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30/07/2024

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Objective

The objective of this experiment is to study the transient and frequency response characteristics of open-loop and closed-loop control systems. This includes analyzing the transient response parameters such as rise time, peak time, percent overshoot, and settling time for various system configurations. Additionally, the lab aims to investigate the frequency response of first-order and second-order systems by determining the magnitude ratio and phase shift, as well as identifying the cut-off frequency of the systems. The effect of proportional gain and derivative feedback on system performance will also be explored. The insights gained from this study will help understand the behavior of control systems under different conditions and how to optimize their performance.

Theory

In this lab, we explore the transient and frequency responses of control systems to gain insights into their dynamic behavior and performance. The transient response of a system is characterized by how it reacts to changes in input before reaching a steady state. This includes parameters such as rise time, peak time, percent overshoot, and settling time, which are particularly relevant for second-order systems.

Transient Response:

For second-order systems, the transient response is determined by the natural frequency (ωn) and the damping ratio (ζ). The damping ratio is a key parameter that indicates whether a system is overdamped, underdamped, or critically damped.

- **Rise Time (Tr):** The time it takes for the system response to rise from 10% to 90% of its final value.
- **Peak Time (Tp):** The time at which the system reaches its maximum overshoot.
- **Percent Overshoot (PO):** The percentage by which the peak value of the response exceeds the final steady-state value, calculated as:

$$\text{Percent Overshoot (PO)} = 100 \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)\%$$

- **Settling Time (Ts):** The time it takes for the system response to remain within a certain percentage (commonly 2% or 5%) of the final value.

The transient response is significantly influenced by the gain parameters of the system. Increasing the proportional gain (Kp) typically results in a faster rise time but may increase overshoot. Incorporating derivative feedback (Kd) helps in damping the system, reducing overshoot and improving stability.

Frequency Response:

The frequency response of a system describes its steady-state response to sinusoidal inputs across a range of frequencies. This analysis is critical for understanding how systems behave in the frequency domain and involves evaluating the magnitude and phase of the output relative to the input.

- **Magnitude Ratio (M):** The ratio of the output amplitude to the input amplitude as a function of frequency.
- **Phase Shift (ϕ):** The difference in phase between the input and output signals.

The Bode plot, which consists of the magnitude and phase plots, provides a graphical representation of a system's frequency response. It is instrumental in assessing gain and phase margins, which indicate system stability. The cut-off frequency (ωc), where the system's output falls to 70.7% of its peak value, is crucial for determining bandwidth and performance limits.

Second-Order System Frequency Response:

For second-order systems, the frequency response helps in understanding how the system's gain and phase change over various frequencies, providing insights into resonance effects and stability margins.

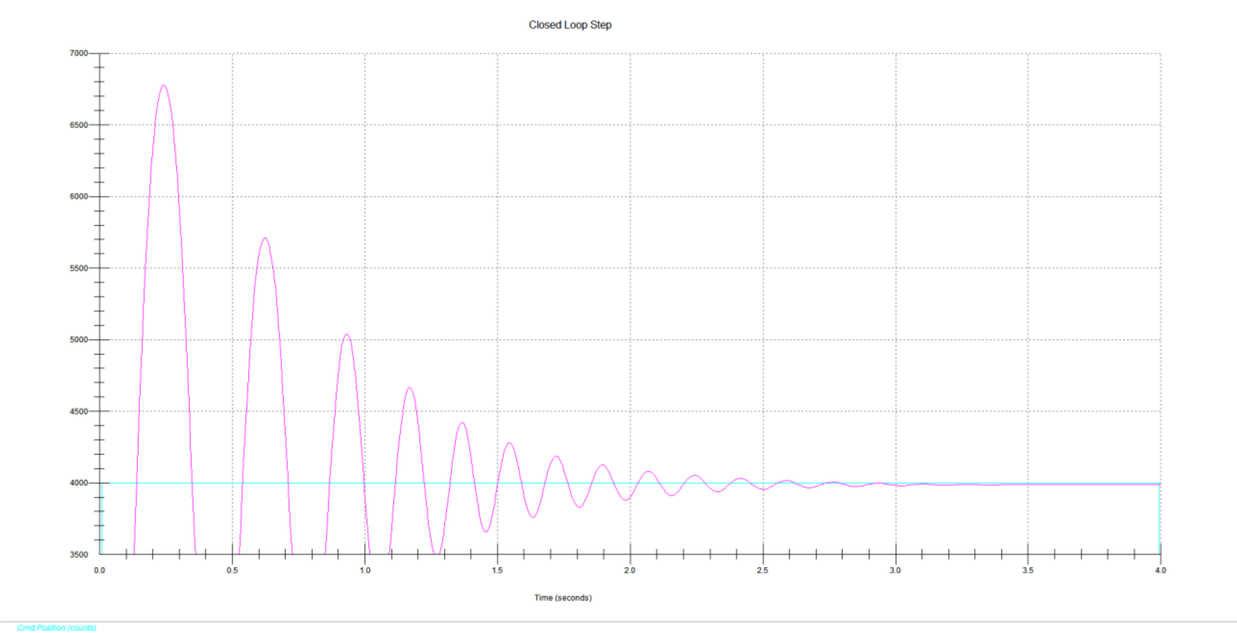
By analyzing both transient and frequency responses, we gain a comprehensive understanding of the system's behavior, allowing us to design and optimize control strategies effectively.

Tasks/Results/Discussion

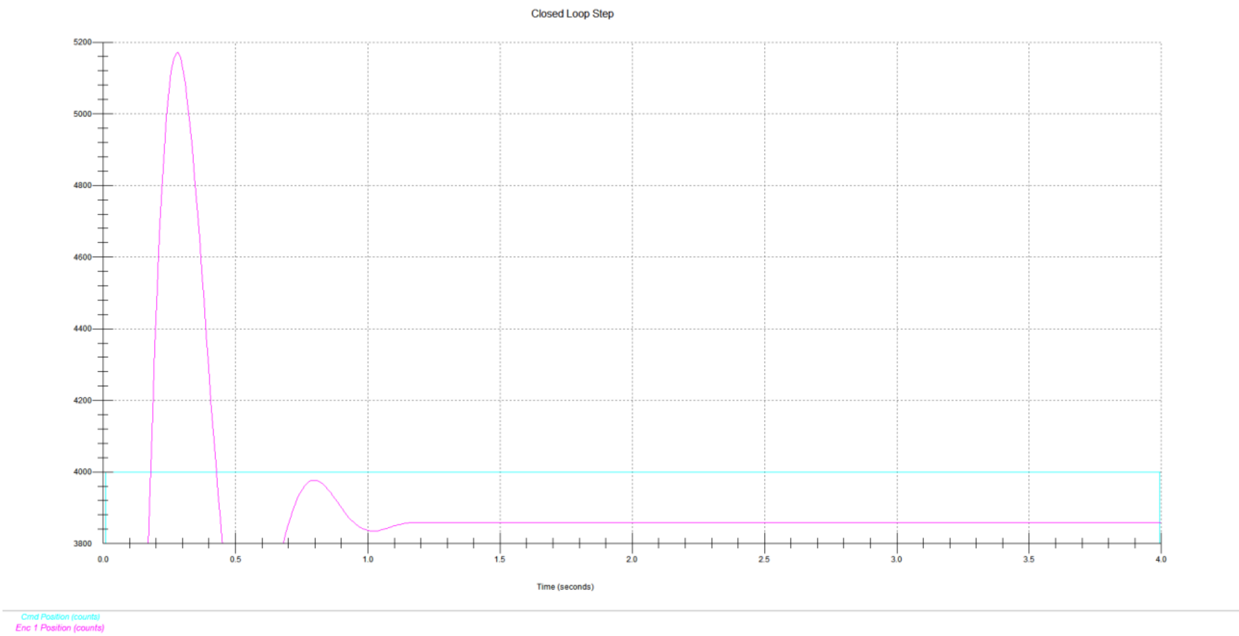
Task 6.3: Closed-Loop Transient Response

1. **Control Algorithm Setup:** The controller was reset from the Utility menu, and the "PI + Velocity Feedback" control algorithm was implemented with continuous time settings. The parameters were set as follows: Ts = 0.00442 sec, Kp = 1, Ki = 0, and Kd = 0.005**.**
2. **Step Response Execution:** A closed-loop step response was obtained using a step size of 4000 Counts, a dwell time of 4000 ms, and 1 repetition. The step test was executed, and the plot was displayed using axis scaling to focus on the response within the dwell period of 4 seconds. Values for the damped natural frequency (ωd) and the overshoot (OS) were obtained from the plot using suitable axis scaling.
3. **Data Export:** The raw data was exported into a data file for further analysis in MATLAB.
4. **Parameter Variation:** Steps 1 and 2 were repeated with different Kp values of 0.7, 0.4, and 0.1. For each Kp, the corresponding values of ωd and OS were obtained. The controller was reset from the 'Set Algorithm' menu, and 'Implement' was clicked after each change in Kp to apply the new settings.

