one looks for the tangent line to one point (tp_m) on the curve, see Fig. 2.

3.3 Genetic Algorithm for Piecewise Linear Approximation Parameters

Using a genetic algorithm (GA) the values of the limits of each interval are established. In both piecewise linear approaches, a GA is used for finding the solution that minimizes the total sum of squared errors.

A solution S is represented by an array composed of rp arrays of binary numbers of the same size lb. Each rp array encodes the position of a required point on the array of integer values, $q_i \in [q_0, q_M]$, that represents the different possible values of the truck load. On the first approach, these points correspond to the intervals limits and on the second one, they correspond to the tangent points, see Fig. 3. The chromosome for each individual of the initial population $P_{t=0}$ is randomly generated using a Bernoulli distribution with parameter $p = 0.5$.

The evaluation function is the sum of the squared errors of function approximation, see (17). For this, each linear function equation and the interval limits are computed.

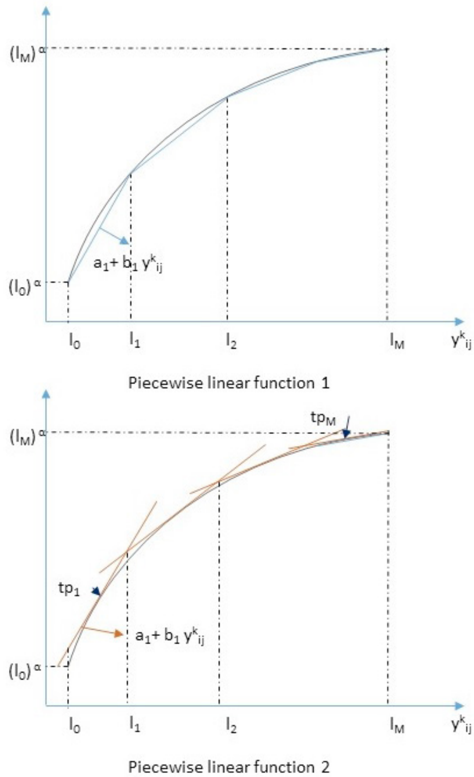

Fig. 2. Piecewise linear approximation functions

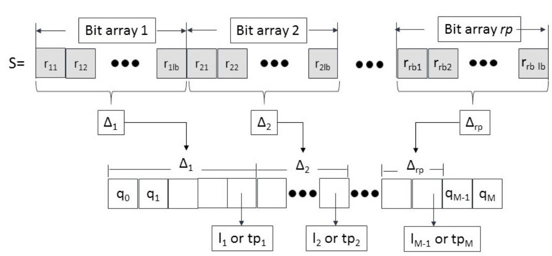

Fig. 3. Genetic Algorithm Solution Encoding

The fitness of a individual is defined as the inverse of the evaluation function value.

$$\sum_{\forall q_i \in [q_0, q_M]} [q_i^{\alpha} - (a_m + b_m \times q_i)]^2, \quad q_i \in [l_{m-1}, l_m] \ \forall m \in \{1, \ldots, M\} \tag{17}$$

Next generations P_{t+1} are produced thanks to recombination and mutation processes.

3.4 Picewise Linear Approximation Modelling

Let t_{ij}^k be the piecewise linear approximation value of $(y_{ij}^k)^{\alpha}$ and $t_0 = (l_0)^{\alpha}$. Then, the piecewise-linear functions of the road segment risk can be transformed into MILP model by introducing binary variables h_{ijk}^m and continuous variables λ_{ijk}^m, $m = 1, \cdots, M$. The h_{ijk}^m indicates the comparison between y_{ij}^k and l_{m-1}, and the λ_{ijk}^m variable evaluates the distance between y_{ij}^k and l_{m-1}, following Padberg (2000). The model for each $(y_{ij}^k)^{\alpha}$ is given as follows:

$$t_{ij}^k = t_0 + \sum_{m=1}^{M} b_m \lambda_{ijk}^m \ \forall i, j \in \mathcal{N} \ \ i \neq j, \ \forall k \in \mathcal{K} \tag{18}$$

$$y_{ij}^k = l_0 + \sum_{m=1}^{M} \lambda_{ijk}^m \ \forall i, j \in \mathcal{N} \ \ i \neq j, \ \forall k \in \mathcal{K} \tag{19}$$

$$\lambda_{ijk}^1 \leq l_1 - l_0 \ \forall i, j \in \mathcal{N} \ \ i \neq j, \ \forall k \in \mathcal{K} \tag{20}$$

$$\lambda_{ijk}^m \geq (l_m - l_{m-1}) h_{ijk}^m \ \forall i, j \in \mathcal{N} \ \ i \neq j, \ \forall k \in \mathcal{K} \quad m = 1, \cdots, M-1 \tag{21}$$

