Mary Robinson

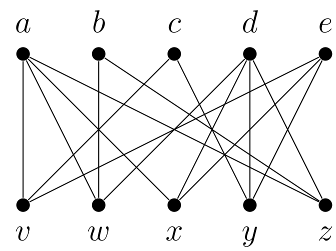
MATH 4750 HW 6

Due **Oct 24** at the start of class. **Answers without justification will receive 0 points.**

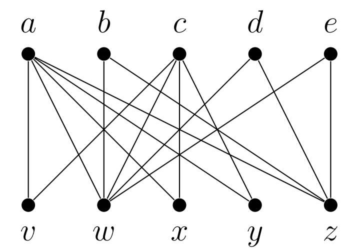
1. Draw your adopted graph. What is the edge independence number of your graph? To justify your answer, highlight a maximum matching and explain why there cannot be a larger one.

2. Give an example of a connected non-Hamiltonian graph that contains two disjoint perfect matchings. Justify your answer.

3. The bipartite graphs G_1 and G_2 below each have parts $U = \{a, b, c, d, e\}$ and $W = \{v, w, x, y, z\}$. In each case, does the graph have a perfect matching? If so, state what the matching is; if not, explain why not.



G_1

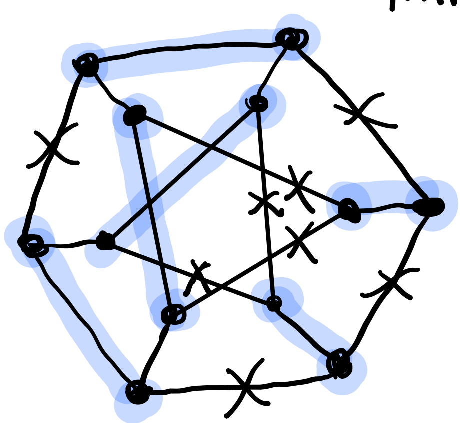


G_2

4. Prove that if the graph G has a Hamiltonian cycle, then the Cartesian product graph $G \square K_2$ has a Hamiltonian cycle.

5. Prove that every r-regular bipartite graph ($r \geq 1$) has a perfect matching.

1. Mine contains a perfect matching!



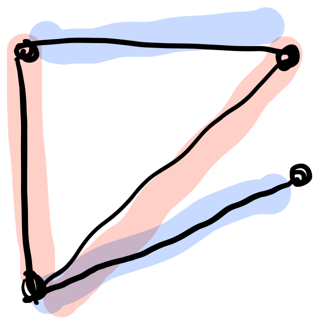
$$\max(G) = 6$$

so

$$\alpha'(G) \leq 6$$

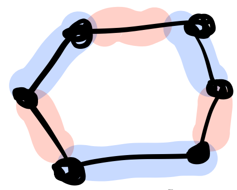
2. Connected, non-hamiltonian, 2 disjoint P.M.

disjoint = no edge in common

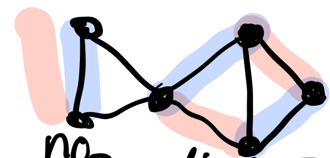


odd Cycle w/ a leaf ✓

non examples:

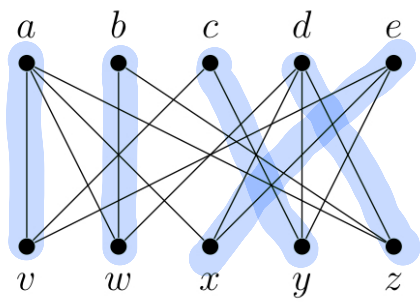


non hamiltonian

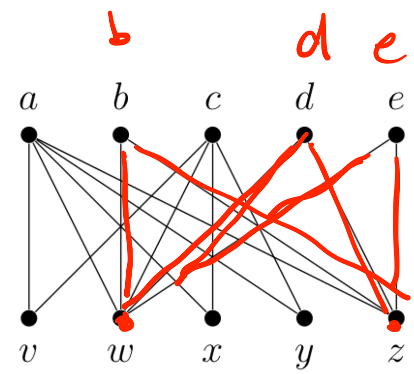


no 2 disj. P.M.

3.



G_1



G_2

There is a perfect matching in the bipartite graph G_1.

av, bw, cy, dz, ex.

S: {b, d, e}

N(S): {w, z}

There is no perfect matching in the bipartite graph since N(S) < S.

4. G is Ham. Dirac's, Ore's $\}$ need degrees of G — construction! K_2 

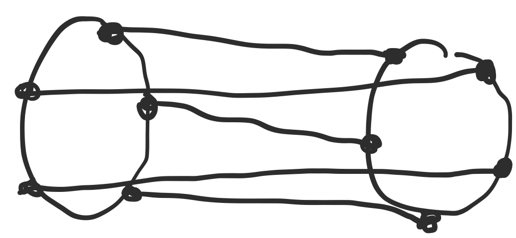
Prove: $G \square K_2$ Ham.

Prove that if the graph G has a Hamiltonian cycle, then the Cartesian product graph $G \square K_2$ has a Hamiltonian cycle.

Proof:

Since G is hamiltonian, it has a hamiltonian cycle $x_1, x_2, x_3, \ldots, x_{n-1}, x_n, x_1$.

Let x_i' be the copy of x_i in $G \square K_2$, so that $x_i x_i' \in E(G \square K_2)$ $\forall$ $1 \le i \le n$.



A hamiltonian cycle in $G \square K_2$ would be $x_1 \ldots x_n, x_n', \ldots, x_1', x_1$. Hence, $G \square K_2$ is Hamiltonian.

$\square$

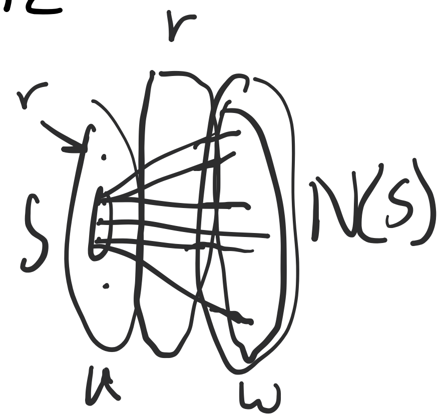
for future ☆ regular bipartite

$\Rightarrow$ same # vxs in 2 parts

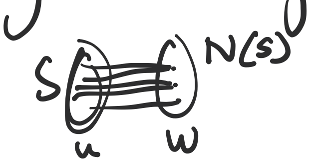
5. r-reg $r \geq 1$ bipartite

Scratch:

has perfect matching



☆ Hall's Thm: A bipartite graph w/ parts U & W has a matching covering U iff $\forall S \subseteq U$ $|N(S)| \geq |S|$.



"$\forall$ set of vxs in U, the # of neighbors of those vxs is $\geq$ the # of those vxs."

Proof:

Let G be a bipartite, r-reg graph with parts U & W. Since $|E(G)| = r|U| = r|W|$, we know $|U| = |W|$. So if we find a matching covering U, it's a perfect matching. Now let $S \subseteq U$. There are $r|S|$ edges that leave S, and all of the $r|S|$ edges enter $N(S)$ by definition of neighborhood. So $r|S| \leq$ the number of edges in $N(S)$. There are $r|N(S)|$ edges that enter S, so $r|S| \leq r|N(S)|$, so $|S| \leq |N(S)|$. Therefore by Hall's Theorem, there exists a matching covering U, which is a perfect matching. $\square$