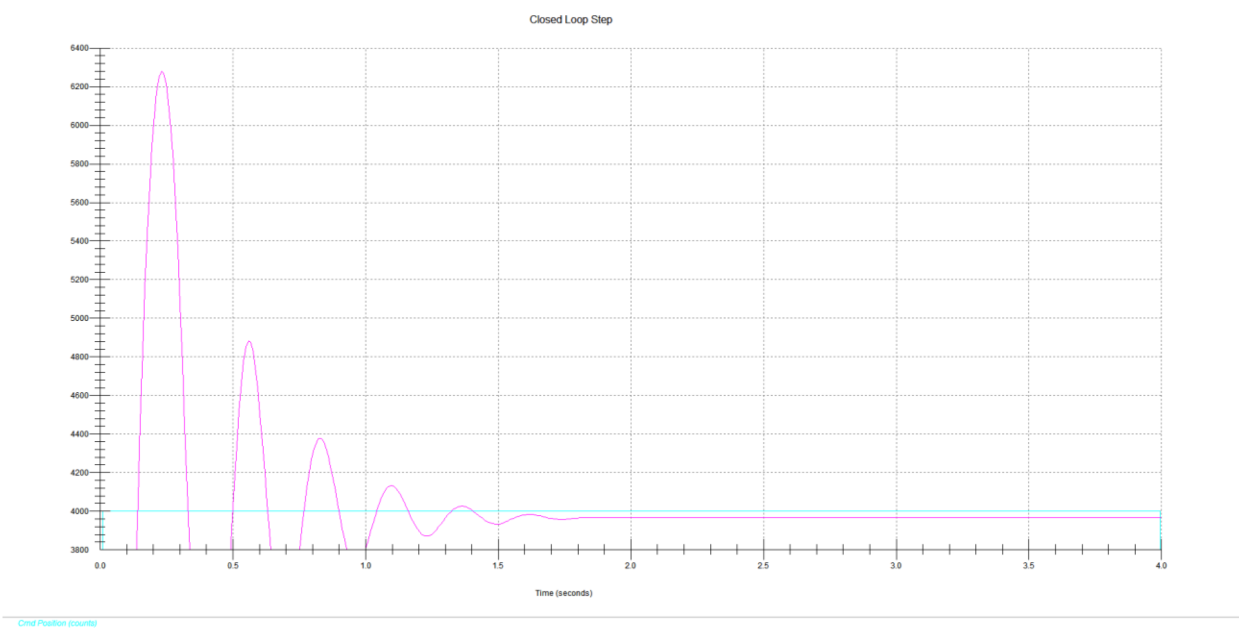
Results and Observations



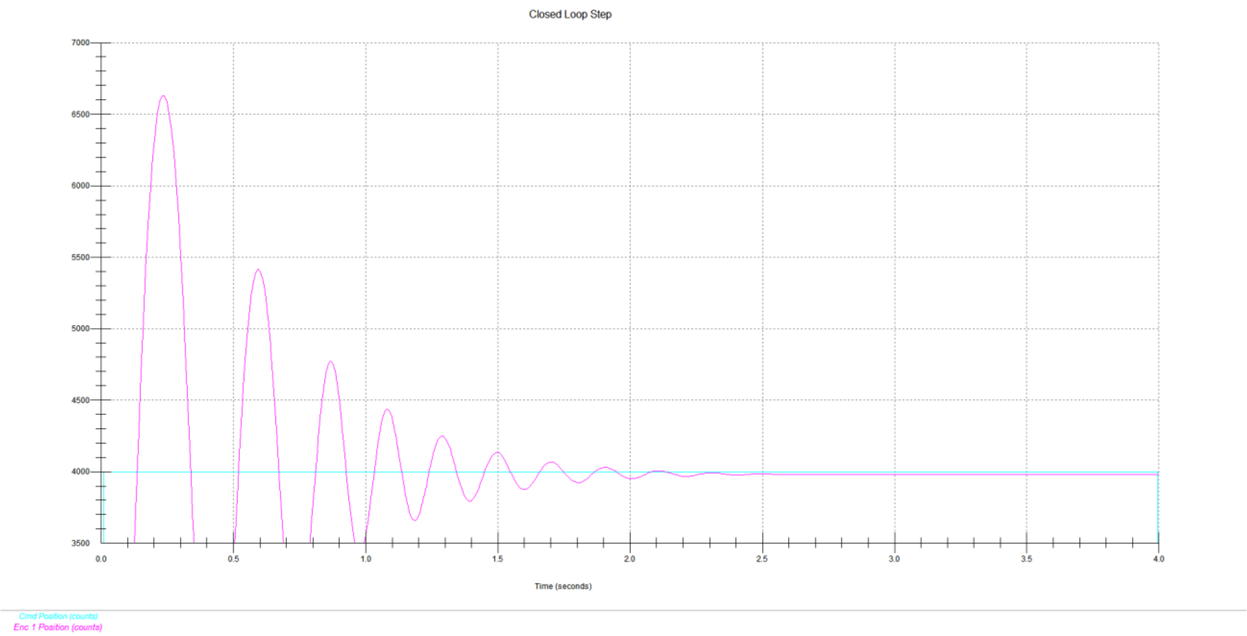
Plot 1: Closed-loop Step Response with Kp = 1



Plot 2: Closed-loop Step Response with Kp = 0.1



Plot 3: Closed-loop Step Response with Kp = 0.4



Plot 4: Closed-loop Step Response with Kp = 0.7

1)

From plot 1: $\%OS = \frac{6750-4000}{4000} \times 100 = 68.75\%$

$$\omega d = \frac{\pi}{Tp} = \frac{\pi}{0.25} \approx 12.57\, rad/s$$

Kp	**Steady-State Value (counts)**	**Overshoot (%)**	**ωd (rad/sec)**
1.0	6750	68.75	12.57
0.7	6600	65	12.57
0.4	6280	57	12.57
0.1	5160	29	12.57

2)

Open-loop systems operate without feedback, meaning they execute control actions based solely on the initial input. They are simple and cost-effective but lack the ability to adjust for errors or changes in the environment, making them less accurate.

Closed-loop systems, on the other hand, use feedback to continuously adjust control actions based on the output. This allows for greater precision and reliability, as the system can correct deviations and respond to disturbances. While more complex and expensive, closed-loop systems are essential for applications requiring high accuracy and adaptability.

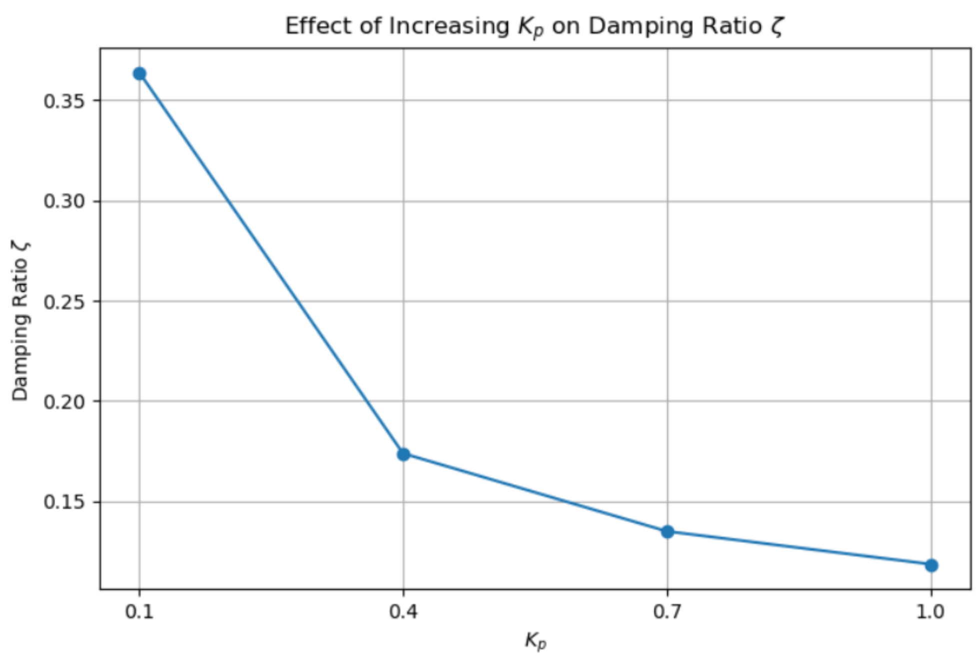
3)

For Kp = 1:

- OS = 68.75%, so p=0.6875
- ln p= ln (0.6875) ≈ −0.3747

$$\zeta = \frac{\sqrt{(-0.3747)^2}}{\sqrt{(9.87 + (-0.3747^2)}} \approx 0.1349$$

Kp	**OS (%)**	ζ
1.0	68.75	0.1185
0.7	65	0.1349
0.4	57	0.1738
0.1	29	0.3640



Plot 5: Effect of Increasing Kp on Damping Ratio ζ

As the proportional gain Kp decreases, the damping ratio ζ increases, indicating a more damped system response. This relationship shows that higher Kp values lead to a more oscillatory response, while lower Kp values result in a system with less oscillation and greater stability. This finding is consistent with the behavior of control systems, where increasing Kp can lead to overshooting and reduced damping.

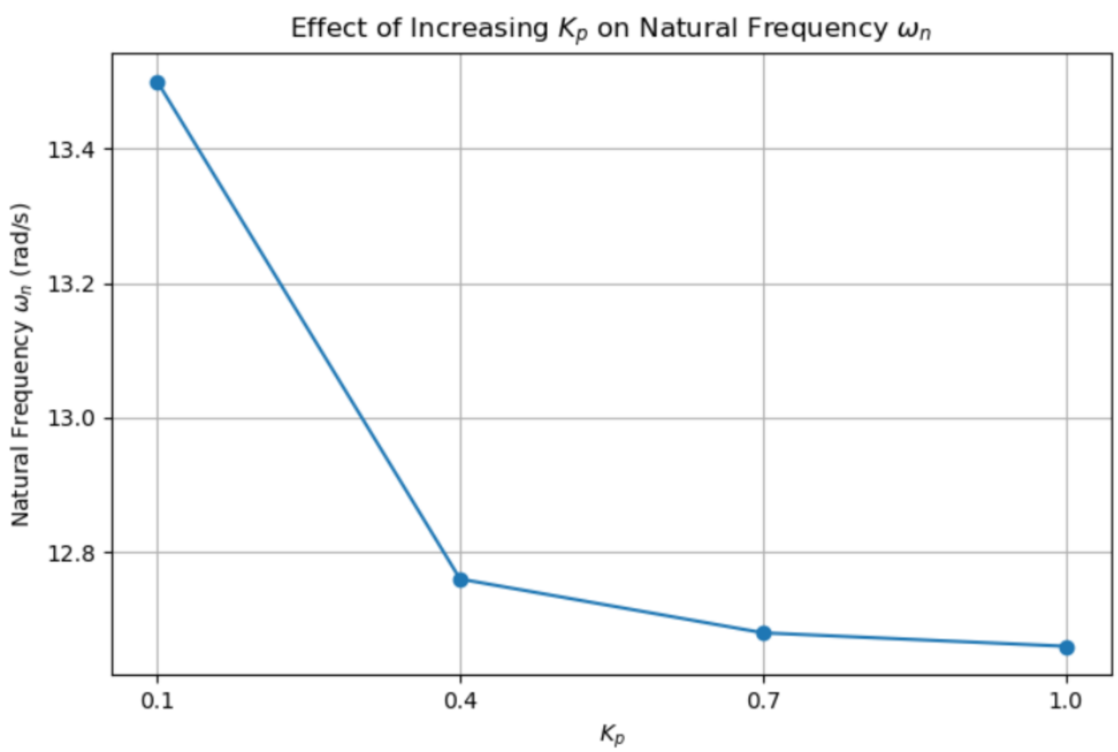
4)

For Kp = 1:

- ωd=12.57 rad/s, ζ=0.1185

$$\omega_n = \frac{12.57}{\sqrt{(1-(0.1185^2)}} \approx 12.66\, rad/s$$

Kp	**ωd (rad/s)**	ζ	ωn (rad/s)
1.0	12.57	0.1185	12.66
0.7	12.57	0.1349	12.68
0.4	12.57	0.1738	12.76
0.1	12.57	0.3640	13.50



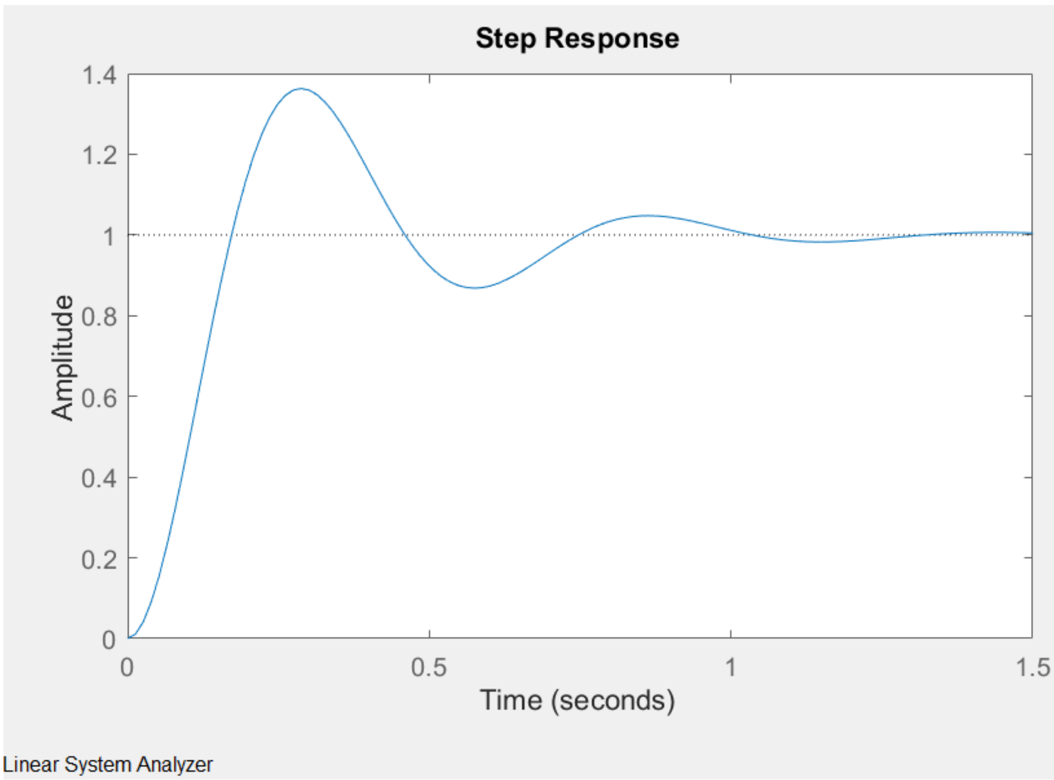
Plot 6: Effect of Increasing Kp on Natural Frequency ωn

The plot shows that as the proportional gain Kp decreases, the natural frequency ωn slightly increases. This trend suggests that reducing Kp results in a system with a higher natural frequency, indicating faster oscillations in response to disturbances. As Kp decreases, the system becomes less stable, as indicated by the increasing natural frequency and decreased damping effect.

