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Poker Hands Project:

This project will explain how the probability of drawing each poker hand is calculated.

Let's start with a Royal Flush. The probability of drawing one is $\frac{\binom{4}{1}}{\binom{52}{5}}$. $\binom{4}{1}$ represents our 4 suits, and we are choosing only one suit, and $\binom{52}{5}$ is the total possible hands we can have in 5-card poker. The simplification of $\frac{\binom{4}{1}}{\binom{52}{5}}$ is 0.000154%, or 649,739 to 1.

Next, Straight Flush. A straight flush is expressed by $\binom{10}{1}\binom{4}{1}-\binom{4}{1}$. For this one, we are excluding royal flushes, hence why we subtract $\binom{4}{1}$. Since we exclude all face cards, we are only choosing 1 card from 10 choices, not 13. We then use product principle to multiply by the number of suits, only choosing 1. Hence, $\binom{10}{1}\binom{4}{1}-\binom{4}{1}$. The calculated probability is $\frac{\binom{10}{1}\binom{4}{1}-\binom{4}{1}}{\binom{52}{5}}$, or 0.00139%, 72,192.33 to 1.

Now for 4 of a Kind. It's expressed as $\binom{13}{1}\binom{4}{1}\binom{12}{1}\binom{4}{4}$. We choose 1 extra card to have, choosing 1 suit, hence $\binom{13}{1}\binom{4}{1}$. Then we choose 4 identical cards that exclude the card we just drew, and since they are all to have different suits, we use $\binom{4}{4}$. By applying product principle, we get $\binom{13}{1}\binom{4}{1}\binom{12}{1}\binom{4}{4}$. The calculated probability is 0.02401%, or 4,164 to 1 odds. This was calculated by doing $\frac{\binom{13}{1}\binom{4}{1}\binom{12}{1}\binom{4}{4}}{\binom{52}{5}}$.

Next is Full House. A full house consists of 3 of a kind and a pair. We can use product principle to multiply those possibilities together. For 3 of a kind, we choose one number or face card, $\binom{13}{1}$, and we choose 3 of the 4 suits. Hence, $\binom{13}{1}\binom{4}{3}$. Then, we choose another card from the numbers and faces, $\binom{12}{1}$, and choose 2 of the 4 suits. Hence, $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$. By calculating $\frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}$, we get our probability is 0.1441%, or 693.1667 to 1 odds.

Now for a Flush. A flush is a 5 card hand with all the same suit, excluding a royal flush. It is expressed as $\binom{13}{5}\binom{4}{1} - \binom{10}{1}\binom{4}{1}$, where $\binom{10}{1}\binom{4}{1}$ is the probability of a royal or straight flush, with no face cards and choosing only one suit. We then have 13 cards to choose 5 from, and 4 suits that we choose one from. Hence, $\binom{13}{5}\binom{4}{1} - \binom{10}{1}\binom{4}{1}$. The calculated probability is 0.1965%, or 507.8019 to 1 odds. I calculated it by dividing it by our overall hands of $\binom{52}{5}$.

A straight is all numbers and/or face cards in order from least to greatest in a 5 card hand. To get the probability, we exclude a royal and straight flush, by subtracting $\binom{10}{1}\binom{4}{1}$, where $\binom{10}{1}$ is all cards excluding face cards, and $\binom{4}{1}$ is choosing 1 suit. Then we choose $\binom{10}{1}$ of cards, again excluding face cards, with $\binom{4}{1}^5$ representing 4 <u>different</u> suits, for 5 cards. Hence,

$$\frac{\binom{10}{1}\binom{4}{1}^5 - \binom{10}{1}\binom{4}{1}}{\binom{52}{5}} = 0.3925\%$$

is the probability, or 253.8 to 1 odds.

Next, a three of a kind. As discussed earlier, the probability is expressed as $\binom{13}{1}\binom{4}{3}$, where we choose any card and take 3 out of the 4 suits. Then our remaining hand is $\binom{12}{2}$, where we choose 2 of 12 remaining cards, and $\binom{4}{1}^2$, where each is a distinct suit. Hence,

$$\frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2}{\binom{52}{5}} = 2.1128\%, \text{ or } 46.33 \text{ to } 1 \text{ odds.}$$

Now a 2 pair is just the probability of one pair, twice, and then choosing another random card. We have $\binom{13}{2}$ for choosing 2 different pairs, and $\binom{4}{2}^2$ for each of the pairs to have <u>different</u> suits. Then our remaining card is chosen from 11 cards left, with any of the 4 suits. Hence,

$$\frac{\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}}{\binom{52}{5}} = 4.7539\% \text{ or } 20.035 \text{ to one odds.}$$

A one pair is just choosing two cards with the same number, with different suits. Thus expressed as $\binom{13}{1}\binom{4}{2}$, where there's 13 cards with 4 suits, and choosing 2 suits. The rest of our hand is $\binom{12}{3}$ where we choose, from 12 cards remaining, 3 of them, that all have different suits, $\binom{4}{1}^3$. Hence,

$$\frac{\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3}{\binom{52}{5}} = 42.2569\%$$ or 1.366 to 1 odds!

Lastly, we have a high card hand, where we don't get any pairs, straights, or flushes. This is expressed as first calculating the card numbers or faces, then multiplying it by the suits possibilities. For the faces and numbers on the cards, we have $\binom{13}{5}$ total, and excluding the case where we repeat a non-face card, of $\binom{10}{1}$. Then we have $\binom{4}{1}^5$ total suits for each card, subtracting the case of a repeat suit. Hence,

$$\frac{\left[\binom{13}{5}-\binom{10}{1}\right]\left[\binom{4}{1}^5-\binom{4}{1}\right]}{\binom{52}{5}} = 50.1177\%$$, or .4953 to 1 odds!

Now that all the probabilities have been explained, let's take a quick look at some payouts for video poker machines. As stated from vpfree2.com, "A pay table is the complete listing of the payouts for each hand for a video poker game."(1) An example of a pay table from an MGM grand video poker machine is this:

$1, $2, $5	1 Play	MG	Bartops	Whiskey Down Bar
99.11% DB	Double Bonus			1-1-3-5-7-9-50-80-160-50-800
98.01% BP	Bonus Poker			1-2-3-4-5-7-25-40-80-50-800

(vpfree2.com, 2)

We see here that the odds for each hand are listed in dashes. So it's a 800 to 1 chance of getting a cumulative royal flush. In poker games, odds are calculated by accumulation, meaning you are dealt 2 cards, then bet, then dealt another, bet, and the cycle continues until there is 5 or 7 cards acheived. So machines have more enticing odds of winning, but it's actually harder to earn larger amounts of money, which is how casinos trick you into playing more.

Sources:

1- https: //www. vpfree2. com/video-poker/pay-tables/help

2- https: //www. vpfree2. com/casino/mgm-grand-las-vegas/games/by-machine/33318