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HOMEWORK 2

Problem 1

a) $C(s) = \frac{1}{s} \cdot \frac{8}{s+8}$

$$\frac{8}{s(s+8)} = \frac{A}{s} + \frac{B}{s+8}$$

$$8 = A(s+8) + Bs$$

$$8A = 8 \Rightarrow A = 1$$

for s: $A + B = 0 \Rightarrow B = -1$

$$C(s) = \frac{8}{s(s+8)} = \frac{1}{s} - \frac{1}{s+8}$$

$$c(t) = L^{-1}\left(\frac{1}{s} - \frac{1}{s+8}\right)$$

$$c(t) = 1 - e^{-8t}$$

time constant $\tau = \frac{1}{a} = \frac{1}{8}$

rise time $t_r = \frac{2.2}{a} = \frac{2.2}{8} \approx 0.275$

settling time (98%) $t_s = \frac{4}{a} = \frac{4}{8} = \frac{1}{2}$

b) $C(s) = \frac{1}{s} \cdot \frac{15}{s+15}$

$$\frac{15}{s(s+15)} = \frac{A}{s} + \frac{B}{s+15}$$

$$15 = A(s+15) + Bs$$

$$15 = 15A \Rightarrow A = 1$$

for $0 = A + B \Rightarrow B = -1$

$$C(s) = \frac{1}{s} - \frac{1}{s+15}$$

$$c(t) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+15}\right)$$

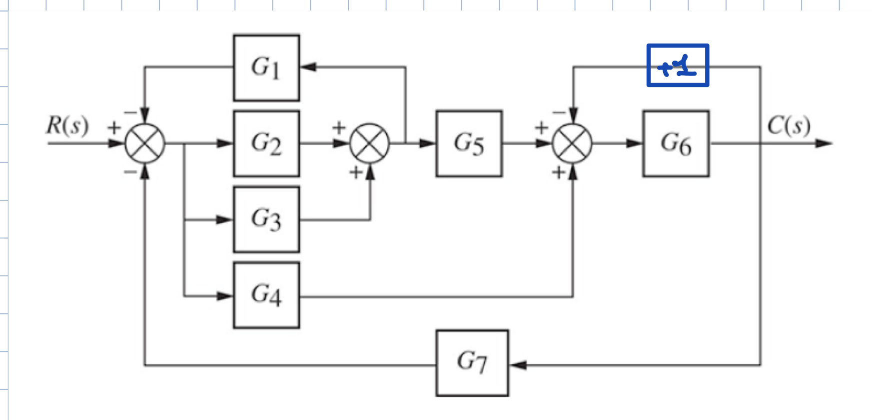
$$c(t) = 1 - e^{-15t}$$

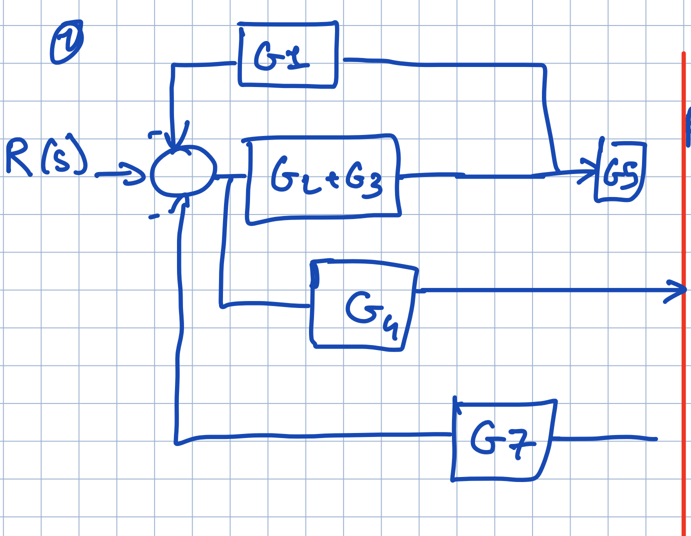
$$\tau = \frac{1}{a} = \frac{1}{15}$$

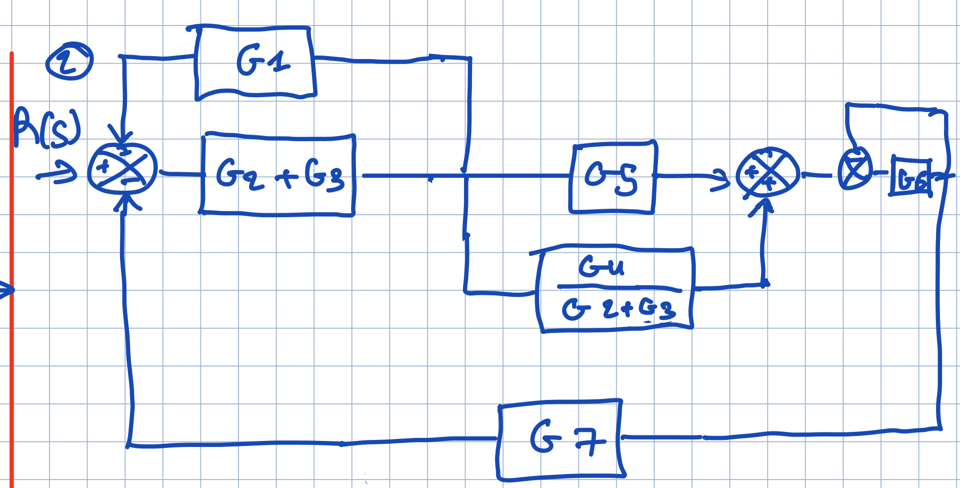
$$t_r = \frac{2.2}{15} \approx 0.147$$

$$t_s(98\%) = \frac{4}{15} \approx 0.267$$

Problem 2







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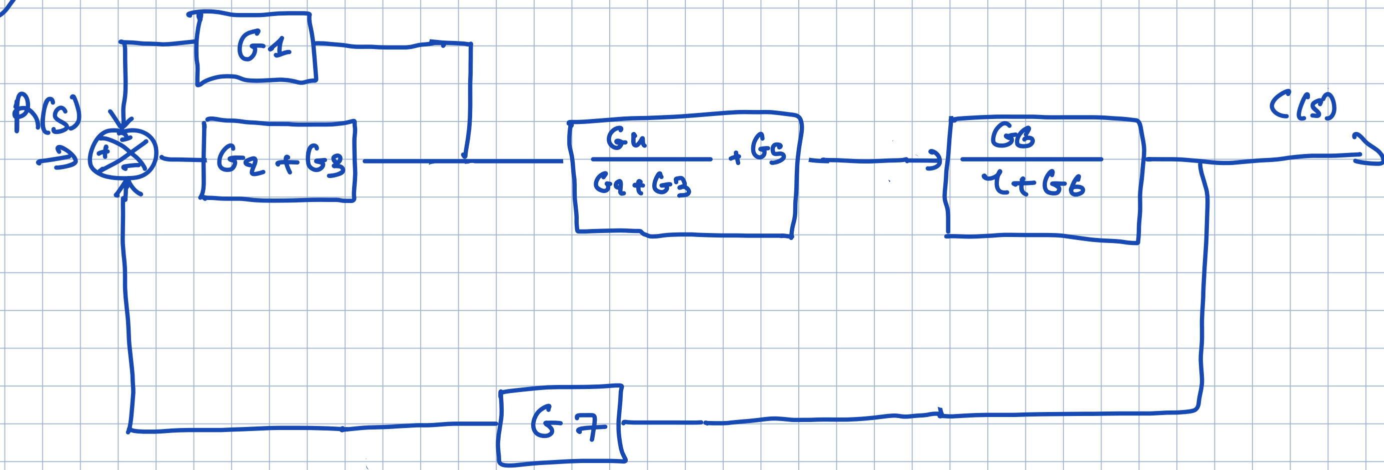

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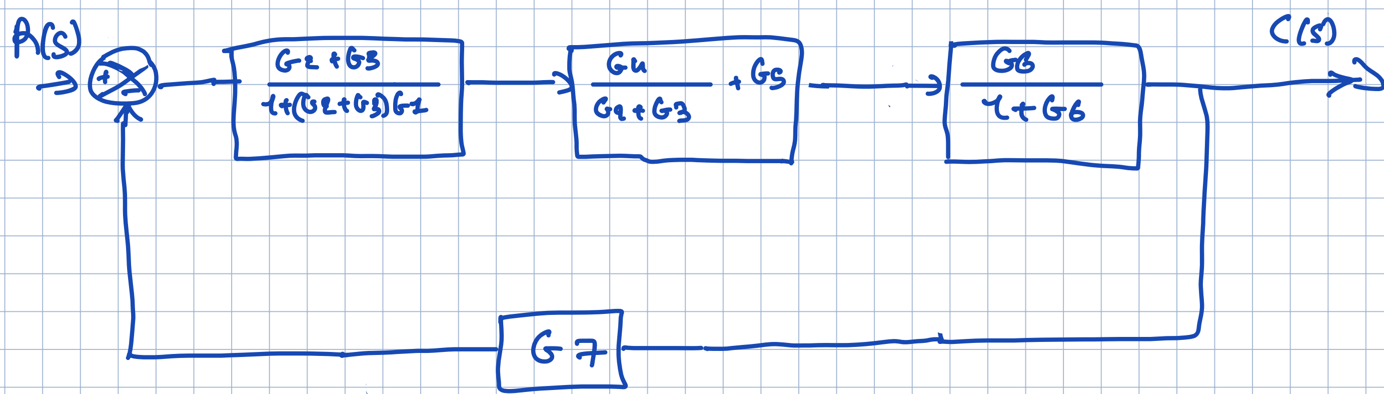