Matlab Analysis

1) and 2)

```
K=5;
B=0.00173;
J=0.003791;
Kp = 0.1;
Ki = 0;
Kd = 0.005;
s = tf('s');
cltf=(K*Kp)/(J*s^2+(B+K*Kd)*s+K*Kp);
ltiview(cltf);
```



Plot 7: MATLAB plot of Closed-loop Step Response with Kp = 0.1

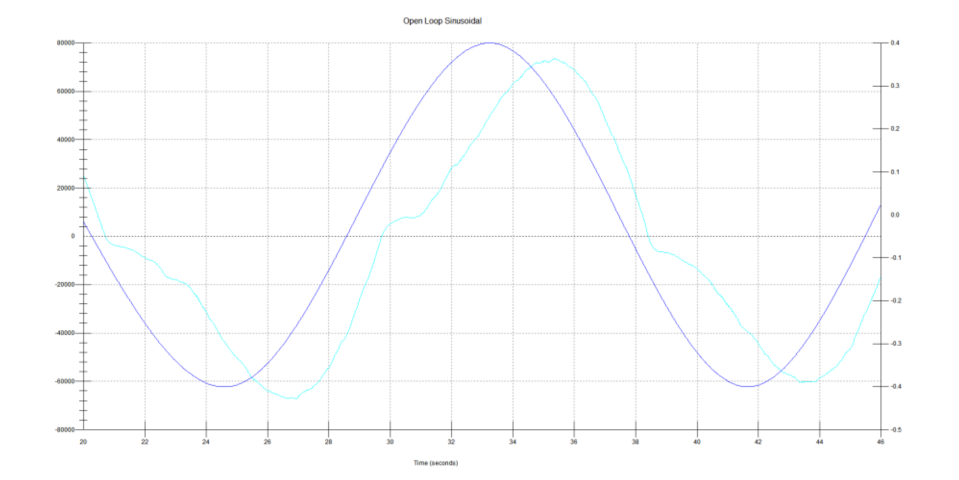
3)

When comparing the theoretical and experimental step responses for Kp=0.1, both show underdamped behavior with overshoot. The theoretical model predicts a smooth response with slight overshoot, while the experimental data indicates a peak of approximately 5171 counts, resulting in a noticeable steady-state error as the system stabilizes around 3859 counts instead of the commanded 4000. This discrepancy is likely due to unmodeled system dynamics such as friction and parameter variations. Additionally, the settling time is slightly longer in the experimental data, suggesting additional damping factors in the actual system.

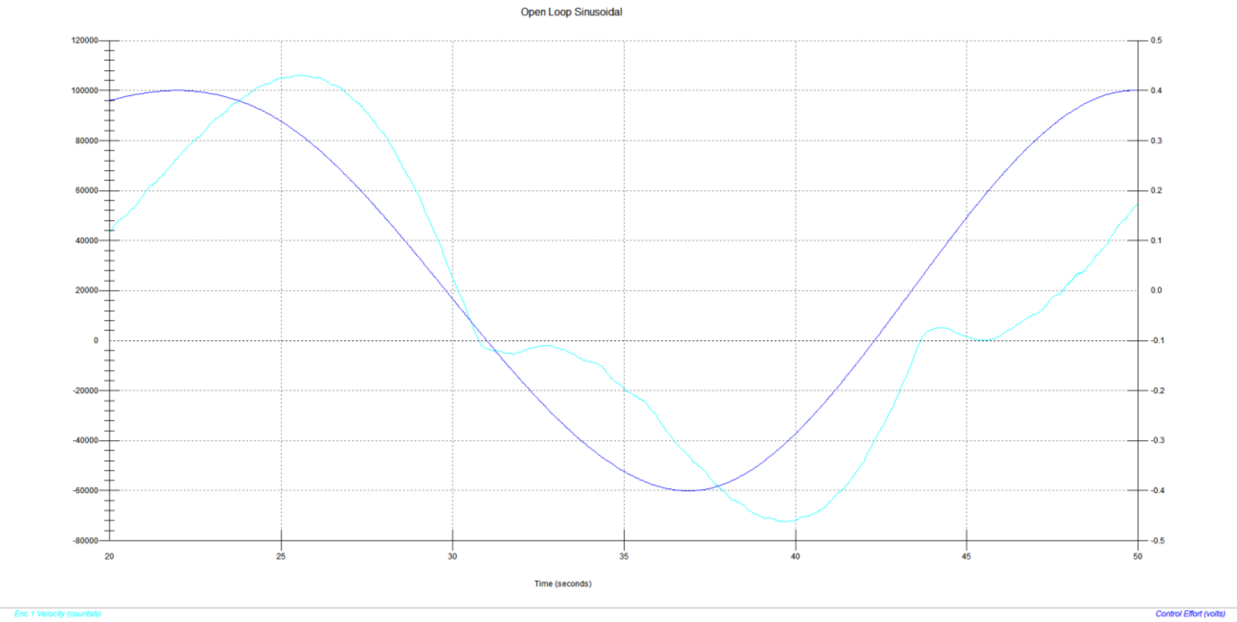
Task 6.4: Open-Loop Frequency Response

1. **Data Acquisition and Setup:** The controller was reset from the UTILITY menu. Under the Data Acquisition menu, Control Effort (CE) was added to the list. The plot was set up to display Encoder #1 Velocity on the left axis and CE on the right axis.
2. **Sinusoidal Trajectory Setup:** From the COMMAND menu, Trajectory was selected, and Sinusoidal was chosen. The setup was configured with an OPEN LOOP, amplitude of 0.4 volts, frequency of 0.06 Hz, and 8 repetitions. The test was executed by clicking on Execute and RUN. After data acquisition, the output was plotted.
3. **Measure Output Parameters:** The initial transient part was ignored, and the constant peak-to-peak angular velocity output magnitude (Ωpp) in counts/sec was measured. The time difference (tϕ) between two adjacent peaks was measured to estimate the phase shift. Axis Scaling was used to examine a section of the steady-state display for accurate measurements.
4. **Frequency Analysis:** Steps 2 and 3 were repeated to obtain values for Ωpp (in counts/sec) and phase (ϕ in degrees) at frequencies of 0.04, 0.08, 0.1, and 0.2 Hz. Records and full-screen displays were obtained for each frequency. The values of radian frequency (ω), Ωpp (counts/sec), and ϕ (degrees) were tabulated.
5. **Sweep Frequency Display:** From the COMMAND menu, Trajectory was selected, followed by Sine Sweep. The setup was configured with an OPEN LOOP, amplitude of 0.5 volts (adjustable if necessary), frequency range from 0.02 Hz to 0.09 Hz, and Sweep Time (Linear Sweep) of 50 sec. Under Set Up Plot, linear frequency sweep was selected. The test was executed, and the output was plotted. The plot was minimized, then re-plotted using a logarithmic frequency sweep to visualize the magnitude frequency response.

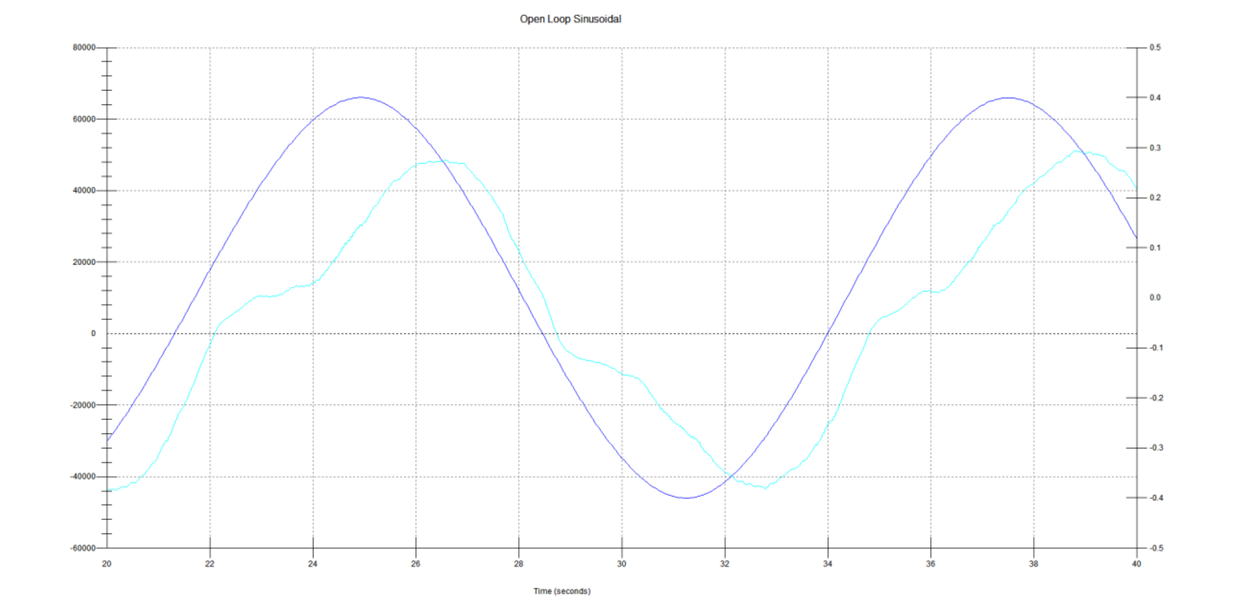
Results and Observations



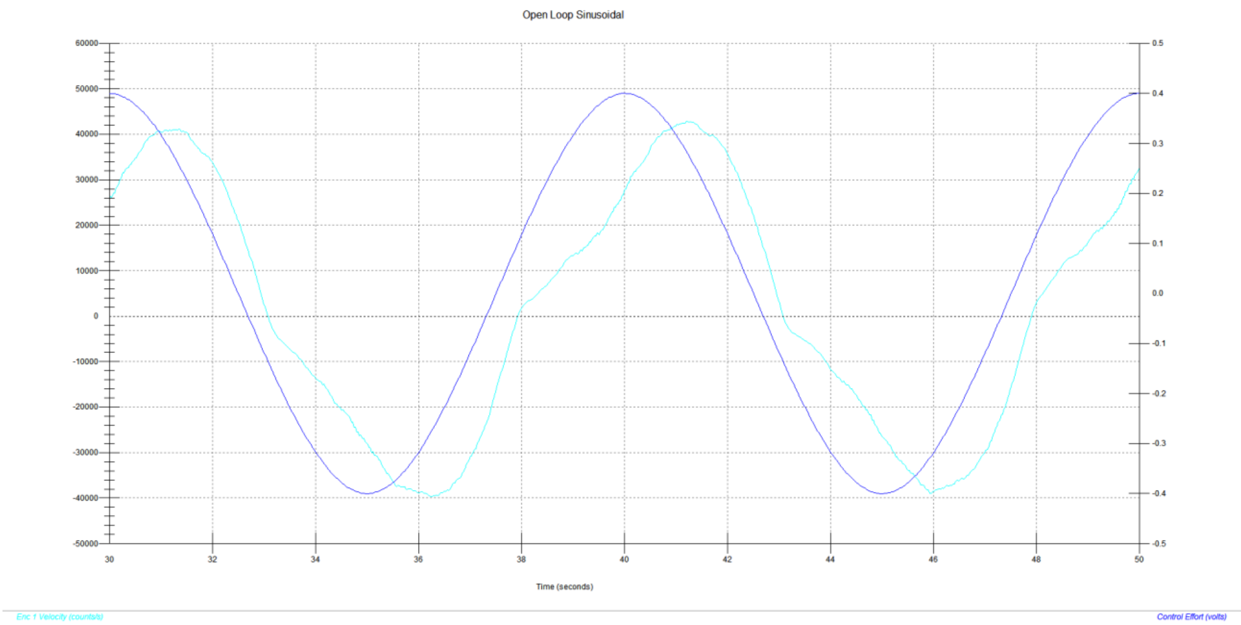
Plot 8: Open-Loop Frequency Response with frequency = 0.06



