4 EXPT #2: SYSTEM IDENTIFICATION

4.1 OBJECTIVE

- To determine the parameters (J, B and K) of a mechanical first-order system and calculate the system model.
- To illustrates the basic principles of system identification using open-loop and closed-loop response.
- To verify the system modeling by using MATLAB functions.

4.2 INTRODUCTION

4.2.1 SYSTEM PARAMETERS

ECP 220 system has the general block diagram shown in Figure 4.1 again, in which, the PI+V controller has been chosen as an example.

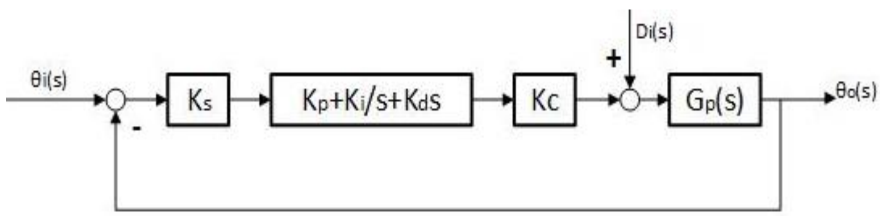


Figure 4.1: A closed-loop configuration

In the above closed-loop configurations, the nominal value of closed-loop forward-path gain K is discussed in Experiment #1:

$$K = K_s K_c K_e K_a K_t = (32)\,[3.05(10)^{-4}]\,(2)\,(0.1)[\,2546.5] \approx \underline{\mathbf{5}\ \text{Nm/radian}}^{*} \qquad (4.1)$$

Notes: The values of K_a and K_t vary from unit to unit and the variation in the actual value of K is ± 20%

The actual values of J and K specific to each unit will be found in this Experiment. The "PI + velocity feedback" configuration (see Figure 4.1 above), which is also the 'default' configuration, will be used since it allows good control of closed-loop system parameters.

4.2.2 THEORETICAL ESTIMATION OF ROTATIONAL INERTIA J

Since the feedback loop is closed around Encoder #1 in the laboratory system, the J and B in the system equations must be values 'reflected' to the location of encoder #1. The arrangement of the belt and pulley coupling between the disks and the drive motor (shown earlier in this manual) is shown schematically in *Figure 4.2: Plan Rotation Inertia*. Assuming 'ideal coupler' or 'transformer' relations, it can be shown that for gears or for toothed-belt and pulleys, the coupling ratio is $\mathbf{n} = \omega_p/\omega_s = \theta_p/\theta_s = N_s/N_p$ where N is the number of teeth and 'p' and 's' refer to primary (driving side) and secondary (driven side). Thus, in fowling Figure 4.2, the overall coupling-ratio is $\mathbf{n} = \omega_1/\omega_2 = \theta_1/\theta_2 = \mathbf{n}_1\mathbf{n}_2 = [\mathbf{N}_{bottom}/\mathbf{N}_D]\,[\mathbf{N}_L/\mathbf{N}_{top}]$ = (24/12) (72/36)=(2)(2)=4. Furthermore, an inertia J or friction B on the secondary side will appear to the primary-side as being multiplied by $(1/\mathbf{n}^2)$. Thus, the total effective inertia at the Encoder#1 location can be expressed as:

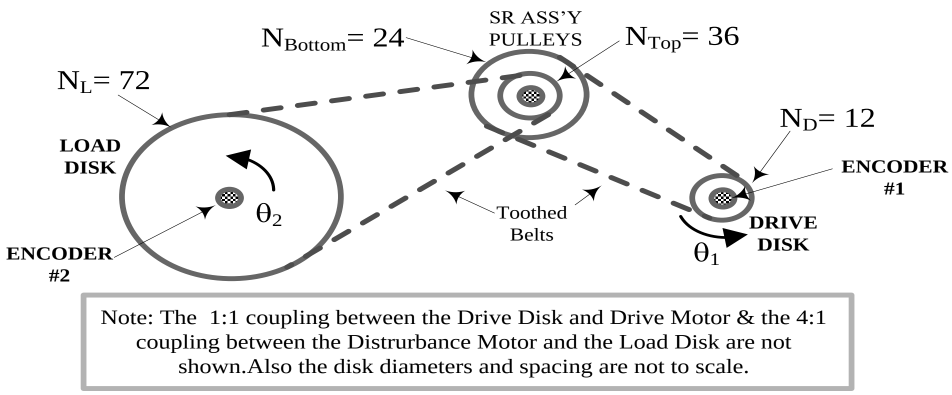


Figure 4.2: Plan Rotation Inertia

$$J = (J_D + J_{WD}) + \frac{J_{SR}}{(n_1)^2} + \frac{J_L + J_{WL}}{(n_1 n_2)^2} \qquad (4.2)$$

where the subscripts D, L and SR refer to the drive, the load and the SR disks and the additional subscript w refers to the additional inertia provide by the weights fixed to the drive and load disks. The inertiasJ_D, J_L, and J_{SR} are specified by the manufacturer as:

- Inertia of the bare drive disk J_D = $4(10)^{-4}$ kgm^2
- Inertia of the bare load disk J_L = $65(10)^{-4}$ kgm^2
- Inertia of the SR assembly J_{SR} = $78(10)^{-6} kgm^2$

(Including top and bottom pulleys, backlash device and screws)

The inertias J_{wD} and J_{wL} must now be calculated: The nominal mass (m) values of the small and larger [diameters 32mm and 50mm respectively] brass weights are 200gm and 500gm respectively. When symmetrically located at a radius R on the disks, their moment of inertia can be calculated as $J_w = mR^2$ at the appropriate location. It is assumed that the small and large weights will be used on the drive and load disks respectively. Further, it is assumed that four identical weights will be used symmetrically on each disk. Since the nominal radii of the drive and load disks are 67mm and 128mm respectively, the radius can be deduced with reasonable accuracy when the masses are moved outwards so that their outside edges are flush with the outside edges of the disks, as seen in *Figure 2.2: Rotational-motion Assembly* of this manual. With the weights permanently bolted at the extremes as explained, the inertia values J_{wD} and J_{wL} are:

$$J_{wD} = 4\,(0.2)\,[0.067 - 0.016]^2 = 2.08(10)^{-3}\ kgm^2$$

and

$$J_{wL} = 4\,(0.5)\,[0.128 - 0.025]^2 = 2.122(10)^{-2}\ kgm^2$$

Equation (3.5) can now be used to calculate the total effective inertia J at the Encoder#1 location as:

$$J = (J_D + J_{wD}) + J_{SR}/(\mathbf{n}_1)^2 + (J_L + J_{wL})/(\mathbf{n}_1\,\mathbf{n}_1)^2$$

$$= 4(10)^{-4} + 2.08(10)^{-3} + (78/4)(10)^{-6} + [65(10)^{-4} + 2.122(10)^{-2}]/16$$

$$= \underline{\mathbf{4.23(10)^{-3}}}\ kgm^2.$$

Note that the largest contribution to J is made by the added brass weights. Also note that the value of J will vary somewhat from unit to unit.

4.2.3 MATLAB IDENTIFICATION TOOLBOX

System Identification Toolbox™ provides MATLAB® functions, Simulink® blocks, and an app for constructing mathematical models of dynamic systems from measured input-output data. It lets you create and use models of dynamic systems not easily modeled from first principles or specifications. You can use time-domain and frequency-domain input-output data to identify continuous-time and discrete-time transfer functions, process models, and state-space models. More information, please visit its website.

Useful Function *tfest* - Transfer function estimation and the System Identification Tool GUI - `ident`.

4.3 PLANT PARAMETER AND SYSTEM MODELING

In this experiment, open-loop tests are performed on the DC motor (plant) to obtain parameters of its transfer function, since the transfer function of the plant is required for analysis of the closed-loop system.

