6 EXPT #4: TRANSIENT AND FREQUENCY RESPONSE

6.1 OBJECTIVE

- To study the transient response of open-loop and closed-loop system.
- To study the frequency response of a first order system (magnitude and phase) and determine the cut-off frequency(ω_c).
- To determine the frequency response (magnitude and phase) of a second order closed-loop system.

6.2 INTRODUCTION

6.2.1 SYSTEM TRANSIENT RESPONSE

The typical unit step response of a second order system shown in Figure 6-1 is used to define various performance criteria such as the 10% to 90% Rise time (T_r), the Peak time (T_P), the percent overshoot (PO), the 2% Settling time (T_s), and the Damped Natural Frequency ($\omega_d = \omega_n\sqrt{1-\zeta^2}$).

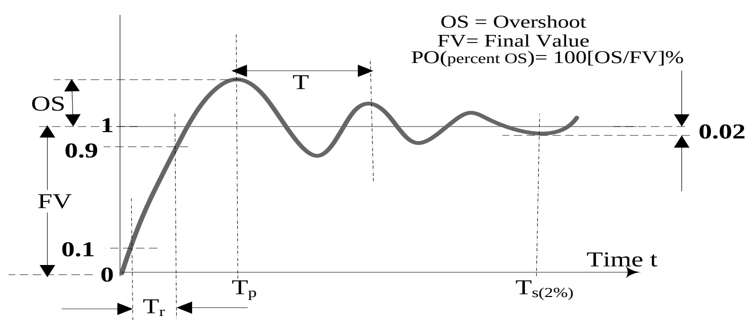


Figure 6.1: Typical unit response of a system

Definition of these parameters allows performance comparisons between different system designs to be made with the objective of selecting a system with the desired performance specifications. The terms OS, T_P, etc. are sometimes loosely applied to a similar response which is not strictly a classical second-order response.

The system parameters damping ratio (ζ) and undamped natural frequency (ω_n) are related to some of the observed parameters shown in the figure above. Some relations are given below:

$$\text{Percent Overshoot (PO)} = 100 \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)\% \qquad (6.1)$$

The relation between *PO* and ζ is shown in Figure 6.2.

If [PO(%) /100] = p, then

$$\zeta = \sqrt{\frac{(\ln p)^2}{\pi^2+(\ln p)^2}} = \sqrt{\frac{(\ln p)^2}{9.87+(\ln p)^2}} \qquad (6.2)$$

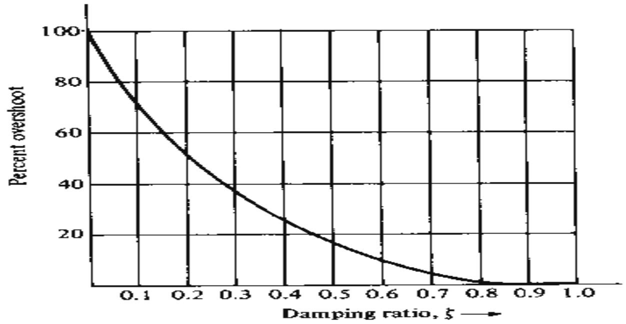


Figure 6.2: The relation between PO and ζ

Equation (6.2) may be used for precise calculation of ζ when the PO is specified.

Also, the peak time and 2% settling time are given by:

$$T_P = \frac{\pi}{\omega_d} \quad \text{and} \quad T_s = \frac{4}{\zeta\omega_n}$$

In this experiment, the effect of changing the proportional gain factor K_p on the step response is observed. The effects of introducing derivative feedback in the forward as well as in the feedback path are then investigated in the second part. The "PI + Velocity FB" configuration in figure 6.3, will be mostly used.

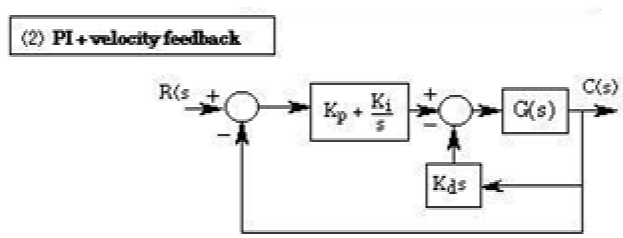


Figure 6.3: System Configuration

If we set $K_i = 0$, the closed-loop transfer function will be a 'standard' second order system with no zeros:

$$T(s) = \frac{C(s)}{R(s)} = \frac{\frac{KK_p}{J}}{s^2 + \left(\frac{B + KK_d}{J}\right)s + \frac{KK_p}{J}} = \frac{\omega_n^2}{s^2 + 2\omega_n\zeta s + \omega_n^2}$$

then

$$\omega_n = \sqrt{\frac{K\,K_p}{J}} \qquad \text{and} \qquad \zeta = \frac{B + K\,K_d}{2\sqrt{K\,K_p J}} \tag{6.3}$$

