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MATH 4750 HW 7

Due **Oct 31** at the start of class. **Answers without justification will receive 0 points.**

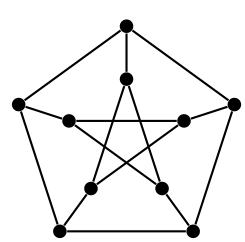
1. Draw your adopted graph. Is your graph planar? Either give a plane drawing of your graph or explain why none exists.

2. Give an example or explain why none exists. If giving an example, be sure to justify why your example satisfies the given criteria.
   (a) A planar graph with degree sequence $s: 4^{(3)}, 5^{(3)}, 6^{(3)}, 7^{(3)}$
   (b) A planar 4-regular graph
   (c) A nonplanar 4-regular graph

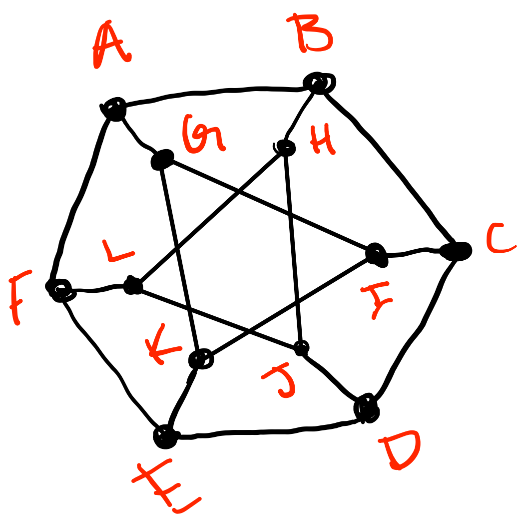
3. Let G be a planar graph of order 20. Show that $\overline{G}$ is not planar.

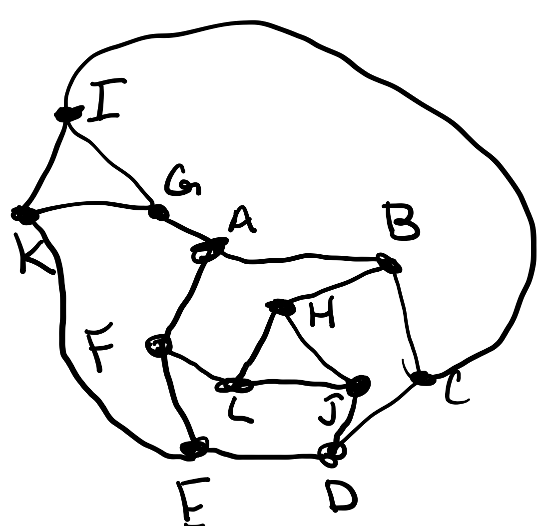
4. Are each of the following graphs planar or nonplanar? Provide justification.
   (a) $K_4 \square K_2$
   (b) $\overline{C_6}$
   (c) $\overline{C_7}$

5. Prove that the Petersen graph G (shown below) is nonplanar in two ways:
   (a) By modifying the counting argument we used to prove the Plane Graph Edge Bound
   (b) By applying Kuratowski's Theorem



1.





Planar!

2. a) 4, 4, 4, 5, 5, 5, 6, 6, 6, 7, 7, 7

PGEB $m > 3n - 6$

order: 12

size: $2m = \Sigma \deg(v)$

$2m = 66$

$m = 33$

nonplanar

$33 > 3(12) - 6$

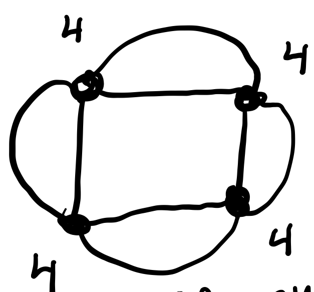
$33 > 36 - 6$

$33 > 30$ ✓

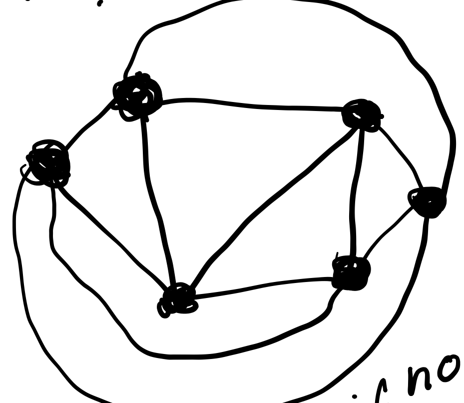
Therefore a planar graph does not exist.

b) Planar 4-reg graph.

Exists!



if multi edges

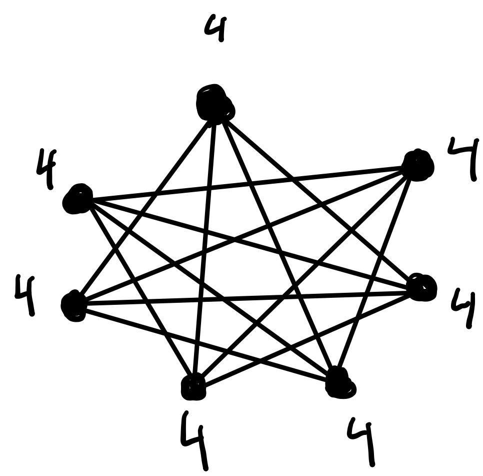


if no multi edges

c) Non Planar 4-reg graph.