4.3.1 EVALUATION OF J AND K USING OPEN-LOOP VELOCITY OUTPUT

PRELIMINARY

In this experiment, the acceleration α_1 developed in response to a step voltage V_i is measured. Next, the inertia is increased by a known amount ΔJ and the acceleration α_2 developed in response to the same step voltage is measured. The step magnitude is selected to provide rapid observable acceleration so that the friction torque may be neglected and the torque equation $T \approx J\,\alpha$ may be used. Since only the partial gain product $K_aK_tK_e$ is involved, we have:

$$(V_i)\{K_aK_tK_e\} = J\,\alpha_1 = (J + \Delta J)\,\alpha_2 \qquad (4.3)$$

Where the acceleration α is expressed in counts/sec^2 and $\alpha_1 > \alpha_2$. If ΔJ is known (as it is, see Step 4 in the Procedure below), J can be determined from the equation

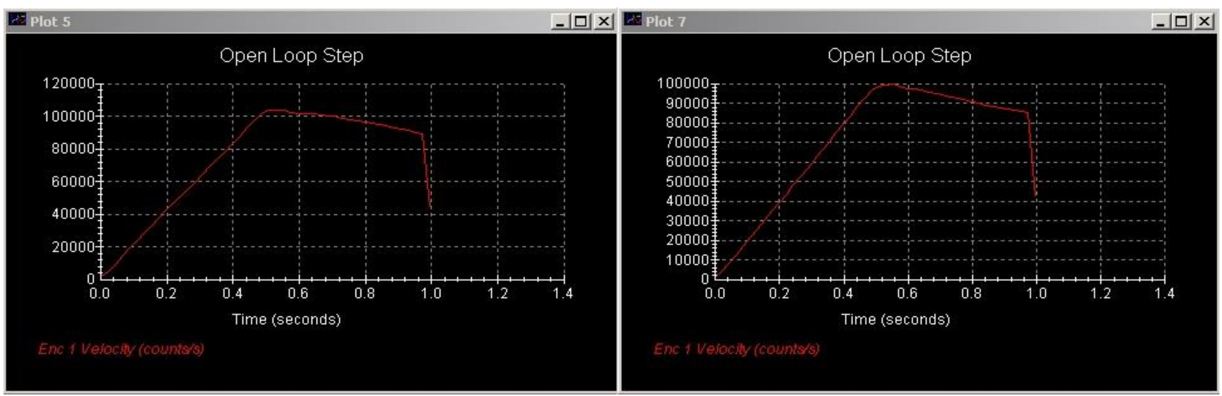
$$J = \left(\frac{\alpha_2}{\alpha_1 - \alpha_2}\right)\Delta J \ \text{kgm}^2 \qquad (4.4)$$

PROCEDURE:

1) Reset the controller from the UTILITY menu. Set up the ‘plot’ to display Encoder #1 Velocity only on the left axis. Under ‘Data Acquisition’ menu, set the sampling period to 5 (ie data to be collected every 5th cycle, = 5 (4.42 ms) ≈ 22ms which is done to reduce the noise inherent in acquiring the velocity signal).
2) From the COMMAND menu, select ‘Trajectory’, STEP and then click on ‘Setup’. Then select OPEN LOOP, Step Size 2 volts, Dwell time 500 ms, and number of Repetitions 1. Next, go to Execute and RUN the test. After the data has been acquired, plot the output. The trace will be similar to the one shown below. (The initial linear part of the curve denotes constant acceleration). Overload occurs

at approximately 0.5 sec when the step reverses and the ‘overload’ circuit will disable the system at this point.

3) Determine* the slope of the initial linear part of the velocity plot, without using any axis scaling. This is the acceleration α_1 in Counts /sec^2 which results in response to the step input voltage V_i = 2 volts.
(*Read the note at the beginning of the experiment about making preliminary calculations)
4) Making sure that the system is still disabled, loosen the cover screws and lift off the plexiglass cover. Carefully insert the additional 69mm diameter brass weight (supplied) in between the four fixed 0.2 kg weights on the drive disk so that it fits in snugly. Make sure that the pins on the weight fit into the radial slots on the disk, and then carefully replace the plexiglass cover.
(**Note**: The extra weight added has been calculated to increase the net inertia effective at the drive disk axis by the amount $\Delta J \approx$ **0.000494** kg.m^2 for all stations)
5) Repeat Steps 1, 2, and 3 above with the extra weight installed. Determine the acceleration obtained with the added weight in place, as α_2 (Note that $\alpha_2 < \alpha_1$). Finally, switch off the controller box, remove the added weight, replace the cover and lightly re-tighten the cover screws.
Two typical open-loop velocity responses obtained, without and with the added weight are seen in the photograph shown below, in ‘background’ and ‘foreground’ traces respectively.