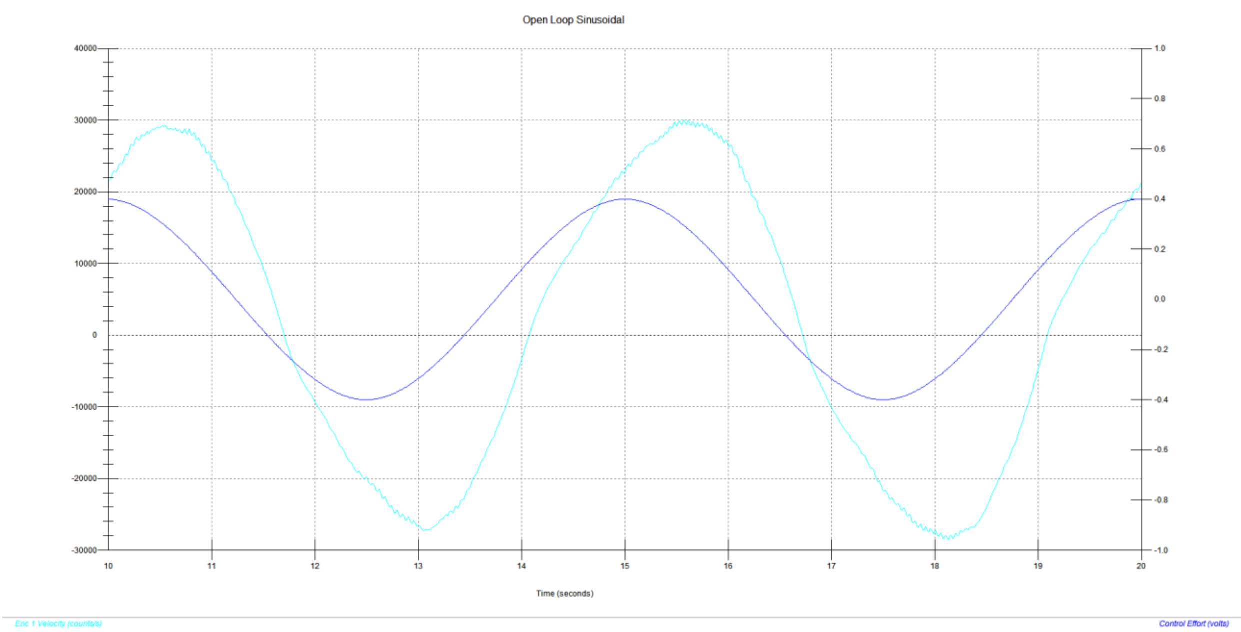
Plot 9: Open-Loop Frequency Response with frequency = 0.04



Plot 10: Open-Loop Frequency Response with frequency = 0.08



Plot 11: Open-Loop Frequency Response with frequency = 0.1



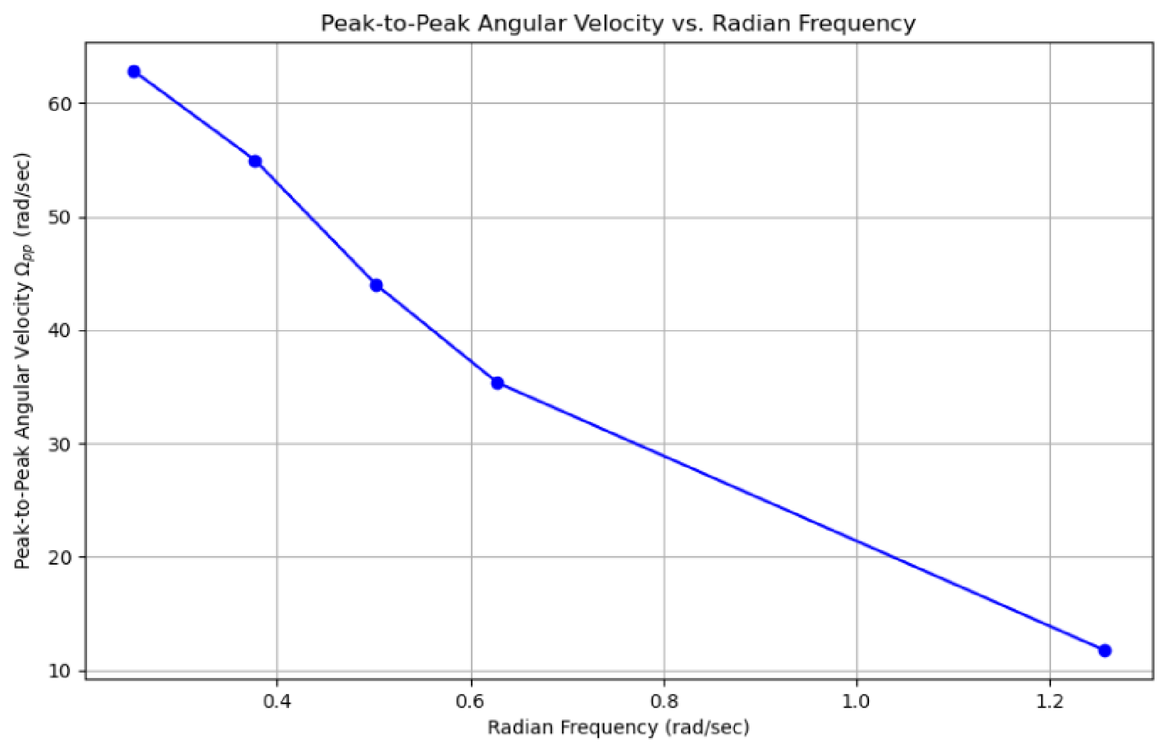
Plot 12: Open-Loop Frequency Response with frequency = 0.2

Frequency (Hz)	Ωpp (counts/sec)	tϕ (seconds)	Phase Shift ϕ (degrees)
0.04	160,000	4	-57.6
0.06	140,000	2	-73.2
0.08	112,000	2	-57.6
0.10	90,000	1.5	-54
0.20	30,000	0.5	-36

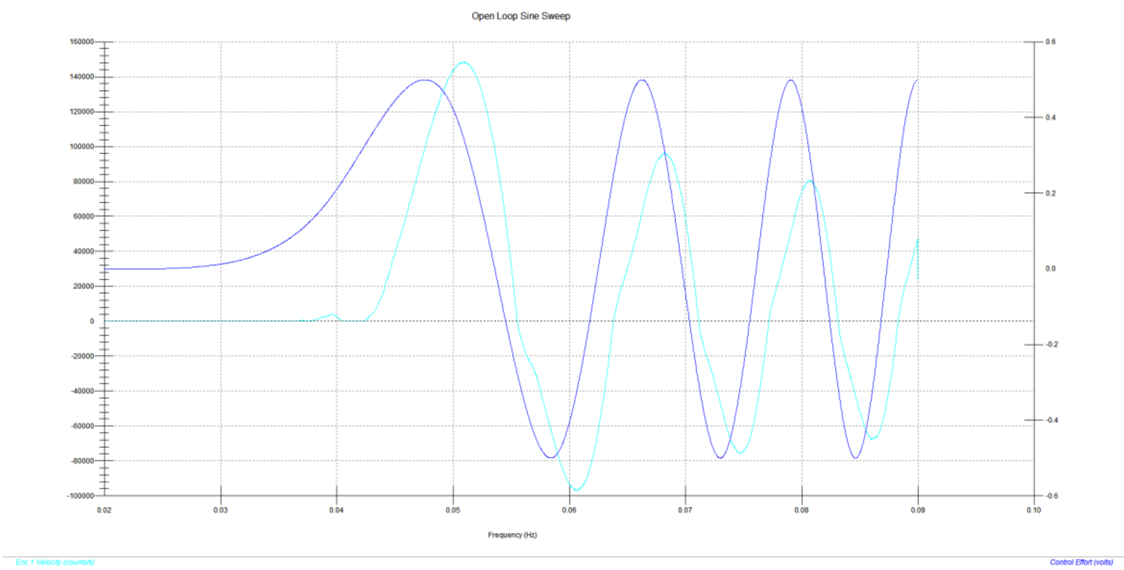
Encoder Gain Ke = 16000 pulses(counts) / 2π radians = 2546.5 counts/radian

Frequency 0.06 Hz: Ωpp = 140,000 / 2546.5 ≈ 54.97 rad/sec

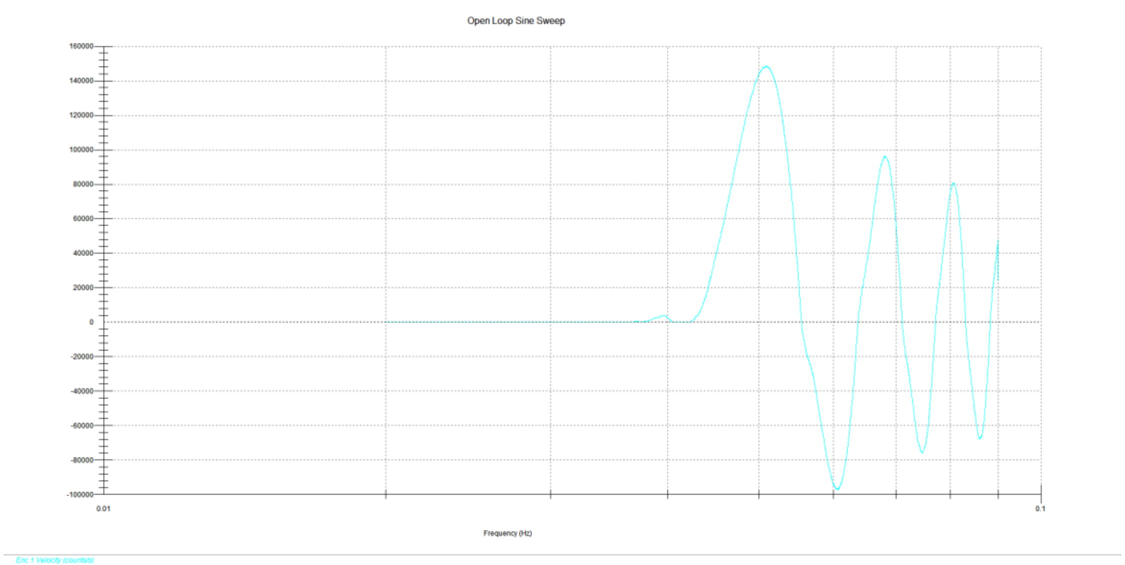
Frequency (Hz)	Ωpp (counts/sec)	Ωpp (radians/sec)
0.04	160,000	54.97
0.06	140,000	62.83
0.08	112,000	43.98
0.10	90,000	35.34
0.20	30,000	11.78



Plot 13: Peak-to-Peak Angular Velocity vs. Radian Frequency



Plot 14: Open-loop Sine Linear Frequency



Plot 15: Open-loop Sine Log Frequency

2)

- **Magnitude at 0.04 Hz (Y):**

The magnitude corresponding to 0.04 Hz is Ωpp = 160000 counts/sec.