$\overline{C_7}$

Exists!



← By PGEB, this is nonplanar. See work on 4c.

3. Proof: Prove by contradiction!

Let G be a planar graph with order 20. By planar graph edge bound, we have $m \leq 3n-6$ where m is number of edges and n is the order. So we have

$$m \leq 3(20)-6$$
$$m \leq 60-6$$
$$m \leq 54$$

Suppose $\overline{G}$ is planar. Then we have

$$\overline{m} \leq 3\overline{n}-6 \rightarrow \overline{m} \leq 3(20)-6$$
$$\overline{m} \leq 60-6$$
$$\overline{m} \leq 54$$

To find the edges, we use K_{20} to get $m + \overline{m} \leq 108$.

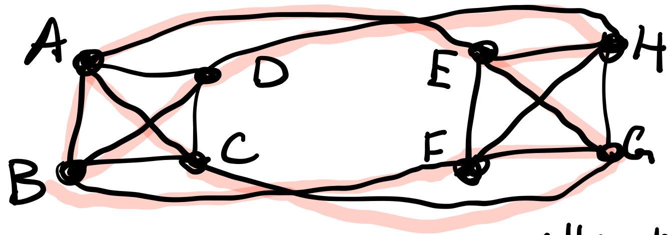
If we use $\binom{n}{2}$ to get K_{20} edges, we get

$$\binom{n}{2} = \frac{n(n-1)}{2} = \frac{20 \cdot 19}{2} = 190 \quad \rightarrow\leftarrow$$

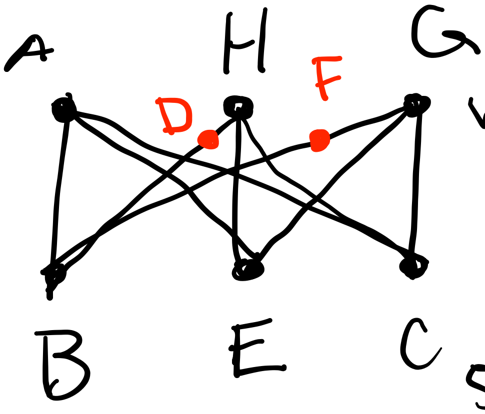
which is a contradiction.

Therefore, $\overline{G}$ is not planar. $\square$

4. a) $K_4 \square K_2$

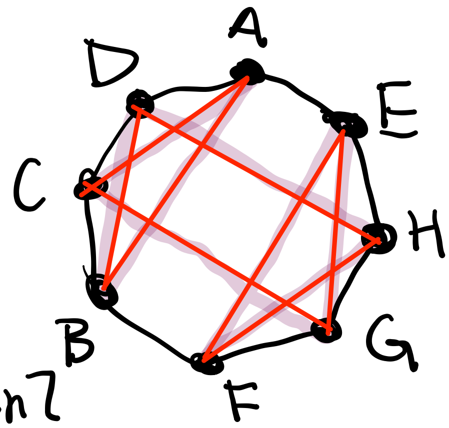


Non Planar by Kuratowski's

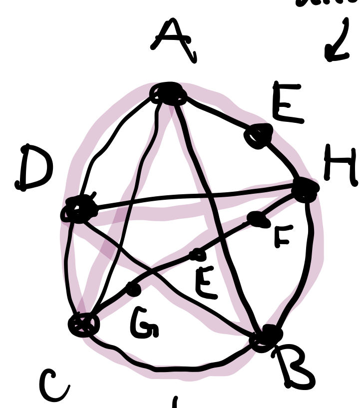


Wrong, I know

$K_{3,3}$ Subgraph?

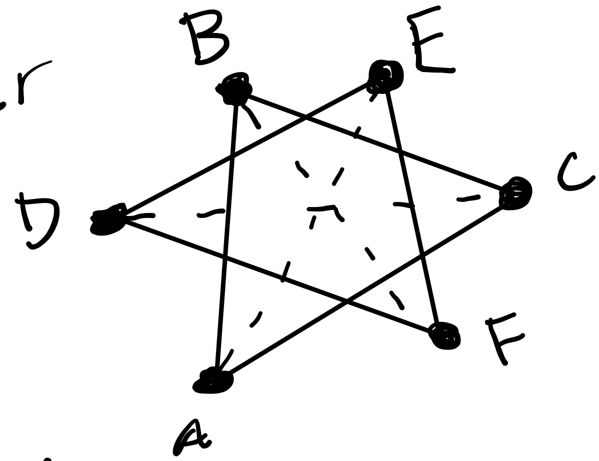


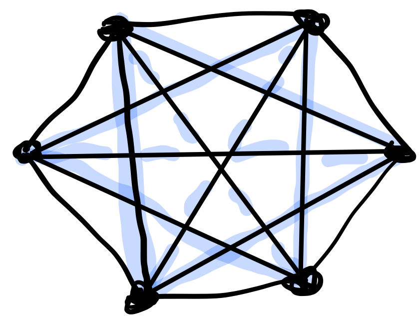
allowed?