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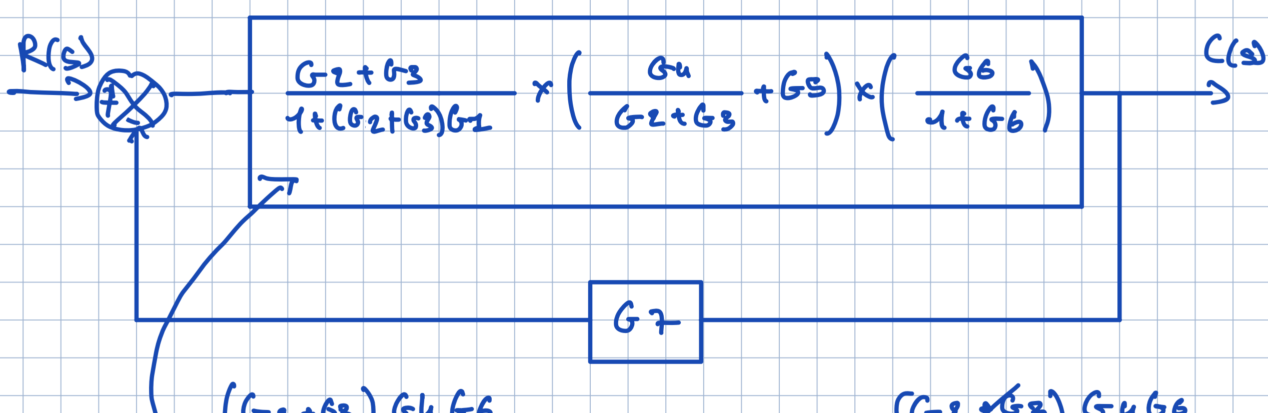
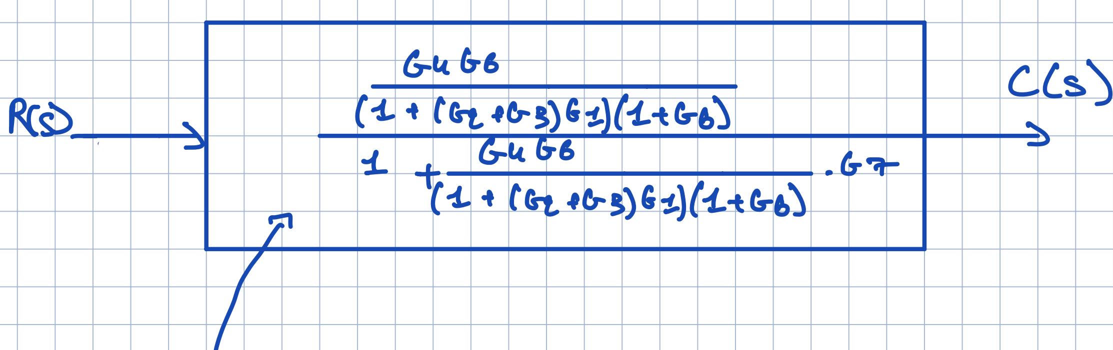


$$\frac{(G_2+G_3)\, G_4 G_6}{\left[1+(G_2+G_3)G_1\right](G_2+G_3)(1+G_6)} = \frac{(G_2 + \cancel{G_3})\, G_4 G_6}{(G_2+\cancel{G_3})\left(1+(G_2+G_3)\right)(1+G_6)}$$

$$= \frac{G_4 G_6}{\left(1+(G_2+G_3)G_1\right)(1+G_6)}$$



$$\frac{(G_2+G_3)\,G_4G_6}{(1+(G_2+G_3)G_1)(1+G_6)+\dfrac{(G_2+G_3)G_4G_6G_7}{(1+(G_2+G_3)G_1)(1+G_6)}}$$