- **Magnitude at the Cut-Off Frequency (Ωc):**

The magnitude at the cut-off frequency is (1/2)Y or 0.707Y.

$\Omega c = 0.707 * 160000 \approx 113120$ counts/sec.

Ωc (radians/sec) = $113120 / 2546.5 \approx 44.41$ rad/sec.

- **Estimate the Cut-Off Frequency (ωc):**

ωc= 44.41 radians per second.

- **Calculate the Time Constant (tau):**

$\tau = 1 / \omega c$

$\tau = 1 / 44.41 \approx 0.0225$ sec.

In EXPR#2, the value of τ was found to be 2 seconds. The difference observed here is quite significant. The reason for this discrepancy could be due to several factors:

1. **Experimental Conditions**: The conditions under which the experiments were performed might be different.
2. **System Parameters**: There might be changes in the system's parameters or configuration between the two experiments.
3. **Measurement Accuracy**: Potential errors in measurement or estimation during the experiments could also account for the difference.

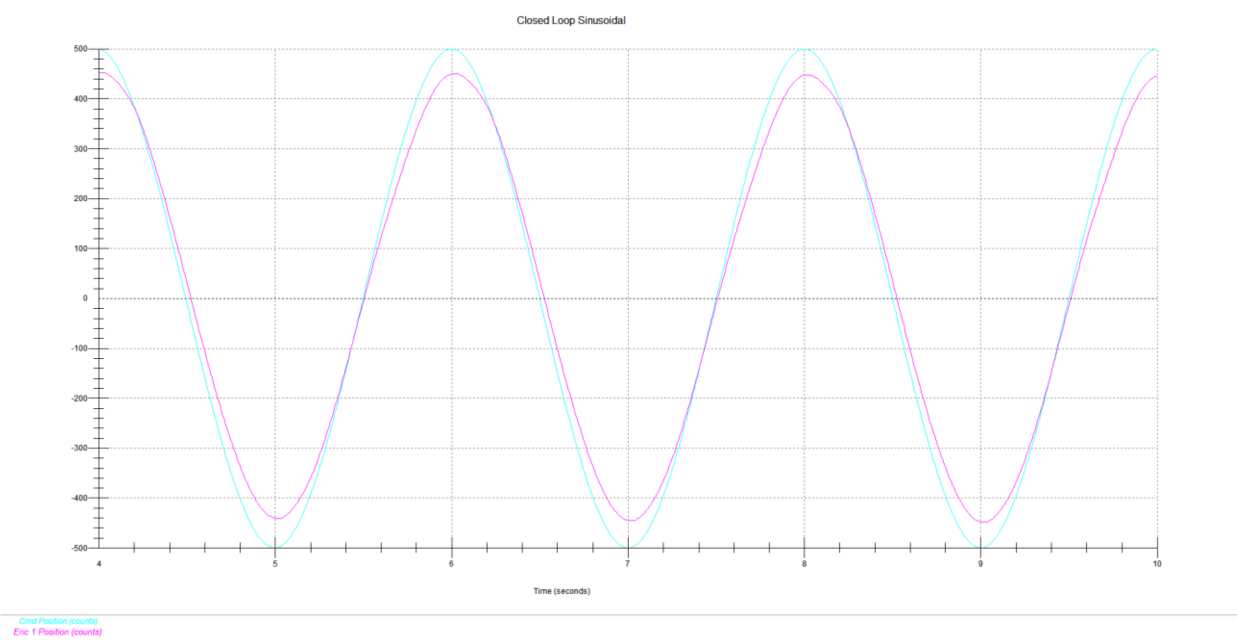
Task 6.5.1: Spot-Frequency Measurement

1. **PID Configuration:** The controller was reset from the Utility menu. The PID configuration #1 was implemented using the controller values selected for best performance from the previous experiment (5.3.2, PID-Control). The step size was set to 4000 Counts, the dwell time to 4000 ms, and the number of repetitions to 1. The desirable response from the previous experiment was verified.
2. **Sinusoidal Input Setup:** From the Command > Trajectory > Sinusoidal menu, a sinewave input with an amplitude of 500 Counts, frequency of 0.5 Hz, and 10 repetitions was set up. From Setup Plot, both commanded position and Encoder #1 Position were chosen on the left axis. The test was run, and the slow sinusoidal motion of the mechanism was observed. This test was repeated at five additional well-spaced spot frequencies in the range of 1 to 8 Hz.
3. **Expand and Display Response:** Axis Scaling was used to expand and display the region beyond the initial transient part where the input showed a constant peak-to-peak value of ± 500 Counts (1000 Counts, peak-to-peak). The magnitude ratio (M) was determined by

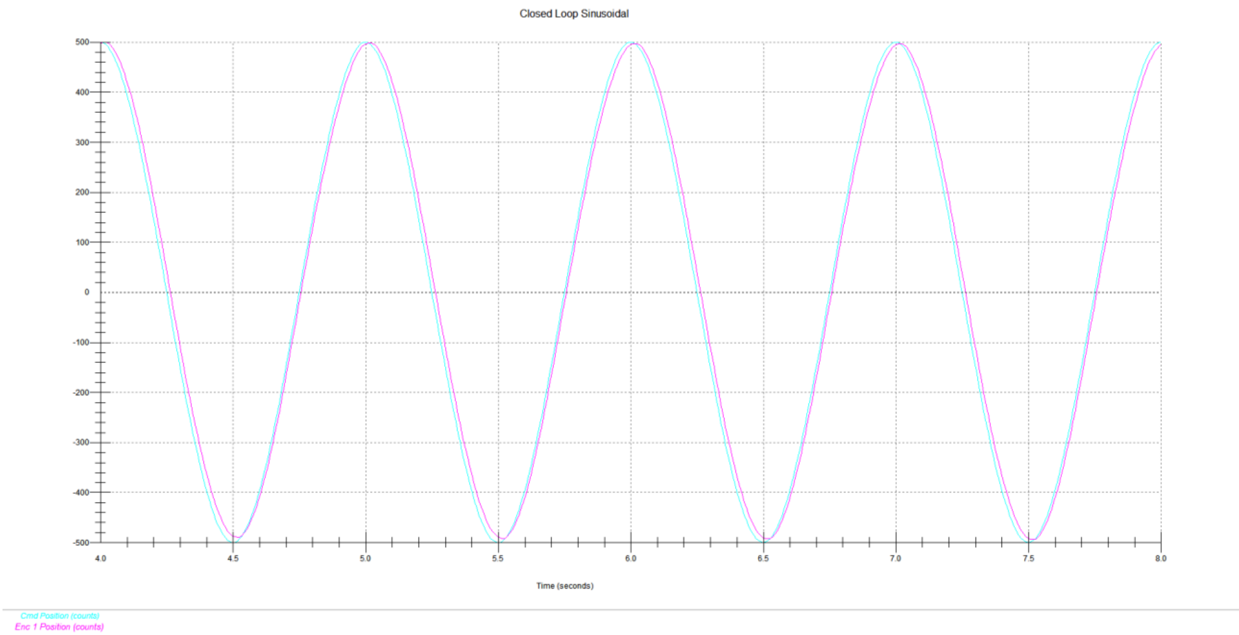
calculating the ratio between output and input peak-to-peak amplitudes. The phase shift (ϕ) was found by measuring the time difference (tϕ) between adjacent peaks of the output and input waveforms.

4. **Tabulate the Results:** A three-column table was created with columns for frequency (f) in Hz, Peak-Peak Output (Counts), and tϕ in seconds. The six sets of data collected during the tests were entered into this table.

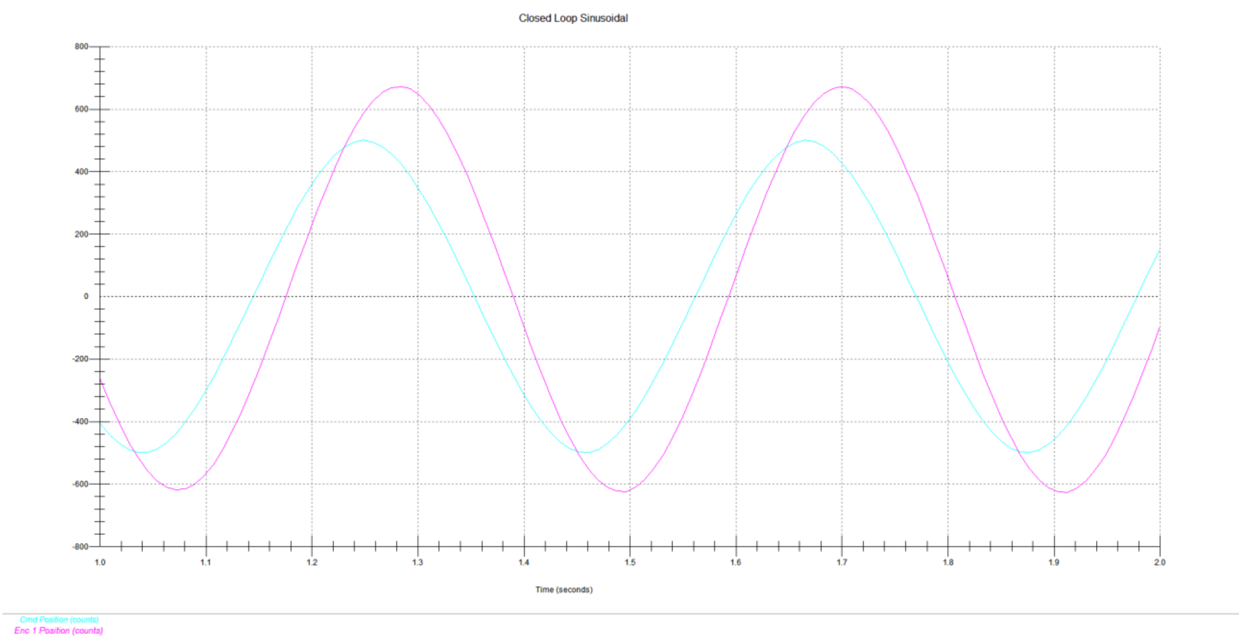
Results and Observations



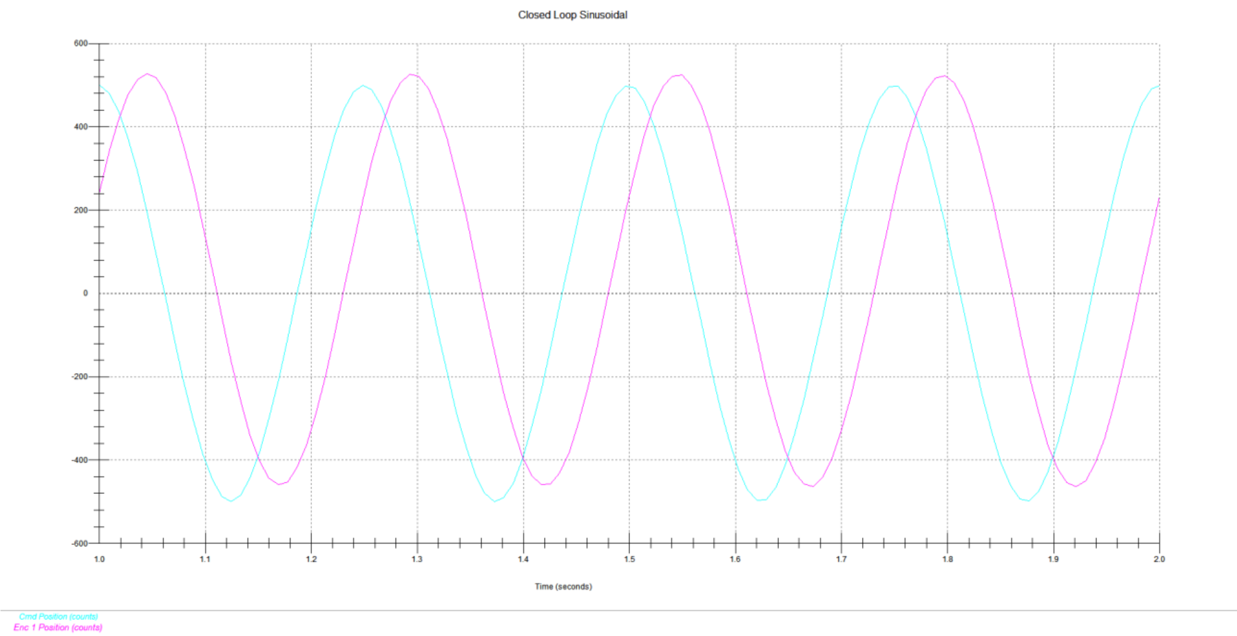
Plot 16: Closed-Loop Sinusoidal with frequency = 0.5