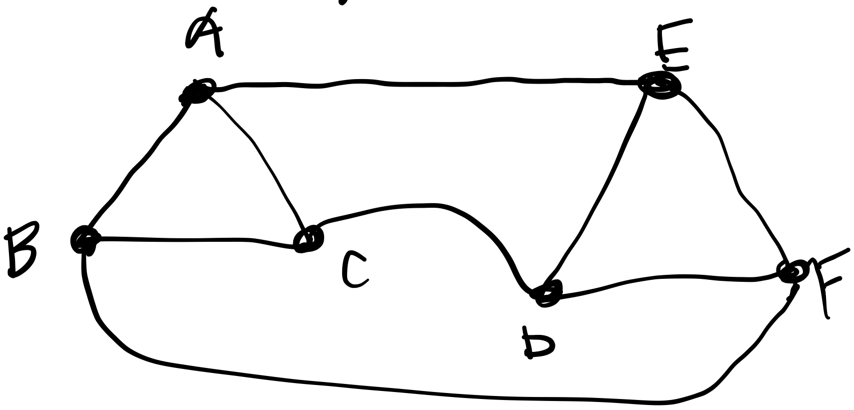


K_5 ?
Subgraph

b) Planar







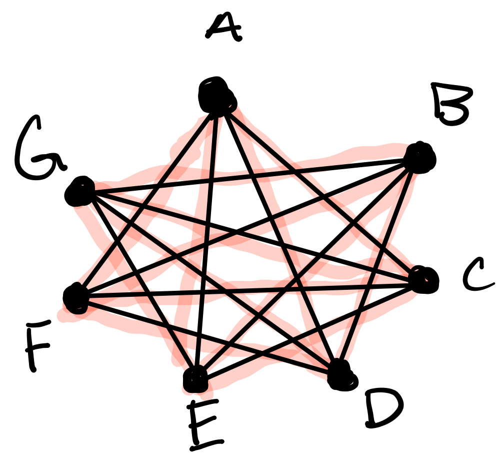
c) Non Planar

$f = m - n + 2$ ← Euler's

$2m \geq 4f$ ← since 4-reg graph

$2m \geq 4(m-n+2)$

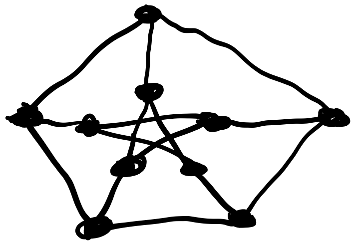
$2n - 4 \geq m$



$14 - 4 \geq 14$

$10 < 14$ ← By PGEB, Non Planar

5.



$n = 10$

$m = 15$

a) PGEB

modify Euler's:

$f = m - n + 2$

$2m \geq 5f$

✱ use 5 since our smallest cycle is C_5.

$2m \geq 5(m - n + 2)$

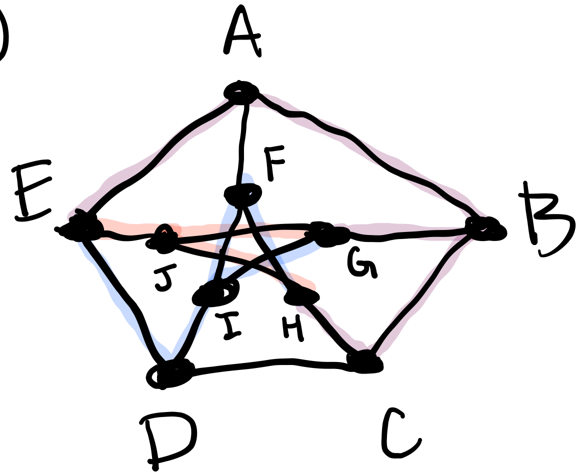
$2m \geq 5m - 5n + 10$

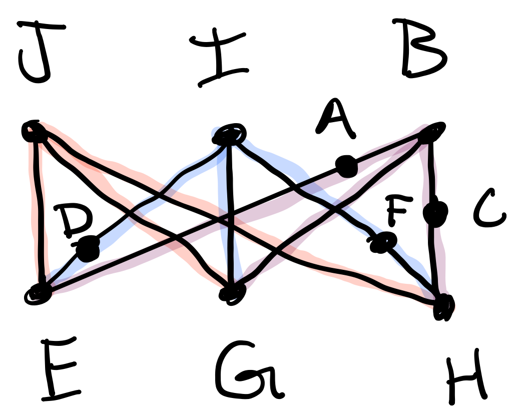
$3m \leq 5n - 10$

$45 \not\leq 40$ →←

By PGEB, this is nonplanar. □

b)





There's a $K_{3,3}$ subgraph so by Kuratowski's, this graph is nonplanar. □