$$=\frac{(G_2+G_3)G_4G_6(1+(G_2+G_3)G_1)(1+G_6)}{(1+(G_2+G_3)G_1)(1+G_6)+(G_2+G_3)G_4G_6G_7}$$

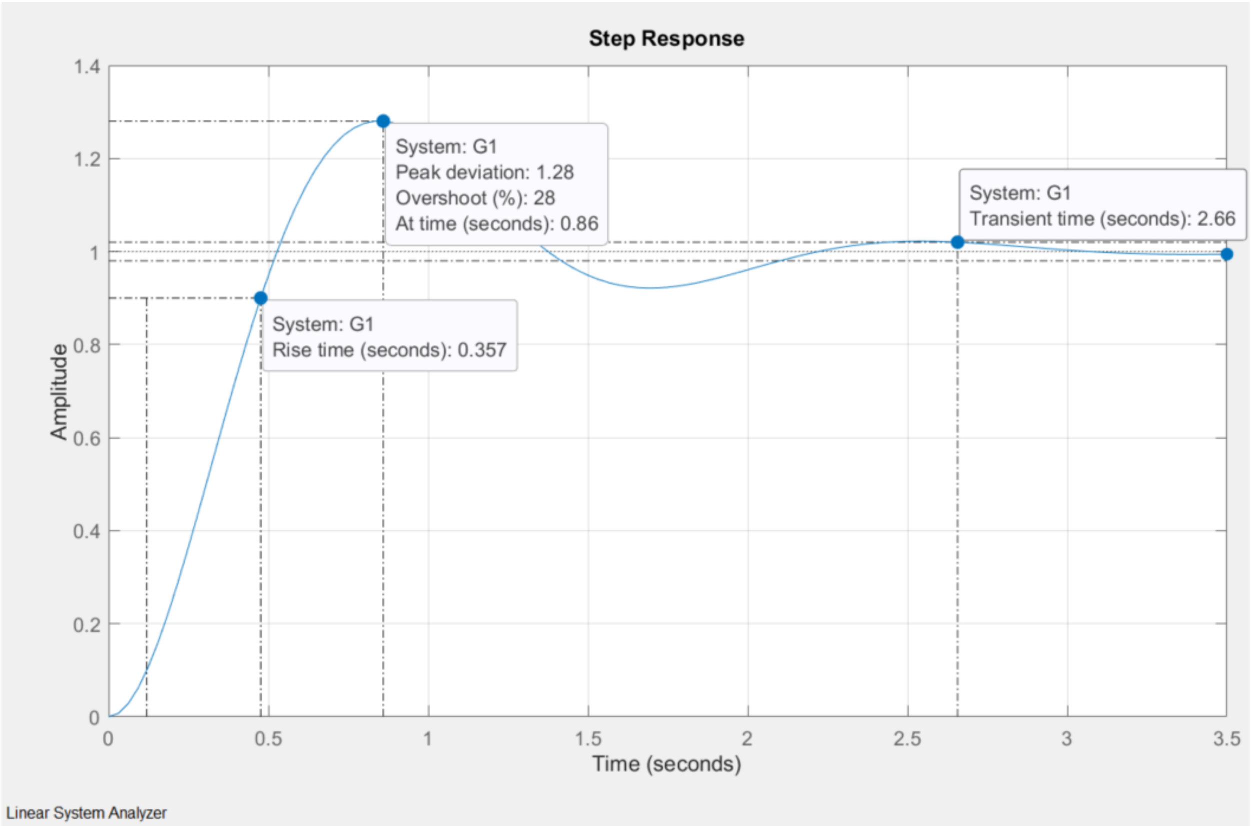
$$C(s) = R(s)\cdot\frac{G_4G_6}{(1+(G_2+G_3)G_1)(1+G_6)+G_4G_6G_7}$$

Problem 3

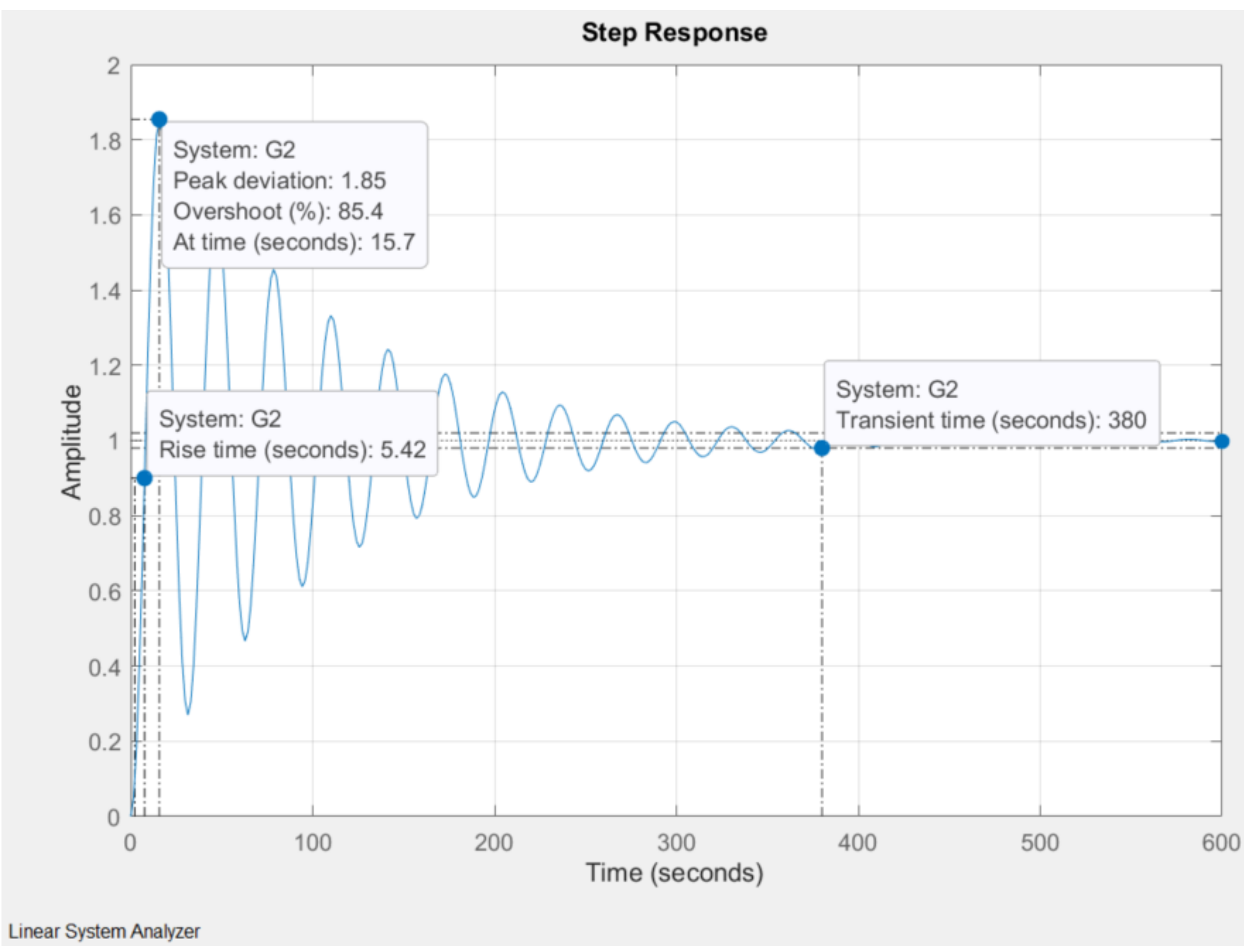
a)

```
>> homework2_question3
System G1:
Damping Ratio: 0.3750
Natural Frequency: 4.0000
Settling Time: 2.6667
Peak Time: 0.8472
Rise Time: 1.5250
Overshoot: 28.0597%

System G2:
Damping Ratio: 0.0500
Natural Frequency: 0.2000
Settling Time: 400.0000
Peak Time: 15.7276
Rise Time: 28.3097
Overshoot: 85.4468%

System G3:
Damping Ratio: 0.2469
Natural Frequency: 3240.3703
Settling Time: 0.0050
Peak Time: 0.0010
Rise Time: 0.0018
Overshoot: 44.9154%
```