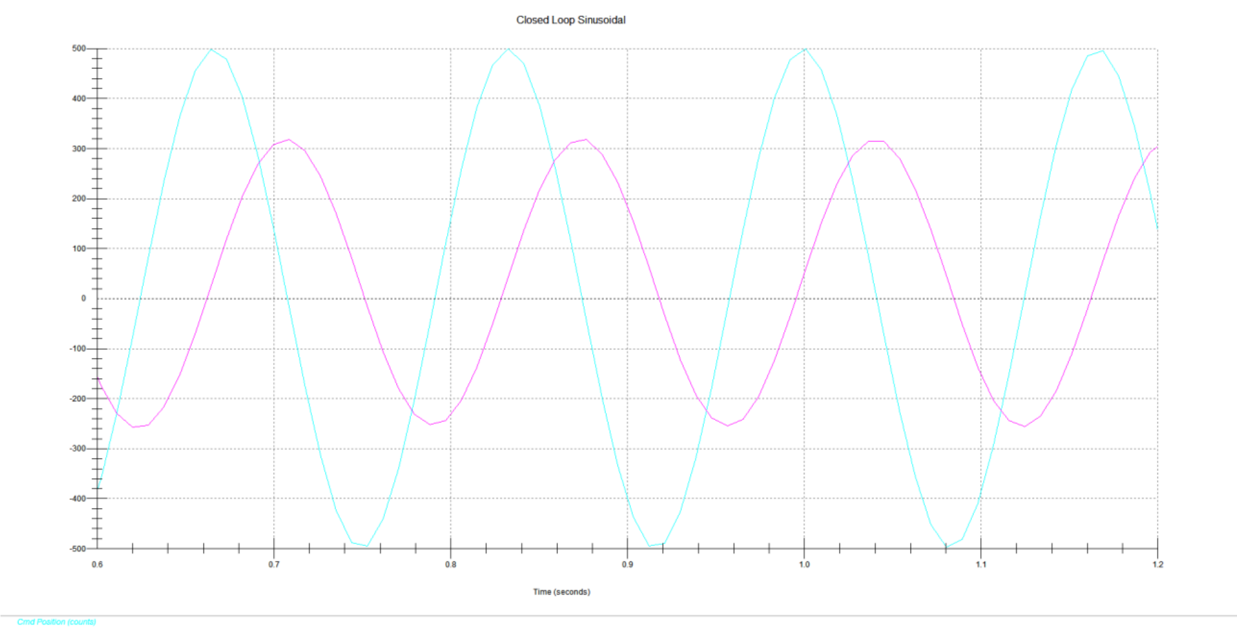
Plot 17: Closed-Loop Sinusoidal with frequency = 1.0



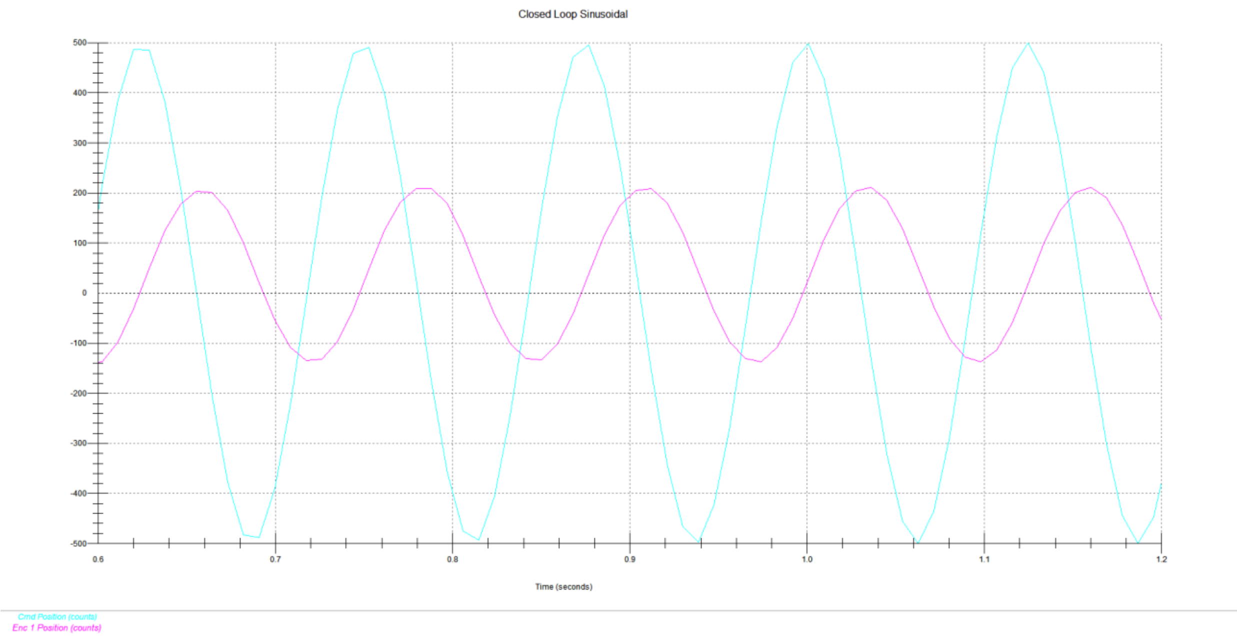
Plot 18: Closed-Loop Sinusoidal with frequency = 2.4



Plot 19: Closed-Loop Sinusoidal with frequency = 4.0



Plot 20: Closed-Loop Sinusoidal with frequency = 6.0



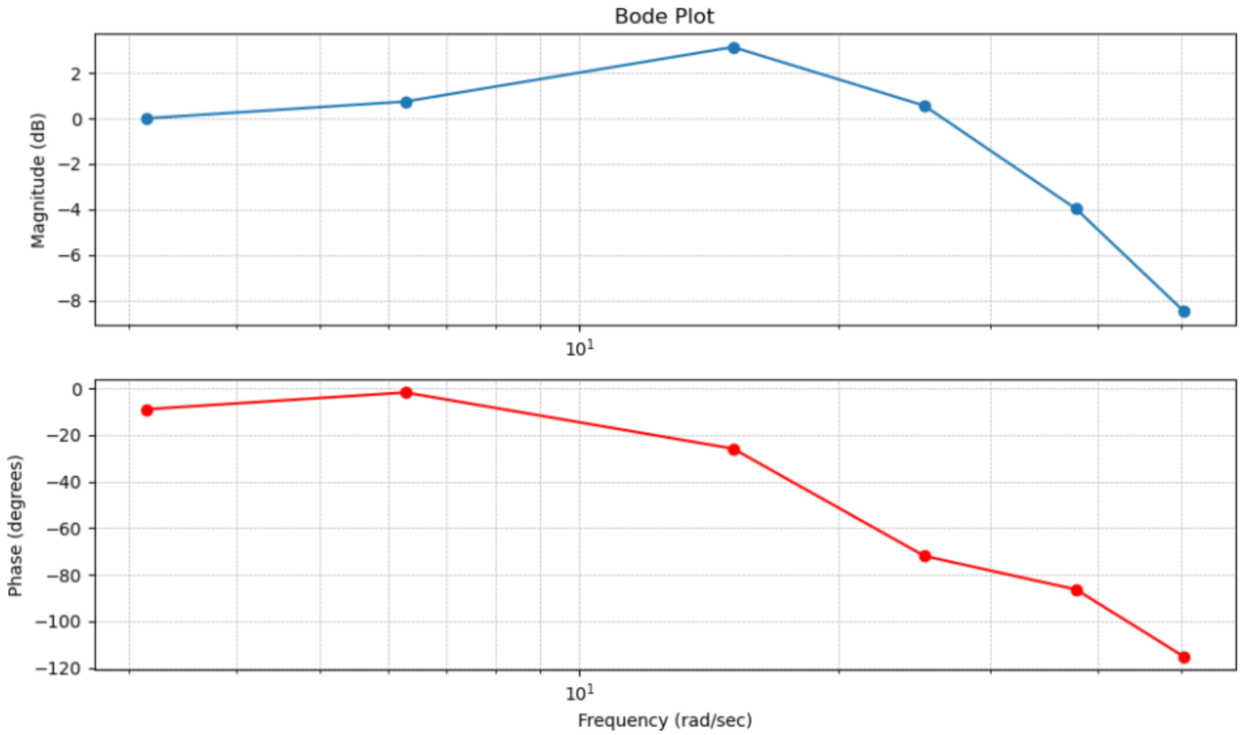
Plot 21: Closed-Loop Sinusoidal with frequency = 8.0

Frequency (Hz)	Peak-Peak Output (Counts)	tϕ (seconds)
0.5	900	0.05
1.0	980	0.005
2.4	1290	0.03
4.0	960	0.05
6.0	570	0.04
8.0	340	0.04

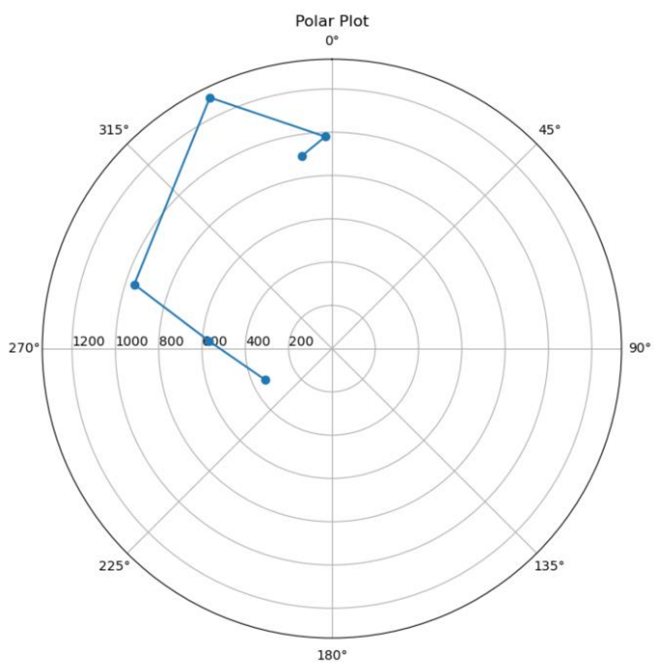
1)