It can be seen that increasing k_p will result in an increase in ω_n (indicative of a faster 'rise time' and hence improved transient response) and a decrease in ζ (indicative of higher 'overshoot'). From the CLTF above, it can be seen that the damping term contains k_d, the 'velocity-feedback' coefficient. The velocity feedback therefore provides an adjustable (artificial) damping which is added to the natural damping *B*. In the second part of the experiment, the effect of this artificial damping is verified. Increasing the overall damping by raising the value of k_d can compensate for the increased OS resulting from increasing k_p. The term 'velocity feedback' arises from the use of the derivative of the output signal, which is velocity, in a position-control system as a feedback signal.

6.2.2 FREQUENCY RESPONSE

The steady state response of a linear system to sinusoidal input signals of varying frequency is generally known as the "frequency response". Frequency response is obtained by applying a fixed-amplitude sinusoidal input signal and measuring the resultant changes in amplitude and phase of the output (with respect to the input), as a function of increasing frequency. More precisely, these changes are expressed, in terms of output and input phasor quantities, as the following quantities:

Magnitude Ratio M = Output amplitude / Input amplitude

and

Phase-shift ϕ = Output phase angle – Input phase angle

Both of which are dependent upon the radian-frequency $\omega=2\pi f$, where f is the frequency in Hz. The most common modes of presentation of frequency-response data are Bode Plots as shown in figure 6-4 and Polar Plots as shown in figure 6-5.

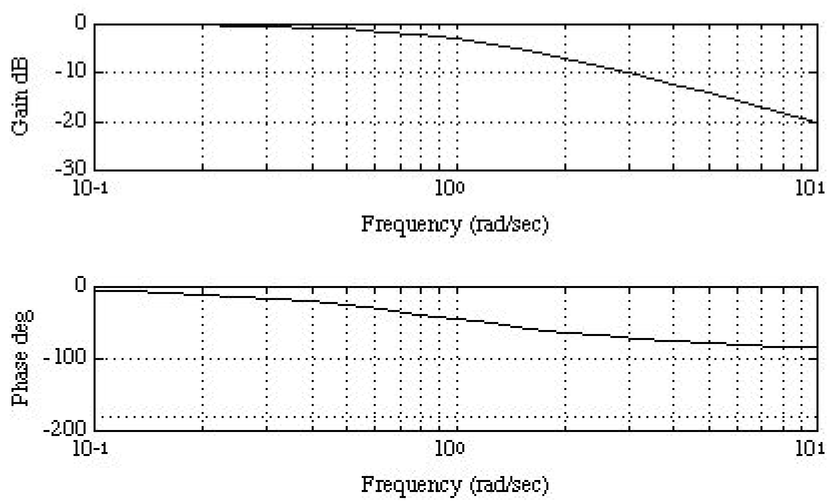


Figure 6.4: Bode Plots

In Bode plots, M (when expressed in decibels, M (dB) = 20 log M) and ϕ (expressed in either degrees or radians) are plotted against a logarithmic frequency scale which will allow a *large range* of frequencies to be shown. The Bode and Polar plots of the OLTF are used in system design, whereas the Bode plots of the CLTF are usually provided as a part of the system specifications. (Historically, design of feedback compensators was done using the Bode plots of the OLTF, with the reference quantities of 'Gain Margin' and 'Phase Margin' which are also indicative of relative system stability. Students should consult the course textbook to understand the meaning of these two terms.)

