ELEC 372 - Homework 1

(To be submitted on Moodle by July 15, 3 PM)

Note:

- By the given deadline, upload a report (in pdf format) containing the solutions to problems below. Late reports will not be accepted.
- For problems solved by hand, include in the report all the steps of the solution.
- For problems requiring the use of Matlab/Simulink, include in the report the obtained results. Upload also the used/developed Matlab and Simlulink files, collecting them in a single zip folder. Plots and figures (e.g., generated in Matlab/Simulink) can be included in the report to better describe the obtained solutions.

Problem 1 An automobile driver uses a control system to maintain the car's speed at a prescribed desired value. 1) Sketch a block diagram to illustrate this feedback system. 2) Discuss how this control system can be fully automated (e.g. autonomous self-driving car)

Problem 2 Why do we prefer to use closed-loop control over open-loop control for most of the physical systems of interest?

Problem 3 When and why is it acceptable to assume that physical systems have a linear model for design and analysis purposes?

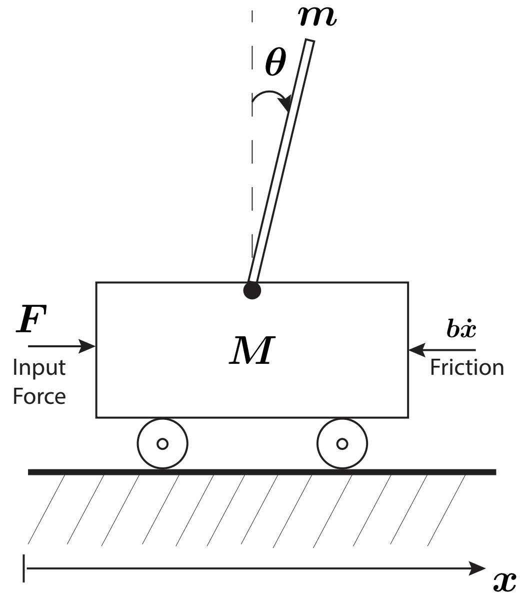

Figure 1: Inverted pendulum

Problem 4 The inverted pendulum cart depicted in Fig. 1 is described by the two following differential equations:

$$\ddot{x} = \frac{-m^2L^2g\cos\theta\sin\theta + mL^2(mL\dot{\theta}^2\sin\theta - b\dot{x}) + mL^2F}{mL^2(M + m(1-\cos^2\theta))} \tag{1}$$

$$\ddot{\theta} = \frac{(m+M)mgL\sin\theta - mL\cos\theta(mL\dot{\theta}^2\sin\theta - b\dot{x}) + mL\cos\theta F}{mL^2(M + m(1-\cos^2\theta))} \tag{2}$$

$$y = x \tag{3}$$

where:

M is the mass of the cart	$M = 2\ [Kg]$
m is the mass of the pendulum	$m = 0.5\ [Kg]$
b is the friction of the cart	$b = 0.1\ [N\frac{s}{m}]$
L is the length of the pendulum bar	$l = 0.3\ [m]$
g is the gravitational acceleration	$g = 9.81\ [\frac{m}{s^2}]$
F is the force applied to the cart	*variable*
x is the cart position coordinate	*variable*
$\dot{\mathbf{x}}$ is the velocity of the cart along the x direction	*variable*
$\ddot{\mathbf{x}}$ is the acceleration of the cart along the x direction	*variable*
$\boldsymbol{\theta}$ is the pendulum angle from the vertical	*variable*
$\dot{\boldsymbol{\theta}}$ is the angular velocity of the pendulum	*variable*
$\ddot{\boldsymbol{\theta}}$ is the angular acceleration of the pendulum	*variable*
y is the output (available sensor measurement)	*variable*

Table 1: Variable and parameters of the inverted pendulum model

With reference to the nonlinear dynamical model (1)-(3):

a) Write down the model into a non-linear state-space representation (i.e., $\dot{z}(t) = f(z(t), u(t))$, $y(t) = g(x(t), u(t))$
(*Hint: substitute* $z_1 = x$, $z_2 = \dot{x}$, $z_3 = \theta$, $z_4 = \dot{\theta}$, $u = F$).

b) Implement, using Simulink, the nonlinear state-space model $\dot{z}(t) = f(z(t), u(t))$ obtained in part a).
(*Hint: edit the provided Simulink template file inverted_pendulum_non_linear.slx*)

c) Create a Matlab script that linearizes the nonlinear system implemented in part b) around the equilibrium configuration $x_{eq} = 0$, $\dot{x}_{eq} = 0$, $\theta_{eq} = 0$, $\dot{\theta}_{eq} = 0$, $F = 0$ (i.e, cart not moving and pendulum in the vertical position). Then, write down the linearized model into the state-space form $\dot{z}(t) = Az(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$ and implement it in Simulink.
(*Hint 1: in the Script, use the Matlab built-in function* ***linmod('model',z_eq,u_eq)****, where* ***model*** *is the simulink file where the nonlinear inverted pendulum is implemented (as in part b). Linmod performs the linearization of a nonlinear system around an arbitrary equilibrium pair (z_eq,u_eq).* For further details on how to use linmod, type **help linmod** on the command window)
(*Hint 2: edit the provided template file inverted_pendulum_linear.slx to implement the linearized model in Simulink*)

d) Find the transfer function of the linearized model derived in part (c) (*Hint: search for a Matlab function that computes the transfer function from a state-space representation*).

e) In Simulink, simulate for 0.5 sec both the nonlinear and linearized model, starting from the equilibrium described in point c), and with the following step-input signal

$$u(t) = 1 \quad t \geq 0$$

Export the results into the Matlab workspace and graphically compare using the "plot" function of Matlab the cart position $z_1(t) = x(t)$ obtained from the nonlinear and linearized models. <u>Comment on the obtained result</u>.
(*Hint: Search for a Simulink block that exports the desired data into the main Matlab workspace*).

Problem 5 Consider the transfer function

$$G(s) = \frac{1.6667(s+0.1)}{(s-6.388)(s+6.398)(s+0.04)}$$

- a) Compute by hand, the Laplace inverse of $Y(s) = G(s)U(s)$, where $u(t)$ is a unit step
- b) Compute by using the Matlab built-in function "*ilaplace*", the Laplace inverse of $Y(s) = G(s)U(s)$, where $u(t)$ is a unit step
- c) By using the Matlab built-in function "*step*", plot $y(t)$ when $u(t)$ is a unit step.