RESULTS

Using Equation (4.4), determine the value of J and compare the value obtained with the calculated nominal value given in sections (4.2.1) and (4.2.2) and find the % error.

Using the determined value of J, find $K_aK_tK_e$ from the equation $2K_aK_tK_e = J\alpha_1$, *Equation (4.3)*. Multiply this value by K_sK_c to obtain K (see section 4.2.1) for the equipment at your lab station.

From data given in introduction,

$$K_sK_c = (10/32768)\ (32) \approx 0.009766 \text{ Volts/count.}$$

Record the determined values of **J and K** found in this part of the experiment.

4.3.2 EVALUATION OF J AND K USING OPEN-LOOP POSITION OUTPUT

PRELIMINARY

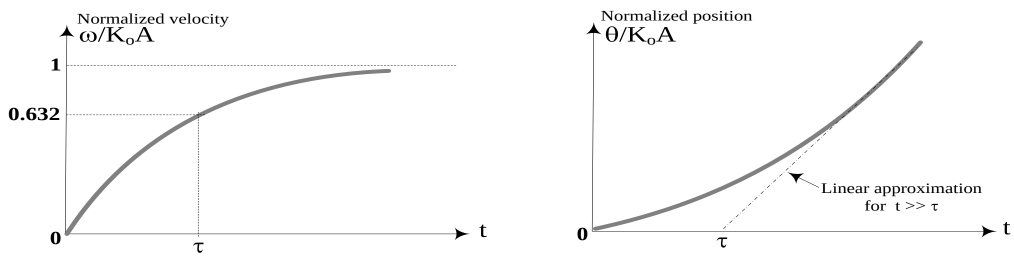
The time responses, $\omega(t)$, and $\theta(t)$ of the system are readily obtained by finding the inverse Laplace transforms of $\omega(s)$ and $\theta_O(s)$, respectively, for any given $V_i(s)$. For an input step voltage magnitude of A volts, $V_i(s) = A/s$, after performing the inverse transformation we obtain:

$$\omega(t) = \mathcal{L}^{-1}\{ K_o A/s(1+ s\tau)\} = K_o A[1- e^{-t/\tau}]$$

and

$$\theta_O(t) = \mathcal{L}^{-1}\{ K_oA / s^2(1+ s\tau)\} = K_oA [t - \tau + \tau e^{-t/\tau}] \qquad (4.5)$$

where the open-loop gain $K_o= K_aK_t/B$. These two step responses are shown below, in *normalized* form, (ω/K_oA) and (θ_O /K_oA) in following figures (a) and (b) respectively.



(a) (b)

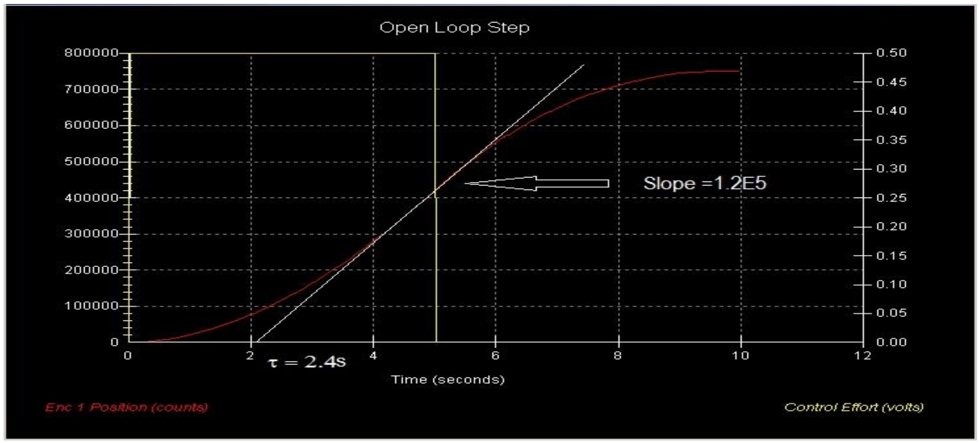
For the angular velocity output, the response is of the classic 'rising exponential type with $\omega(t)$ approaching K_oA for large t and the response reaching 0.632 K_oA at $t = \tau$. For the angular position output in equation (4.5), a linear approximation may be made by drawing a tangent to the curve for large t: This tangent will intersect the time axis ($\theta = 0$) at $t = \tau$. The slope of the tangent is K_oA (radians/sec). The latter response will be used in Experiment #2 to evaluate the 'plant' parameters in the laboratory system.