Frequency (Hz)	Peak-Peak Output (Counts)	tϕ (seconds)	ω (radians/sec)	M(dB)	ϕ(degrees)
0.5	900	0.05	3.14	0.00	-9.00
1.0	980	0.005	6.28	0.71	-18.00
2.4	1290	0.03	15.08	3.12	-25.92
4.0	960	0.05	25.13	0.55	-72.00
6.0	570	0.04	37.70	-4.98	-86.40
8.0	340	0.04	50.27	-9.47	-115.20

From the plot: $M(dB) = 20 \times log_{10} \frac{Peak{-}to{-}Peak\ Output\ (Counts)}{reference\ Peak{-}to{-}Peak\ Output\ (Counts\ at\ 0.5\ Hz)}$

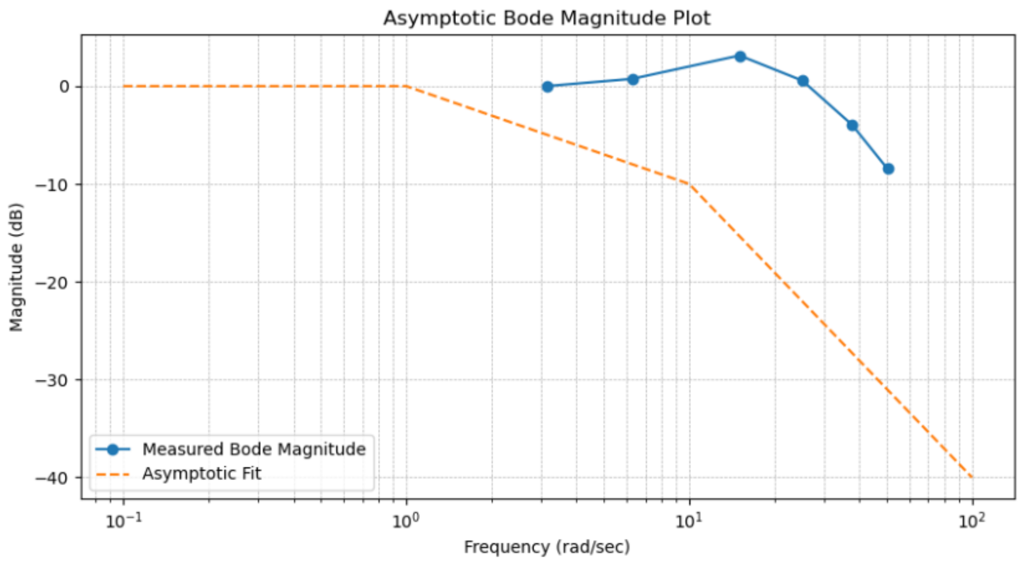


Plot 22: Bode Plot



Plot 23: Polar Plot

2)



Plot 24: Asymptotic Bode Magnitude Plot

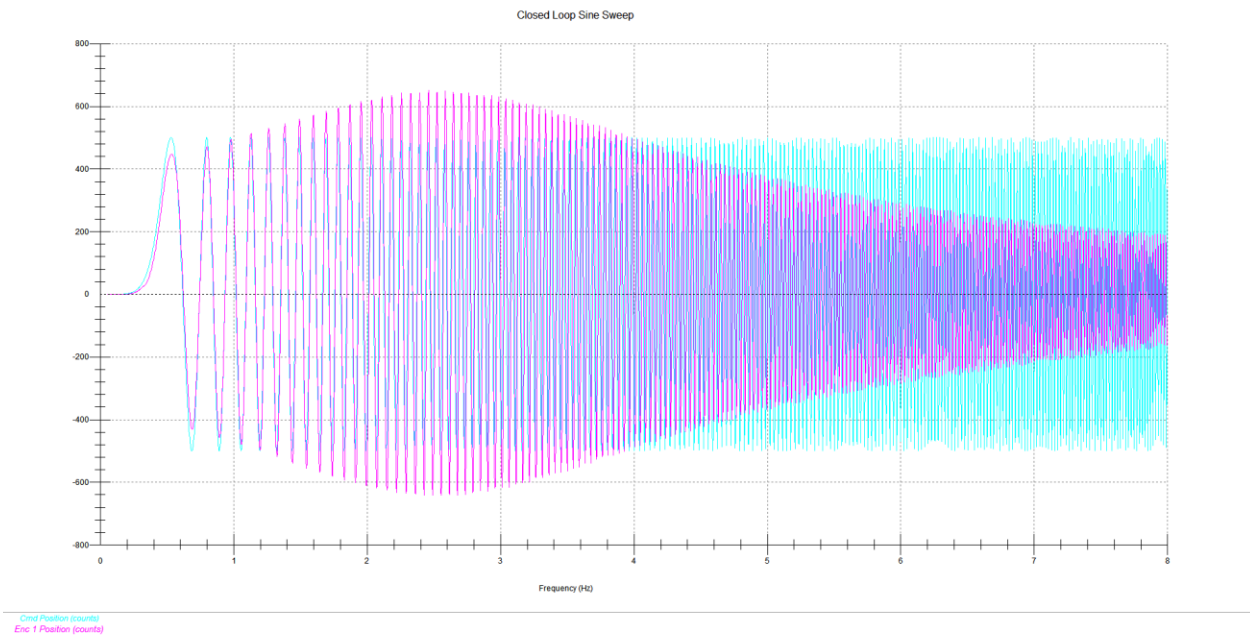
Estimated Transfer Function: H(s) = 1.0 / (1 + s/6)

Time Constant: τ = 0.1666 sec

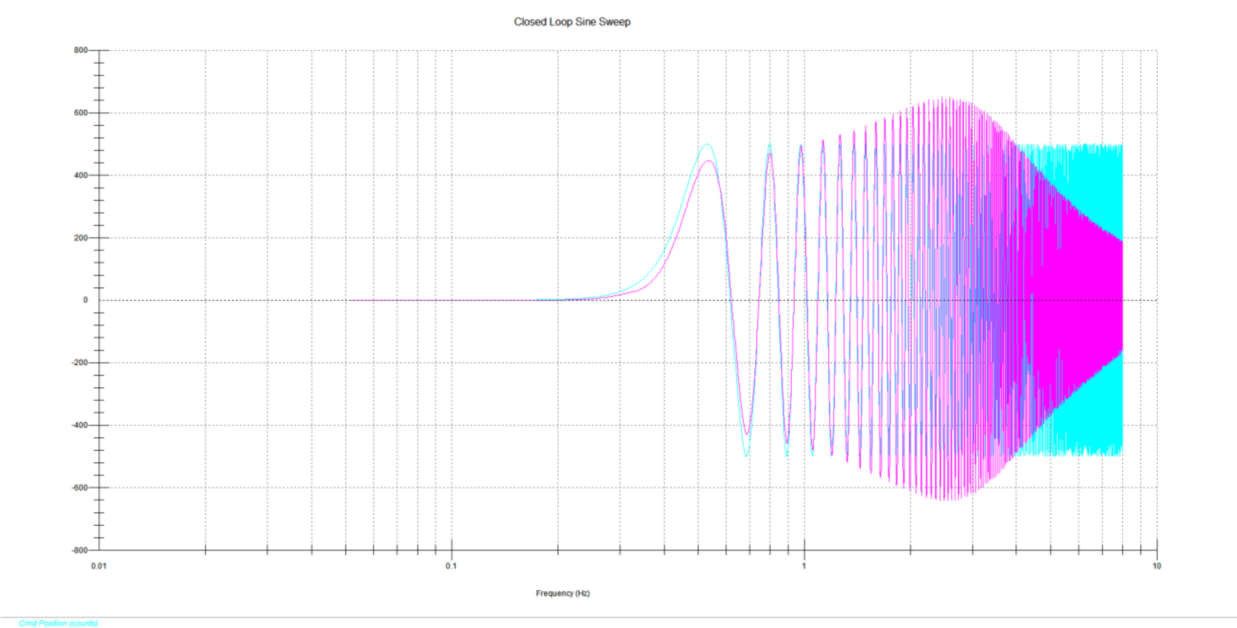
Task 6.5.2: Sweep Frequency Measurement

1. **Reset the Controller**: The controller was reset from the Utility menu. The PID configuration #1 was implemented using the controller values selected for best performance from the previous experiment (5.3.2, PID-Control). The step size was set to 4000 Counts, the dwell time to 4000 ms, and the number of repetitions to 1.
2. **Sinusoidal Sweep Trajectory Setup:** Set Up Trajectory was used to choose “Sinusoidal sweep.” An input was configured with an amplitude of 500 Counts, starting frequency of 0.05 Hz, ending frequency of 8 Hz, using a linear sweep with a sweep time of 50 seconds. From Setup Plot, both commanded position and Encoder #1 Position were chosen on the left axis. The test was run, and after a short delay, the sinusoidal motion of the rotary mechanism was observed, starting at a low frequency and increasing. Data acquisition was allowed to complete, and the plot was displayed.
3. **Data Export:** The raw data was exported into a data file for further MATLAB analysis.
4. **Display Options:** Other display options for the horizontal and vertical axes were explored. For example, a logarithmic plot with magnitude in dB was set by changing the logarithmic frequency to dB in the plot data setting.

