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CS 3345

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1) Suppose your calculator only did base 10 logarithms.
Write an expression to compute log base 2 of 64 using only log base 10.

log_10 (64) / log_10 (2)

2) Express the following summation in closed form (an expression that can be directly computed from k). 3 + 5 + 7 + 9 + … + 2k+1

2(1+2+3+4…+k) + (1+1+1+1)
= 2(k(k+1)/2) + k
k(k+1) + k = **k^2 + 2k**

3) Proof by counterexample; Prove that the following statement is false: 2^n > n! for any n >= 1
Let n = 4
2^4 = 16 < 4! = 24

4) Proof by contradiction

Suppose we know the shortest path from A to D is A to B to C to D. Using the technique of proof by contradiction, show why the shortest path from A to C must be A to B to C.
Assume that the shortest path from A to C is not A to B to C. Then, this means that there is a shorter path consisting of A to C and then to D, but this contradicts the given condition, which means the shortest path from A to C must be A to B to C.

5) Induction proofs.

a. Prove by induction:
n
sum (2i-1) = n^2
i=1

Let P(n) be the proposition that the above sum holds for a given n >= 1
Base step:
Let n = 1; 2(1) – 1 = 1^2 = 2-1 = 1
n = 2; (2(2) – 1) + 1 = 2^2 = 3+1 = 4
Inductive step:
Show P(n) → P(n+1)
1 + 3 + 5 + … + 2n-1 = n^2
1 + 3 + 5 + … + 2n-1 + (2(n+1)-1) = 1 + 3 + 5 + … + 2n-1 + 2n + 1
= n^2 + 2n + 1 = (n+1)^2

6) Given: $T(1) = 3$
$T(N) = T(N\text{-}1) + 3, N>1$
What would the value of T(10) be?

$T(10) = T(9) + 3 = (T(8) + (2)3 = T(7) + 3(3)\ldots = T(1) + 9(3)$
$3 + (9)(3) = 30$

7) For the problem above, is there a formula that could directly calculate T(N)?
$T(N) = n(3)$ // we are consecutively adding more threes, n times.

8) Using induction, prove that your formula for the previous problem is correct.
Prove by induction that 3N is the closed-form equation for $T(n) = T(n\text{-}1) + 3, n > 1$

Basis step:
given $T(1) = 3$; $T(1) = 3(1) = 3$ (correct)
let $n = 2$; $T(2) = T(1) + 3 = 6$; $3(2) = 6$
Inductive step:
Let $T(N) = 3N$
show $T(n) \rightarrow T(n+1)$
$3(n) \rightarrow 3(n+1)$
$3(n+1) = 3n + 3 = 3n+3$ which is true