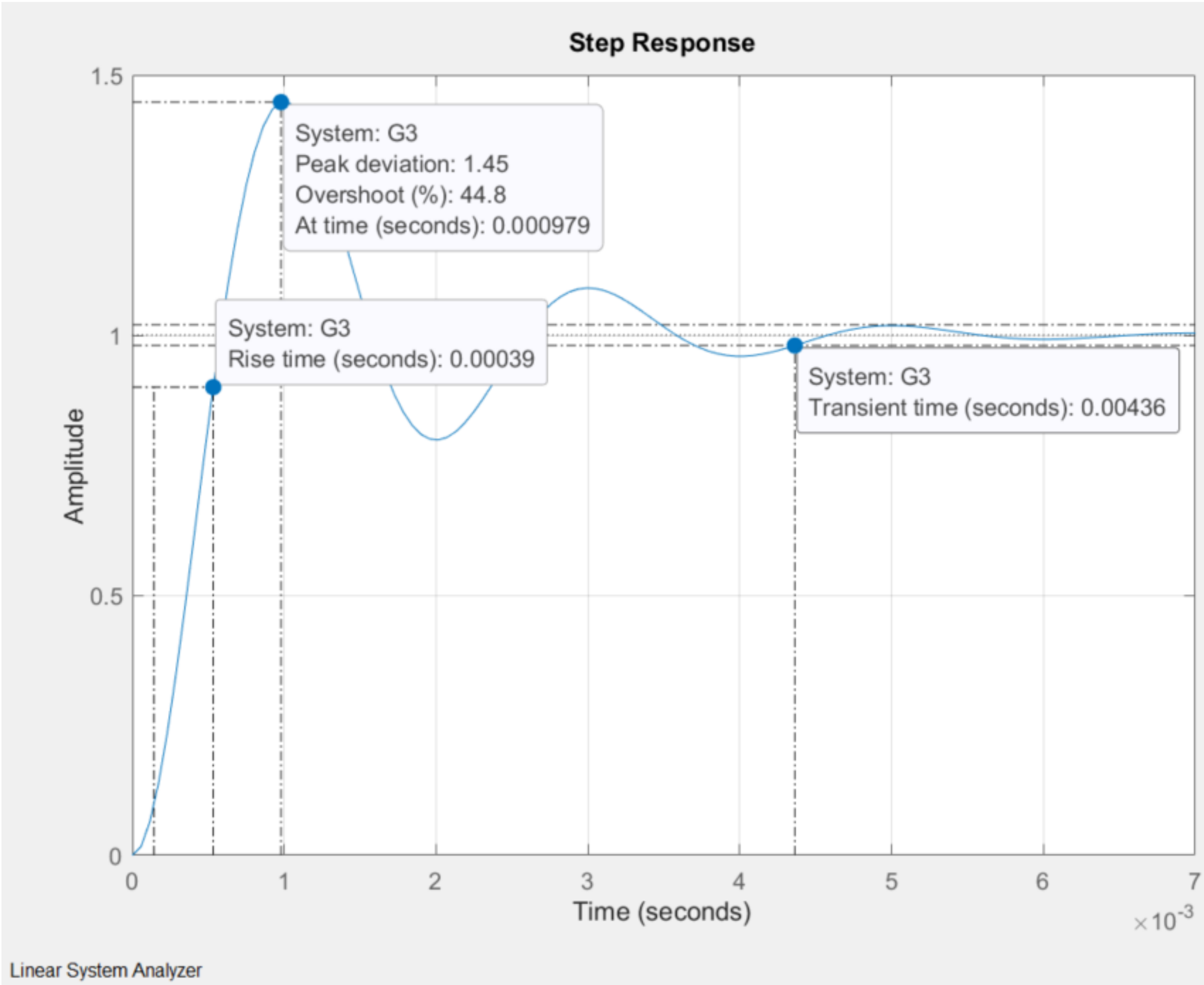
b)