$$\lambda_{ijk}^{m+1} \leq (l_{m+1} - l_m) h_{ijk}^m \ \ \forall i, j \in \mathcal{N} \ \ i \neq j, \ \forall k \in \mathcal{K} \quad m = 1, \cdots, M-1 \tag{22}$$

$$\lambda_{ijk}^m \geq 0, \ \forall k \in \mathcal{K}, \forall (i,j) \in \mathcal{L}, \ m = 1, \cdots, M \tag{23}$$

$$h_{ijk}^m \ \in \{0, 1\}, \ \forall k \in \mathcal{K}, \forall (i,j) \in \mathcal{L}, \ m = 1, \cdots, M \tag{24}$$

$$t_{ij}^k \geq 0, \ \forall k \in \mathcal{K}, \forall (i,j) \in \mathcal{L} \tag{25}$$

total risk function for a set of routes is now defined as:

$$z = P_{release} \times \beta \times \sum_{(i,j) \in \mathcal{L}} \sum_{k \in \mathcal{K}} TTAR^k \times al_{ij} \times PD_{ij} \times t_{ij}^k \tag{26}$$

Equations (18)-(25) are included as constraints into the model defined in subsection 2.1.

4. NUMERICAL TESTS

4.1 Problem Instances

To assess the quality of the approximation risk function, we computed the total risk of best known solution routes for HVRP instances proposed by Golden et al. (1984). [1]

In this work we assumed that the nodes demand is expressed in hundreds of gallons of gasoline, gasoline density is $2.805 Kg/Gallon$, and one unit of distance is equal of $100m$. A release probability equals to 0.02487845, from Kazantzi et al. (2011a) and Button and Reilly (2000), and values of 0.72 and 0.00027 for α and β are used, see Ronza et al. (2007). The risk objective (26) becomes:

$$z = 0.02487845 \times 0.0027 \times \sum_{(i,j) \in \mathcal{L}} \sum_{k \in \mathcal{K}} TTAR^k \times \frac{al_{ij} \times PD_{ij}}{10} \times t_{ij}^k \tag{27}$$

The exposed population surrounding an arc (i,j), PD_{ij}, is represented by a quantity in the square grid (dimension

[1] The cost-optimal solution for instances 3, 4, 5, 14 and 15 see Gendreau et al. (1999); instances 13 and 17 see Brandão (2009); instance 16 see Penna et al. (2013); instance 18 see Prins (2009); and the best know solution for instance 20 see Subramanian et al. (2012).

4×4 distance units) that contains the arc. A decay function from the center to the exterior is used in order to represent an urban area where the population density is decreasing towards peripheral zones. PD_{ij} value generator is shown in (28).

$$PD_{ij} = 9500u(1 - \frac{maxDist - popDist}{maxDist}) + 500 \quad (28)$$

where u is generated using a uniform distribution, $u \sim U(0.4, 0.6)$, $maxDist$ is the maximum between the length and the width of the rectangle that contains the squared grid, and $popDist$ is the maximum value between the difference of the abscise and ordinate coordinates of the left superior corner of the current square grid and the coordinates of left superior corner of the central square grid.

The HazMat incident rate of a vehicle on an arc is about 1×10^{-6} per $(vehicle - Km)$, see Button and Reilly (2000) and Kazantzi et al. (2011b). In this case we use $TTAR_k \sim U[0.6, 1.0]10^{-6}$ per $(vehicle - Km)$ $\forall k \in \mathcal{K}$.

The GA was code in *Java SE 8* and executed in an *Intel Core i7* Processor 2.4 GHz with 16 GB of RAM running *Windows 10*. The MILP formulation was implemented using a comercial solver (*Gurobi*) for *Python 2.7*.

For the piece-wise linear approximation of the incident probability function, the number of linear functions (M), is fixed at four corresponding to small load, a medium-size load, a large load and a very-large load. After a parameter tuning for a truck load that goes from 1 (minimum demand) to 400 (maximum vehicle capacity), the parameters of GA were: a population size of 100 individuals, a mutation rate of 0.02, and 500 generations. The value assigned to lb is 10.

After running the algorithm, the results for the first piecewise linear approximation function are:

$$b_1 = 0.07349097;\ 280.5 \leq (280.5y_{ij}^k) \leq 9256.5$$
$$b_2 = 0.04590467;\ 9256.5 \leq (280.5y_{ij}^k) \leq 30574.5$$
$$b_3 = 0.03555368;\ 30574.5 \leq (280.5y_{ij}^k) \leq 64795.5$$
$$b_4 = 0.02979203;\ 64795.5 \leq (280.5y_{ij}^k) \leq 112200.0$$

and for the second function are:

$$b_1 = 0.06835302;\ 280.5 \leq (280.5y_{ij}^k) \leq 9995.2$$
$$b_2 = 0.04468697;\ 9995.2 \leq (280.5y_{ij}^k) \leq 31855.6$$
$$b_3 = 0.03515308;\ 31855.6 \leq (280.5y_{ij}^k) \leq 65838.6$$
$$b_4 = 0.02964824;\ 65838.6 \leq (280.5y_{ij}^k) \leq 112200.0$$

Given that the truck load varies according to the demand of the clients to visit, and the range of these values is the same for all the studied instances, the same piecewise linear approximation was used in all of them.