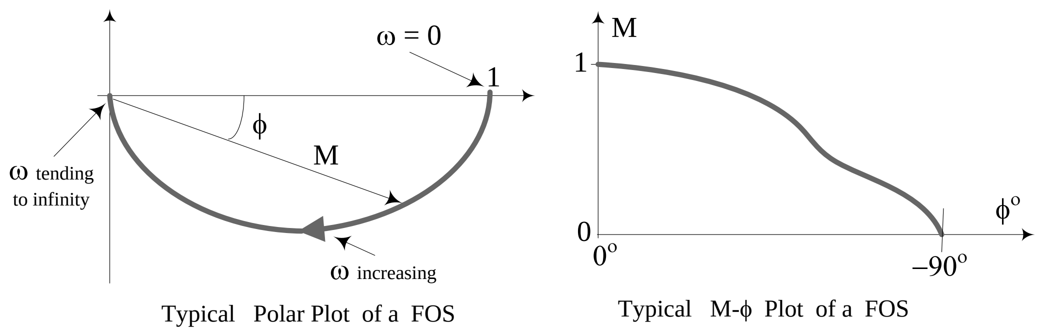


Figure 6.5: Polar Plots

Straight-line segmental approximations of the Bode plots (also called "asymptotic plots" of a transfer function) can be quickly sketched, if the transfer function can be analytically obtained in factored form (See 'asymptotic plots' in your course textbook). Conversely, an asymptotic plot can be 'fitted' to an experimentally-obtained magnitude plot in order to obtain the transfer-function.

6.2.3 Results

Write the results of transient response and frequency response in separate papers, which will be signed by your lab instructor at the end of the lab.

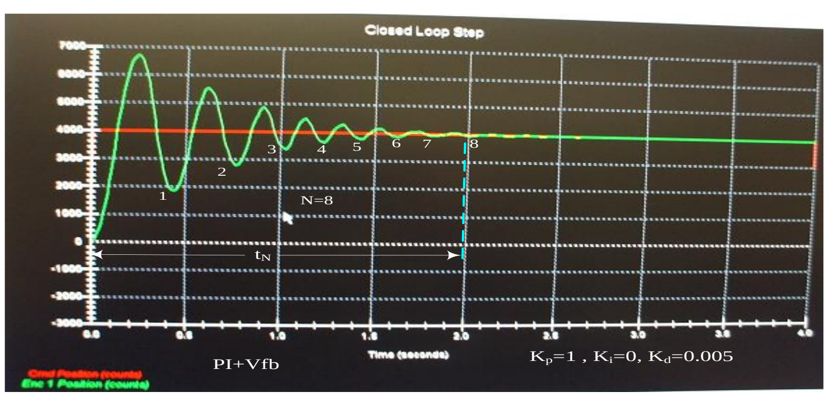
6.3 CLOSED-LOOP TRANSIENT RESPONSE

PROCEDURE

1) Reset the controller from the Utility menu. From the Control Algorithm menu, implement the configuration "PI + velocity feedback", the type of Continuous time with the following settings:

$$T_s = 0.00442 \text{ sec}, \quad K_p = 1 \quad K_i = 0 \quad \text{and} \quad K_d = 0.005$$

2) Obtain the closed-loop step response using a Step Size of 4000 Counts, Dwell Time of 4000 ms, and number of Repetitions 1. RUN the step test and display the plot. Use axis- scaling to display only the section of the response within the dwell period of 4 seconds to make the display appear as a 'step response' as shown below for an instance.

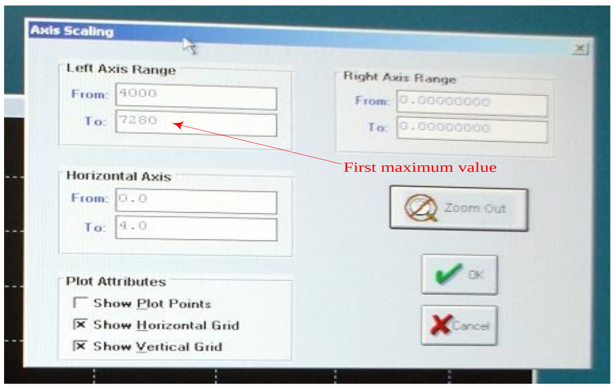


Obtain* values for: the damped natural frequency ω_d and the overshoot (OS) and using a suitable axis scaling of the plot.

Finding ω_d : The frequency ω_d can be found by finding the peak time t_p. Then, the corresponding radian frequency is given by $\omega_d = \frac{\pi}{T_p}$.

Finding OS: The OS can be found by re-scaling the left axis to display values from 4000 Counts. The magnitude difference between the first positive peak[1] X_1 and 4000 is the OS. The percent overshoot (PO) is then given by

$$PO = 100 \left[\frac{X_1 - 4000}{4000} \right] \% \qquad (6.4)$$



3) Export raw data into a data file for further MATLAB analysis.