In this test, a smaller step voltage V_i is used as input and the position output is measured. Attempting to use an input that appears as a 'step', however, will trigger the built-in 'overload' circuit which will automatically 'abort' the test. Therefore, a small magnitude 'bounded' step is used which will tend to make the output 'converge' to a finite value, and half of the response time period can then be taken to correspond to Equation (3.8). It is easily seen that a linear approximation of the above response has a slope S given by K_oV_i (i.e $= K_aK_t K_e V_i /B$) and the straight-line describing this linear approximation will intercept the x-axis (time-axis) at $t = \tau = J/B$. Thus, measurement of the slope S and the time-axis intercept τ may be used to find both J and B.

PROCEDURE

1) Under 'Data Acquisition' menu, make sure that Control Effort (CE) has been added to the list. Reset the controller from the UTILITY menu. Set up the 'plot' to display the Encoder #1 Position on the left axis and the CE on the right axis.
2) From the COMMAND menu, select 'Trajectory', STEP and then click on 'Set Up'. Then select OPEN LOOP, Step Size **0.5** volts, Dwell time 5000 ms, and number of repetitions 1. Next, go to "Execute" and RUN the test. After the data has been acquired, plot the output. The trace will be similar to the one shown below. (The first half of the response, 0 to 5 sec, is due to the positive step). Of course, the slope and intercept values will have some variation depending upon the station equipment.

The straight-line approximation (the line is tangential to the S-shaped curve at midpoint, approx. 5sec.) has been inserted in the plot shown below. Determine* the slope S of the line (in Counts/sec) and the time-axis intercept τ (sec).



RESULTS:

From the displayed slope S (in counts/sec), determine*

$$S\ (\text{radians/sec}) = S\ (\text{counts/sec}) / K_e$$

$$= S\ (\text{counts/sec}) / 2546.5 = 3.927(10)^{-4}\ S\ (\text{counts/sec})$$

Since V_i = 0.5 volt, S (radians/sec) = $K_a K_t V_i / B$= (2) (0.1) (0.5)/B

Hence, determine

$$B\ (\text{Nms/radian}) = \frac{0.1}{S(radians/\sec)} = \frac{256.65}{\text{S(counts / sec)}}$$

From the displayed time-axis intercept, determine the time-constant τ = J/B from x-axis intercept. Finally determine J:

$$J(\text{kg.m}^2) = B\tau$$

Record the determined values of J and B found in this part of the experiment. Compare the value of J found with the value determined in section 4.3.1 and obtain the average value. Compare the average value of J with the calculated nominal value given in section 4.2.2 and find the percentage error.