4.2 Results

Tables 1 and 2 show the expected consequences estimated using (15) and both PLF. In these tables, *Inst.* denotes the name of the test-problem, n denotes the number of customers, *BKS* represents the best known solution (optimal solutions in some cases). The approximation and the exact value of the expected consequences for each cost-optimal solutions is presented, and the gap between both approaches and the exact value. All the possible route sequence combinations were considered in order to find the minimum risk for a solution.

Table 1. Total routing risk for optimal-cost solution using the first piecewise linear approximation

Inst.	n	BKS cost	Risk		
			Exact value (15) $\times 10^{-6}$	Approximation (27) Value $\times 10^{-6}$	Approximation (27) Gap %
3	20	961.026	307.65	296.526	3.62
4	20	6437.331	248.85	233.875	6.02
5	20	1007.051	431.687	418.693	3.01
6	20	6516.468	261.625	248.483	5.02
13	50	2406.361	768.897	754.072	1.93
14	50	9119.28	571.197	554.861	2.86
15	50	2586.37	700.91	673.843	3.86
16	50	2720.433	790.004	762.671	3.46
17	75	1734.531	1479.142	1454.417	1.67
18	75	2369.646	1229.726	1198.711	2.52
19	100	8661.808	1171.318	1131.362	3.41
20	100	4032.81	1189.733	1148.8	3.44

Table 2. Total routing risk for optimal-cost solution using the second piecewise linear approximation

Inst.	n	BKS cost	Risk		
			Exact value (15) $\times 10^{-6}$	Approximation (27) Value $\times 10^{-6}$	Approximation (27) Gap %
3	20	961.026	307.65	313.025	1.75
4	20	6437.331	248.85	252.622	1.52
5	20	1007.051	431.687	437.342	1.31
6	20	6516.468	261.625	266.198	1.75
13	50	2406.361	768.897	775.776	0.89
14	50	9119.28	571.197	577.295	1.07
15	50	2586.37	700.91	710.516	1.37
16	50	2720.433	790.004	801.02	1.39
17	75	1734.531	1479.142	1491.461	0.83
18	75	2369.646	1229.726	1242.815	1.06
19	100	8661.808	1171.318	1187.278	1.36
20	100	4032.81	1189.733	1205.337	1.31

Given that the average difference in percentage between the exact value (by using (15)) and the piecewise linear approximation using the first function (3.4%) is greater than the average difference (1.3%) for the second function, this last approach is used on optimization problem.

The optimal risk for the first four instances is obtained using the above presented MILP model of the risk problem optimization shown in table 3. For other instances the CPU time is more than 15 hours. From table 3, it is remarkable that for instance 5, the total cost for the optimal-risk solution is more than twice the optimal-cost solution. Also, the optimal risk value obtained for each instance is greater than the total risk value obtained when the optimal-risk solution is evaluated by using (15).

5. CONCLUSIONS

The present study addresses the risk minimization problem for vehicle routing in hazardous materials (HazMat) transportation using heterogeneous fleet of vehicles. We

Table 3. Optimal risk values for some instances of HVRP with unlimited fleet

Inst.	n	Risk Optimization			
		MILP $\times 10^{-6}$	Exact value (15) $\times 10^{-6}$	Cost value	Time Sec.
3	20	156.817	154.08	2047.102	17
4	20	168.238	165.597	33556.056	68
5	20	251.837	248.793	1305.316	133
6	20	196.801	193.361	14541.962	1839

propose a mixed integer linear programming model that incorporates a piecewise linear approximation of the transportation risk objective. The objective function captures some variations in risk which are not considered in previous models in HazMat transportation. This function includes an estimation of the HazMat transportation incident probability as a function of the truck load and type. The proposed model is a more realistic approach and it can be used for reducing the risk in HazMat distribution. Comparing the risk of the cost-optimal solution to the risk-optimal solution for the small instances of Heterogeneous Vehicle Routing Problem with Fixed Costs and unlimited fleet, the minimization of total cost and the minimization of total risk appear as to be conflicting objectives.

Given the NP-hard nature of VRP problems, the development of heuristics and meta-heuristics techniques that can deal with problems with a greater number of clients is a promising aspect of future research in using heterogeneous fleet of truck for HazMat transportation.

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