4) Repeat Steps 1 and 2 for K_p values of 0.7, 0.4, 0.1 and obtain the corresponding values of ω_d and the OS. Note that in each case the controller may be reset from the 'Set Algorithm' menu by clicking on 'Implement' after each change in the K_p value is made.

RESULTS

1) Tabulate ω_d and OS values for the above cases.
2) Write a brief summary about the difference between open-loop and closed-loop system?
3) From the OS values, use Equation (6.2) to calculate and tabulate the values of ζ, corresponding to the four K_p values used. Obtain a plot showing the effect of increasing K_p on the damping ratio ζ and comment on the result.
4) From the tabulated values of ζ and ω_d, determine the corresponding ω_n values. Obtain a plot showing the effect of increasing K_p on the natural frequency ω_n and comment on the result.

ANALYSIS in MATLAB*

1) Use the following settings: T_s = 0.00442 sec, K_p = 0.1 K_i = 0 and K_d = 0.005 and the obtained model in Experiment #2, write MATLAB code to calculate CLTF in the configuration of "PI + velocity feedback" , see formula 6.3.

---

[1] After the plot is displayed, selecting 'Axis Scaling' will show the 'left axis' scales as 'From..., To...'. The value indicated in the 'To...' box is an accurate figure for the first maximum value X_1 from which the PO can be readily deduced. This is shown in the next figure.

Sample code is listed below, you may have to change the parameter to yours (obtained in Experiment #2):

```
K=5, B=0.002, J=0.02;
Kp = 0.1, Kd = 0.005;
s = tf('s');
cltf = (K*Kp)/(J*s^2+(B+K*Kd)*s + K*Kp);
```

2) Refer to sample code in Experiment #1, using MATLAB LTI Viewer to analysis system step response.
3) Compare the result with the exported raw data, for at least one K_p.

6.4 OPEN-LOOP FREQUENCY RESPONSE

The frequency response of the first order system $G(s)=C(s)/R(s)= 1/(1+\tau s)$ can be expressed by the magnitude-ratio and phase shift equations, which are reproduced below:

$$\text{Output/input magnitude ratio: } M(\omega) = C(\omega)/R(\omega) = 1/(1+ \omega^2\tau^2)^{0.5}$$

$$\text{Phase-shift: } \phi(\omega) = -\tan^{-1}(\omega\tau)$$

and are plotted with respect to a logarithmic frequency-axis. In this experiment, the frequency response of the velocity (rather than position) is considered. The transfer function will be of first-order and also the response will be plotted against a linear frequency-axis, to correspond to more basic form of expression.

When applied to the first-order plant of the laboratory system (with τ =J/B), for the velocity will be given by

$$\boldsymbol{\Omega}(\omega) / V_i(\omega) = K_o/ (1+ j\omega\tau)$$

Where $K_o= K_eK_aK_t/B$. In the above equation, the symbol $\boldsymbol{\Omega}$ has been used here to denote the angular velocity variable in order to avoid confusion with the sinusoidal radian frequency ω. This corresponds to a magnitude of

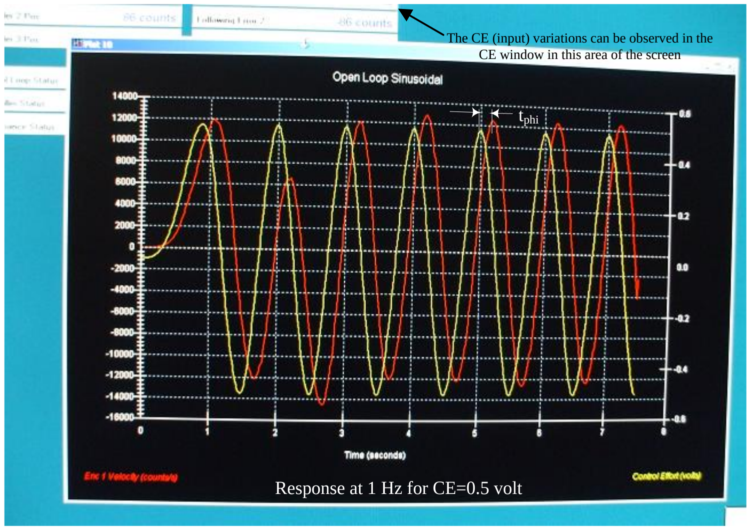
$$M(\omega) = |\boldsymbol{\Omega}(\omega)/ V_i(\omega)| = (K_eK_aK_t/B) /(1+ \omega^2\tau^2)^{0.5} \qquad (6.5)$$

and a phase shift given by:

$$\phi(\omega) = -\tan^{-1}(\omega\tau) \qquad (6.6)$$

For a given open-loop sinusoid input voltage V_i (i.e. Control Effort, CE), the experimental variation of $\Omega(\omega)$ and $\phi(\omega)$ with frequency may be obtained. At any given frequency, the amplitude of the angular velocity Ω (in counts) can be obtained from the displayed value of the Encoder #1 Velocity.