Step Response
System: G1
Peak deviation: 1.28
Overshoot (%): 28
At time (seconds): 0.86
System: G1
Transient time (seconds): 2.66
System: G1
Rise time (seconds): 0.357
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0.5
1
1.5
2
2.5
3
3.5
Linear System Analyzer



Step Response
System: G2
Peak deviation: 1.85
Overshoot (%): 85.4
At time (seconds): 15.7
System: G2
Transient time (seconds): 380
System: G2
Rise time (seconds): 5.42
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
100
200
300
400
500
600
Linear System Analyzer


Step Response
System: G3
Peak deviation: 1.45
Overshoot (%): 44.8
At time (seconds): 0.000979
System: G3
Rise time (seconds): 0.00039
System: G3
Transient time (seconds): 0.00436
Amplitude
Time (seconds)
×10^-3
Linear System Analyzer

Problem 4

a) $\% OS = 15\%$, $T_r = 0.5$ sec

$$0.15 = e^{\frac{-\delta\pi}{\sqrt{1-\delta^2}}}$$

$$\ln(0.15) = \frac{-\delta\pi}{\sqrt{1-\delta^2}}$$

$$(-\delta\pi)^2 = \left(\ln(0.15)\left(\sqrt{1-\delta^2}\right)\right)^2$$

$$\delta^2\pi^2 = \ln(0.15)^2(1-\delta^2)$$

$$\delta^2\pi^2 = \ln(0.15)^2 - \ln(0.15)^2\delta^2$$

$$\delta^2(\pi^2 + \ln(0.15)^2) = \ln(0.15)^2$$

$$\delta^2 = \frac{\ln(0.15)^2}{(\pi^2 + \ln(0.15)^2)}$$

$$\delta = \frac{\ln(0.15)}{\sqrt{(\pi^2 + \ln(0.15)^2)}}$$

$$\delta \approx 0.516$$

$$T_r = \frac{2.16\delta + 0.6}{\omega_n}$$

$$0.5 \approx \frac{2.16(0.516) + 0.6}{\omega_n}$$

$$\omega_n \approx \frac{1.71456}{0.5}$$

$$\omega_n \approx 3.43$$

$$p_1, p_2 = -\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$$

$$= -0.516(3.43) \pm j(3.43)\sqrt{1-(0.516)^2}$$

$$p_1, p_2 = -1.77 \pm 2.94j$$

b) $\%OS = 8\%$, $T_p = 10 sec$

$$0.08 = e^{\frac{-\delta\pi}{\sqrt{1-\delta^2}}}$$

$$\ln(0.08) = \frac{-\delta\pi}{\sqrt{1-\delta^2}}$$

$$(\delta\pi)^2 = \ln(0.08)^2(1-\delta^2)$$

$$\delta^2\pi^2 = 6.38(1-\delta^2)$$

$$\delta^2(\pi^2+6.38) = 6.38$$

$$\delta^2 = \frac{6.38}{(\pi^2+6.38)}$$

$$\delta = \sqrt{\frac{6.38}{(\pi^2+6.38)}}$$

$$\delta \approx 0.626$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\delta^2}}$$