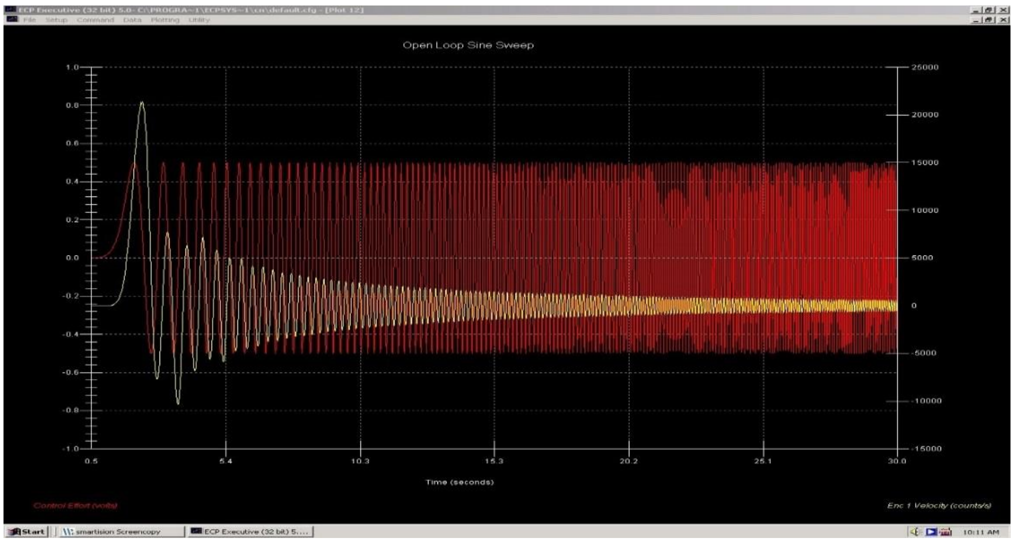
Record the average value of J, and the values of B and K obtained, these values are to be used in subsequent lab work.

4.3.3 SYSTEM IDENTIFICATION VIA THE OPEN-LOOP RESPONSE TO A SINE SWEEP INPUT

The system position output $\theta_o(s)$ is given by Equation (4.5), shown the Figure 4.3. As mentioned in the description of the system, selecting 'open loop' under the Trajectory Setup menu allows a voltage input to be applied to the plant. Among the inputs, the step and sinusoidal inputs are available for open-loop tests.

Here sinusoidal signal is chosen to evaluate the system parameters.

1) Set up the controller. Under the Set-up menu, set T_s=0.00442sec and select Continuous Time Control. Select PI With Velocity Feedback and Set-up Algorithm. Enter the k_p = 1 and k_d = 0, k_i = 0 and select Implement Algorithm, then OK.
2) Under Setup Plot, choose the Control effort (CE) to be displayed on the Left axis and Encoder#1 Velocity to be displayed on the Right axis (If the CE is not available, it should be added to the list using the Data Acquisition setup menu).
3) From Trajectory, choose sinusoidal linear sweep input of 0.1 to 10 Hz, 0.5 volt, and sweep time of 20 sec, and choose Open Loop mode, choose Open Loop mode. Then RUN the system. A plot similar to the one seen below should result.



4) Export raw data to a file. From the menu DATA, choose "*Export Raw Data*", name the data file, e.g. *raw_data_freq.txt*, and then you can transfer data file to MATLAB. More details to see part 2.6.2. Modify the raw data file *raw_data_freq.txt* to a MATLAB m-file separately.
5) Load it into MATLAB workspace. Try to use the functions, e.g. 'iddata', 'tfest' of MATLAB System Identification Toolbox, to obtain DC motor first order and second-order transform functions.

RESULTS

Compare the results from MATLAB and the model obtained from experiment.

```
>> time = data(:,2); %read Time
>> y=data(:,4);      %set Encode1 Pos as y
>> u=data(:,6);      % Control Effort as input
>> dy= diff(y);                %We need speed as Output
>> dy(end +1) = dy(end);       %restore sector size to N
>> zf = iddata(dy,u,0.00884); %set Object iddata
>> tf1 = tfest(zf,1,0) ;       %for First order
```

4.3.4 SYSTEM IDENTIFICATION VIA CLOSED-LOOP SYSTEM RESPONSE

PRELIMINARY

In the Figure 4.1, $G(s) = \frac{K}{s(B+Js)}$, set K_i=0, then the closed-loop transfer function (CLTF) will be:

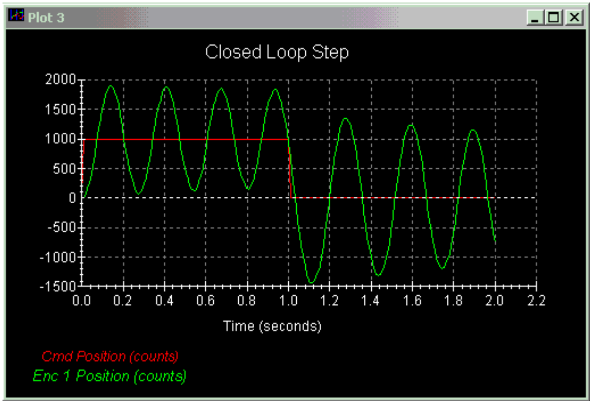
$$T(s) = \frac{KK_p}{Js^2 + (B + KK_d)s + KK_p} \qquad (4.6)$$

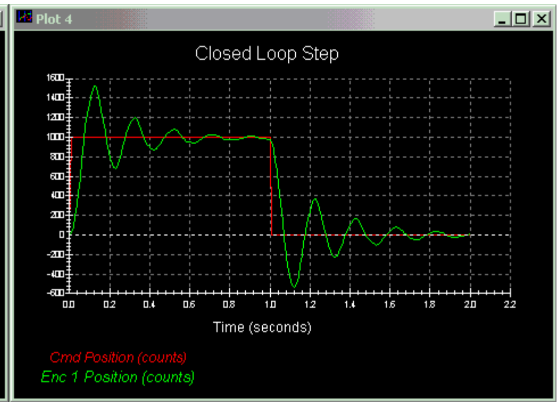
When set K_p =1.0 and K_d=0, the system output will be highly oscillatory. In this case, set a value, e.g. K_d =0.01, just after data acquisition is complete. Click on Plot and view the waveform. If the waveform is good, the raw-data could be save into a file for MATLAB analysis.

Procedure:

1) Set up the controller. Under the Set-up menu and set Ts=0.00442 s and select Continuous Time Control. Select PI With Velocity Feedback and Set-up Algorithm. Enter K_p=1, K_d=0, and K_i=0 and select Implement Algorithm, then OK.
2) Go to Set up Data Acquisition in the Data menu and select encoder #1, Commanded Position as data to acquire and specify data sampling every 2 servo cycles. Select Zero Position from the Utility menu to zero the encoder positions.
3) Under Setup Plot, choose both Commanded Position and Encoder#1 Position to be displayed on the left axis.
4) Enter the Command menu, go to Trajectory and select Step. Select Closed Loop Step and input a step size of 1000 counts, duration of 2000ms and 1 repetition. Select Execute from the Command menu and select Run. The drive disk will step, oscillate, and attenuate, then return. Encoder data is collected to record this response. Select OK after data is uploaded.
5) Click on Plot and view the waveform. If the data is not satisfactory, repeat the above steps or modify K_p and K_d if necessary.
6) Export raw data to a file. From the menu DATA, choose "Export Raw Data", name the data file, e.g. *raw_data_cl_step1.txt*, then you can transfer data file to MATLAB. See the section 2.6.2 for details.
7) Enter the Control Algorithm box, enter the K_p = 1 and K_d = 0.01, K_i = 0 and run steps 4-6 again, save another data file.
8) Convert the raw data files, *raw_data_cl_step1.txt* and *raw_data_step2.txt*, to a MATLAB m-files.
9) Load it into MATLAB workspace. Try to use the functions of MATLAB System Identification Toolbox, such as '***iddata', 'tfest'***, to obtain the second-order transfer functions of the closed-loop system
10) Then calculate the DC motor first-order transfer function G(s) (K, J and B).

```
>> time = data(:,2); %read Time
>> y=data(:,4);      %set Encode1 Pos as y
>> u=data(:,3);      % Control Position as input
>> zf = iddata(y,u,0.00884);  %set Object iddata
>> Tfd2 = tfest(zf,2,0);      % for second order
```





RESULTS

1) Calculate system the open-loop transfer function (OLTF) with the determined values of K, J and B from open-loop tests in section 4.3.2 and 4.3.3.
2) Obtain the OLTF in MATLAB by the data of the closed-loop test., then calculate K, B and J.
3) Compare these models.
4) Write a brief summary about how the lab equipment might facilitate a better understanding of control systems.

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