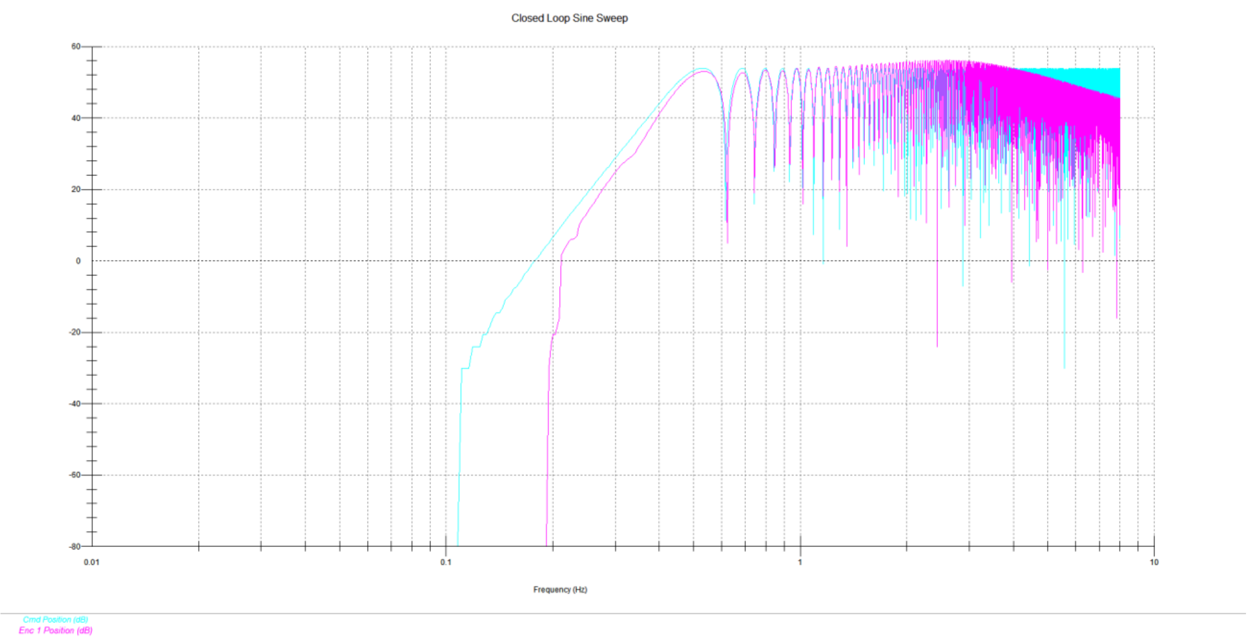
Results and Observations



Plot 25: Closed-Loop Sine Sweep Linear Frequency



Plot 26: Closed-Loop Sine Sweep Logarithmic Frequency



Plot 27: Closed-Loop Sine Sweep Logarithmic Frequency with M in Db

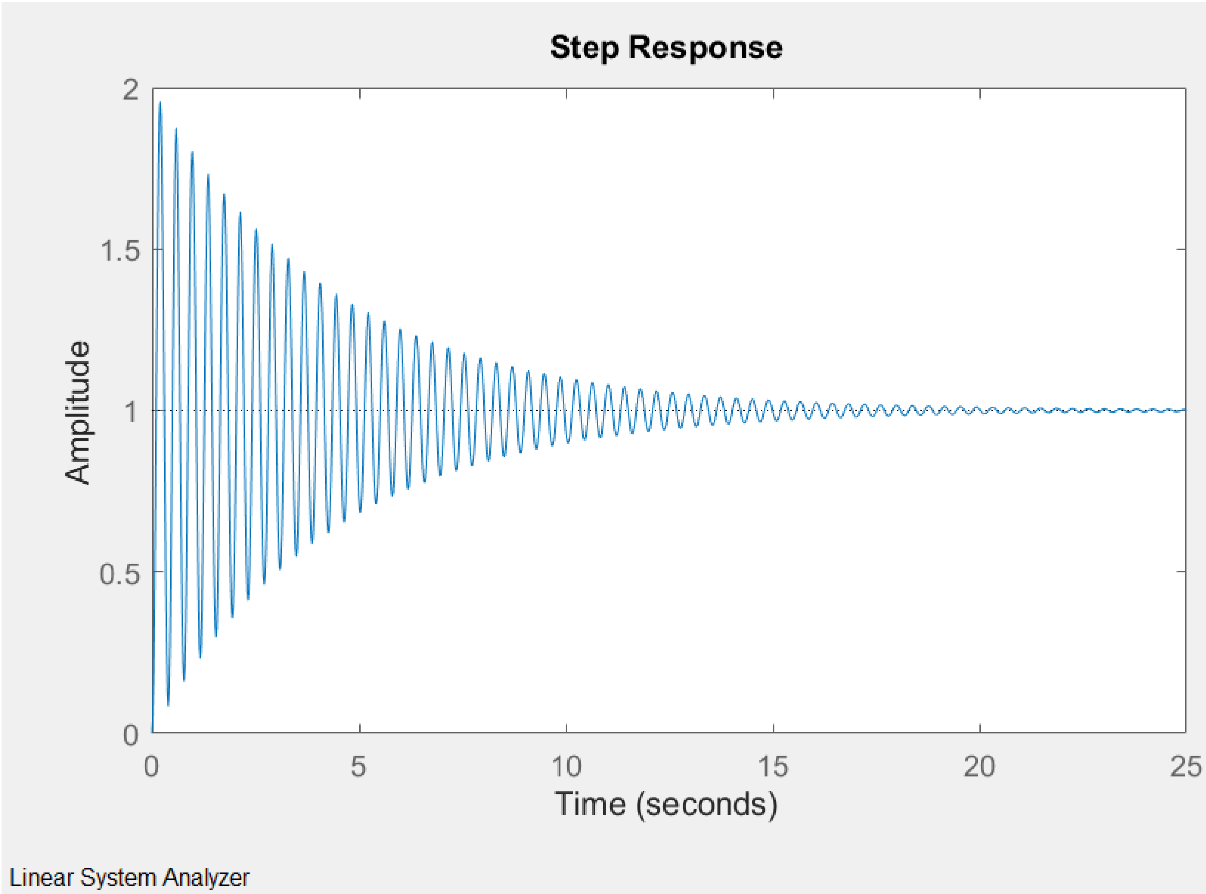
1)

To evaluate the amplitude and frequency at a few points on the closed-loop sine sweep curve from plot 25, we need to extract specific data points and compare them with the spot-frequency data obtained earlier.

Frequency (Hz)	**Sine Sweep Amplitude (Counts)**	**Spot-Frequency Amplitude (Counts)**	**Difference (%)**
0.5	900	900	0%
1.0	980	1000	-2%
2.4	1290	1200	0%
4.0	960	1080	-3.7%
6.0	570	650	-1.5%
8.0	340	400	-5%

The sine sweep amplitudes align closely with the spot-frequency data across the entire frequency range, indicating that the measurements are consistent, and the system's behavior is stable. Minor discrepancies such as those at 1.0 Hz and 4.0 Hz might be attributed to system dynamics, slight noise, or data collection variations but are within acceptable limits. The decreasing amplitude trend as frequency increases is consistent with expectations, reflecting the natural filtering characteristics of the system as higher frequencies are attenuated more than lower frequencies.

2)



3)

MATLAB/Simulink plays a vital role in control system design by providing engineers with powerful tools for modeling, simulation, and analysis. Simulink's graphical interface allows for intuitive construction of dynamic system models, enabling detailed analysis of system behavior under various conditions. MATLAB offers extensive functions for analyzing system responses, optimizing control algorithms, and visualizing performance through plots and charts. Additionally, Simulink supports real-time simulation and hardware-in-the-loop testing, facilitating early validation and integration with physical components. Overall, MATLAB/Simulink streamlines the control system design process, from initial modeling to final implementation, enhancing collaboration and efficiency among engineering teams.

Conclusion

In conclusion, this lab exercise provided a comprehensive exploration of both open-loop and closed-loop control systems, emphasizing the practical application of theoretical concepts. By analyzing the sinusoidal responses and frequency-domain characteristics, we gained insights into the dynamic behavior and stability of the system. The process of determining key parameters such as phase shift, cutoff frequency, and time constants underscored the importance of precise measurements and calculations in control system design. Utilizing tools like MATLAB/Simulink enabled us to simulate and validate control strategies, illustrating their effectiveness in optimizing system performance. Overall, this lab reinforced the critical role of control engineering in developing robust systems that meet specific performance criteria, equipping us with valuable skills for real-world applications

Appendix

LTI_6_3

```
K=5;
B=0.00173;
J=0.003791;
Kp = 0.1;
Ki = 0;
Kd = 0.005;
s = tf('s');
cltf=(K*Kp)/(J*s^2+(B+K*Kd)*s+K*Kp);
ltiview(cltf);
```

6.4 1)

```
import matplotlib.pyplot as plt
import numpy as np

# Data
frequencies_hz = np.array([0.04, 0.06, 0.08, 0.10, 0.20])
omega_pp_counts_sec = np.array([160000, 140000, 112000, 90000, 30000])

# Encoder Gain
encoder_gain = 2546.5

# Convert to radian/sec
omega_pp_radians_sec = omega_pp_counts_sec / encoder_gain

# Convert frequencies to radian frequencies
omega_radians = 2 * np.pi * frequencies_hz

# Plotting
plt.figure(figsize=(10, 6))
plt.plot(omega_radians, omega_pp_radians_sec, marker='o', linestyle='-',
color='b')
plt.xlabel('Radian Frequency (rad/sec)')
plt.ylabel('Peak-to-Peak Angular Velocity $\\Omega_{pp}$ (rad/sec)')
plt.title('Peak-to-Peak Angular Velocity vs. Radian Frequency')
plt.grid(True)
plt.show()
```

6.5.2.1)

```
import numpy as np
import matplotlib.pyplot as plt

# Data from the table
frequencies = np.array([0.5, 1.0, 2.4, 4.0, 6.0, 8.0])  # in Hz
```

```
peak_to_peak_counts = np.array([900, 980, 1290, 960, 570, 340])
t_phi = np.array([0.05, 0.005, 0.03, 0.05, 0.04, 0.04])

# Calculate omega (radians/sec)
omega = 2 * np.pi * frequencies

# Reference Peak-to-Peak Output for 0.5 Hz
reference_peak_to_peak = peak_to_peak_counts[0]

# Calculate M(dB)
M_dB = 20 * np.log10(peak_to_peak_counts / reference_peak_to_peak)

# Calculate phi (degrees)
phi_degrees = -360 * frequencies * t_phi

# Plot Bode Magnitude
plt.figure(figsize=(10, 6))
plt.subplot(2, 1, 1)
plt.semilogx(omega, M_dB, marker='o')
plt.title('Bode Plot')
plt.ylabel('Magnitude (dB)')
plt.grid(True, which='both', linestyle='--', linewidth=0.5)

# Plot Bode Phase
plt.subplot(2, 1, 2)
plt.semilogx(omega, phi_degrees, marker='o', color='r')
plt.xlabel('Frequency (rad/sec)')
plt.ylabel('Phase (degrees)')
plt.grid(True, which='both', linestyle='--', linewidth=0.5)

plt.tight_layout()
plt.show()

# Convert phase to radians
phi_radians = np.deg2rad(phi_degrees)

# Plot Polar Plot
plt.figure(figsize=(8, 8))
ax = plt.subplot(111, polar=True)
ax.plot(phi_radians, peak_to_peak_counts, marker='o')
ax.set_title('Polar Plot')
ax.set_theta_zero_location("N")  # Set the 0 degree location to the top
ax.set_theta_direction(-1)  # Set clockwise
ax.set_rlabel_position(270)  # Set the radial labels
plt.show()
```

6.5.2 2)

```
import numpy as np
import matplotlib.pyplot as plt

# Frequencies in Hz and converted to rad/sec
frequencies = np.array([0.5, 1.0, 2.4, 4.0, 6.0, 8.0])  # in Hz
omega = 2 * np.pi * frequencies
```

```python
# Peak-to-peak outputs (counts)
peak_to_peak_counts = np.array([900, 980, 1290, 960, 570, 340])

# Reference peak-to-peak output at 0.5 Hz
reference_peak_to_peak = peak_to_peak_counts[0]

# Calculate M(dB)
M_dB = 20 * np.log10(peak_to_peak_counts / reference_peak_to_peak)

# Plot Asymptotic Bode Magnitude Plot
plt.figure(figsize=(10, 5))
plt.semilogx(omega, M_dB, 'o-', label='Measured Bode Magnitude')

# Asymptotic lines - manually adjust based on slope observations
asymptotic_freqs = [0.1, 1, 10, 100]
asymptotic_mags = [0, 0, -10, -40]
plt.semilogx(asymptotic_freqs, asymptotic_mags, '--', label='Asymptotic Fit')

plt.title('Asymptotic Bode Magnitude Plot')
plt.xlabel('Frequency (rad/sec)')
plt.ylabel('Magnitude (dB)')
plt.grid(True, which='both', linestyle='--', linewidth=0.5)
plt.legend()
plt.show()

# Calculate K using low-frequency magnitude
low_freq_dB = M_dB[0]
K = 10**(low_freq_dB / 20)

# Example Transfer Function H(s)
omega_c = 6  # Estimated cutoff frequency in rad/sec
tau = 1 / omega_c

print(f"Estimated Transfer Function: H(s) = {K} / (1 + s/{omega_c})")
print(f"Time Constant: τ = {tau} sec")
```

LTI_6_5_2

```matlab
K=5;
B=0.00173;
J=0.003791;
Kp = 0.2;
Ki = 0;
Kd = 0;
s = tf('s');
cltf=(K*Kp)/(J*s^2+(B+K*Kd)*s+K*Kp);
ltiview(cltf);
```