$$10 \approx \frac{\pi}{\omega_n\sqrt{1-0.626^2}}$$

$$\omega_n = \frac{\pi}{10(0.779)}$$

$$\omega_n \approx 0.403$$

$$p_1, p_2 = -0.626(0.403) \pm j\,0.403\sqrt{1-(0.626)^2}$$

$$p_1, p_2 = -0.252 \pm 0.314j$$

c) $T_s(98\%) = 1 sec$, $T_p = 1.1 sec$

$$T_s = \frac{4}{\delta\omega_n}$$

$$1 = \frac{4}{\delta\omega_n} \Rightarrow \delta\omega_n = 4$$

$$\omega_n = \frac{4}{\delta}$$

$$1.1 = \frac{\pi}{\omega_n\sqrt{1-\delta^2}}$$

$$\omega_n(\sqrt{1-\delta^2}) = \frac{\pi}{1.1} = 2.86$$

$$\omega_n^2(1-\delta^2) = 8.16$$

We have $\omega_n = \frac{4}{\delta}$

$$\left(\frac{4}{\delta}\right)^2(1-\delta^2) = 8.16$$

$$\frac{16}{\delta^2} - \frac{16\delta^2}{\delta^2} = 8.16$$

$$\frac{16}{\delta^2} - 16 = 8.16$$

$$16\left(\frac{1}{\delta^2} - 1\right) = 8.16$$

$$\frac{1}{\delta^2} - 1 = 0.51$$

$$\frac{1}{\delta^2} = 1.51$$

$$\delta^2 = 0.662$$

$$\delta \approx 0.814$$

back to $\omega_n = \frac{4}{\delta}$,

$$\omega_n = \frac{4}{0.814} \approx 4.92$$

$$p_1, p_2 = -(0.814)(4.92) \pm j(4.92)\sqrt{1-(0.814)^2}$$

$$p_1, p_2 = -4.00 \pm 2.86j$$

Problem 5

a) $m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$

$$5\ddot{x}(t) + 1\dot{x}(t) + 10x(t) = f(t)$$

$$L(f(t)) = 5s^2X(s) + sX(s) + 10X(s)$$

$$X(s)(5s^2 + s + 10) = F(s)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{5s^2+s+10}$$

Problem 5

a)

```
>> homework2_question5
  tf with properties:

       Numerator: {[0 0 1]}
     Denominator: {[5 1 10]}
        Variable: 's'
         IODelay: [0]
      InputDelay: [0]
     OutputDelay: [0]
       InputName: {''}
       InputUnit: {''}
      InputGroup: [1×1 struct]
      OutputName: {''}
      OutputUnit: {''}
     OutputGroup: [1×1 struct]
           Notes: [0×1 string]
        UserData: []
            Name: ''
              Ts: [0]
        TimeUnit: 'seconds'
    SamplingGrid: [1×1 struct]
```

b)

```
Poles of the transfer function:
  -0.1000 + 1.4107i
  -0.1000 - 1.4107i

Damping Ratio: 1.4142
Natural Frequency: 0.0707
Percent Overshoot: 80.0329%
Settling Time (Ts): 38.2106 sec
Peak Time (Tp): 2.2214 sec
Rise Time (Tr): 0.7786 sec
```

b) $G(s) = \dfrac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$

$$\frac{1}{5s^2 + s + 10} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$$5s^2 + s + 10 \rightarrow s^2 + \frac{1}{5}s + 2$$

$$\omega_n^2 = 2 \Rightarrow \omega_n = \sqrt{2}$$

$$2\delta\omega_n = \frac{1}{5}$$

$$2\delta\sqrt{2} = \frac{1}{5}$$

$$\delta = \frac{0.2}{2\sqrt{2}} \approx 0.0707$$

$$\%OS = e^{\frac{-0.0707\pi}{\sqrt{1-0.0707^2}}} \times 100$$

$$\approx 81.87\%$$

Settling time (98%)

$$T_s \approx \frac{4}{\delta\omega_n} = \frac{4}{0.0707\sqrt{2}} \approx 40.02 \text{ sec}$$

Peak time

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} = \frac{\pi}{\sqrt{2}\sqrt{1-0.0707^2}} \approx 2.22 \text{ sec}$$

Rise time

$$T_R \approx \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{2}} \approx 1.27 \text{ s}$$

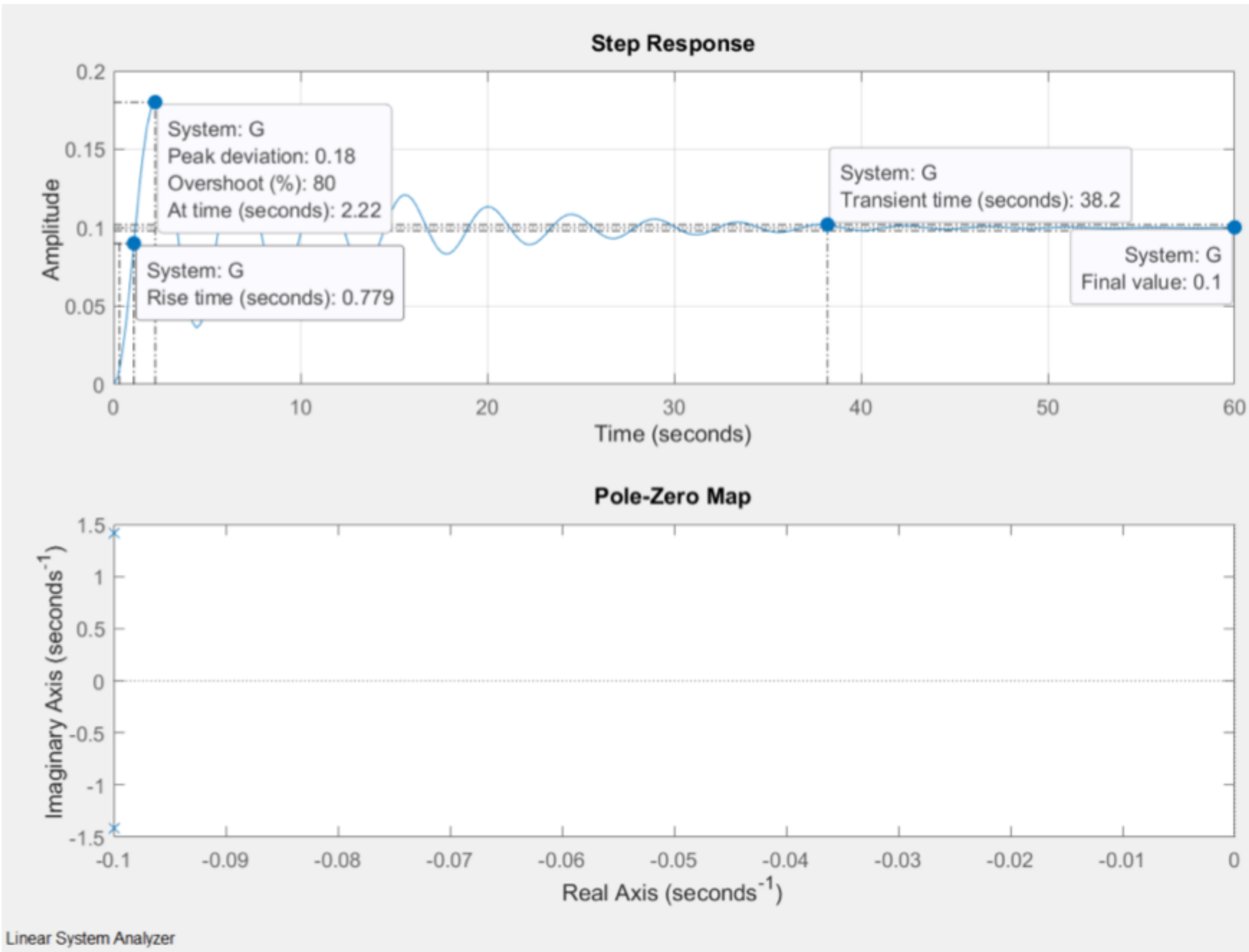
Final value

$$x(\infty) = \lim_{s \to 0} s \cdot X(s) \Rightarrow X(s) = G(s) \cdot \frac{1}{s}$$

$$X(s) = \frac{1}{5s^2 + s + 10} \cdot \frac{1}{s}$$

$$x(\infty) = \lim_{s \to 0} \frac{1}{5s^2 + s + 10} = 0.1$$

c)



Problem 6

$$\% OS = e^{\frac{-\delta\pi}{\sqrt{1-\delta^2}}} \times 100$$

to achieve $\% OS < 5\%$

$$5 > e^{\frac{-\delta\pi}{\sqrt{1-\delta^2}}} \times 100$$

$$\ln(0.05) > \frac{-\delta\pi}{\sqrt{1-\delta^2}}$$

$$-2.9957 > -\frac{\pi\delta}{\sqrt{1-\delta^2}}$$

$$2.9957 < \frac{\delta\pi}{\sqrt{1-\delta^2}}$$

$$\frac{2.9957\sqrt{1-\delta^2}}{\pi} < \frac{\delta\cancel{\pi}}{\cancel{\pi}}$$

$$0.953 < \frac{\delta}{\sqrt{1-\delta^2}}$$

$$\frac{\sqrt{1-\delta^2}}{\delta} < \frac{1}{0.953}$$

$$(\sqrt{1-\delta^2})^2 < (1.049\,\delta)^2$$

$$1-\delta^2 < 1.100\,\delta^2$$

$$1 < 1.100\,\delta^2 + \delta^2$$

$$1 < 2.100\,\delta^2$$

$$\delta^2 > \frac{1}{2.100}$$

$$\delta^2 > 0.476$$

$$\delta > \sqrt{0.476} \approx 0.69$$

$$T_s = \frac{4}{\delta \omega_n}$$

to achieve $T_s < 4$

$$4 > \frac{4}{\delta \omega_n}$$

$$\delta \omega_n > 1$$

$$\omega_n > \frac{1}{\delta}$$

$$\omega_n > \frac{1}{0.69} \approx 1.45$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\delta^2}}$$

to achieve $T_p < 1$

$$1 > \frac{\pi}{\omega_n \sqrt{1-\delta^2}}$$

$$\omega_n \sqrt{1-\delta^2} > \pi$$

$$\omega_n \sqrt{1-0.69^2} > \pi$$

$$\omega_n \times 0.72 > \pi$$

$$\omega_n > \frac{\pi}{0.724} \approx 4.34$$

$$\omega_n > \mathrm{Max}(1.45, 4.36) = 4.34$$

$$s = -\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$$

$$Re(s) = -\delta\omega_n$$

$$Re(s) < -0.69 \times 4.36$$

$$Re(s) < -3.01$$

real part of the poles must be less than -3.01

$$Im(s) = \omega_n \sqrt{1-\delta^2}$$

$$> 4.34 \sqrt{1-0.69^2}$$

$$> 4.34\,(0.724)$$

$$Im(s) > 3.14 \approx \pi$$