<u>Test frequency range:</u> For the laboratory system, the upper frequency region where ϕ approaches a 90^o value is found to be around 1Hz. For example the Encoder #1 velocity output at 1Hz for a 0.5 volt CE is shown below.



Ignoring the initial (transient) part of the display, the phase shift ϕ is found from the time difference t_{phi} between two adjacent peaks:

$$\phi = -360\, f\, t_{phi} \quad \text{(degrees)}$$

where f is the frequency in Hz and the negative sign is included to indicate that the output lags the input. In the above figure, for example, $t_{phi} \approx 0.25$ sec yield to $\phi \approx -90^o$ which is the high-frequency limit for the first-order transfer function under consideration.

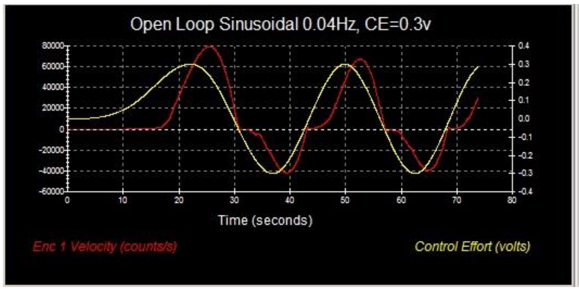
Another point regarding the test that must be mentioned is the initial (or transient) part. The system shows a certain delay before turntable action is observed, and this delay is greater for lower values of the test frequency. However, system activity is indicated by the changing numbers in the 'Control Effort' window at the top left of the screen, which shows the sinusoidal variation of V_i.

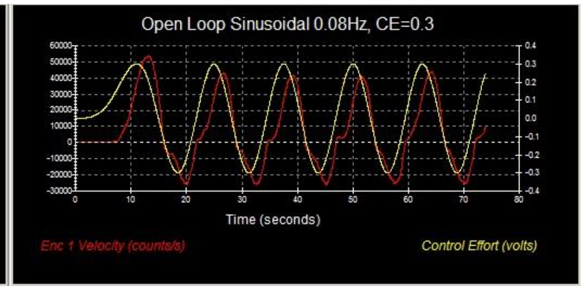
PROCEDURE:

1) Under 'Data Acquisition' menu, make sure that Control Effort (CE) has been added to the list. Reset the controller from the UTILITY menu. Set up the 'plot' to display the Encoder #1 <u>Velocity</u> on the left axis and the CE on the right axis.
2) From the COMMAND menu, select 'Trajectory' and choose Sinusoidal and then click on 'Set Up'. Then select OPEN LOOP, Amplitude **0.4** volts, Frequency **0.06** Hz, and number of Repetitions **8**. Next, go to 'Execute' and RUN the test. (Remember the point mentioned above about a delay in action.) After the data has been acquired, plot the output. (The trace will be similar to the one shown earlier for 1 Hz.)
3) Ignoring the initial transient part*, measure the constant peak-to-peak angular velocity output magnitude Ω_{pp} in counts/sec. Also measure the time difference t_{phi} between two adjacent peaks in order to estimate the phase shift.
   Use the right end of the display. Axis Scaling may be used to examine a section of the steady state display.

4) Using Steps 2-3 above as a guide, obtain values for Ω_{pp} (in counts/sec.) and ϕ (in degrees) for frequencies of 0.04, 0.08, 0.1 and 0.2 Hz.

   Two typical displays are shown below.

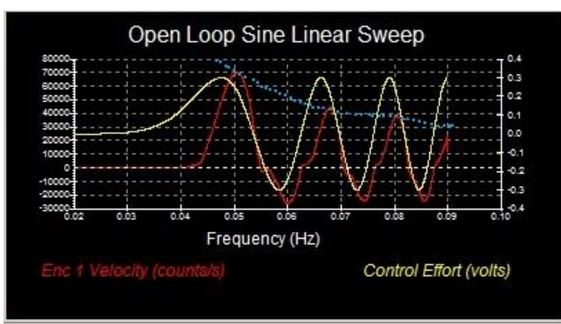


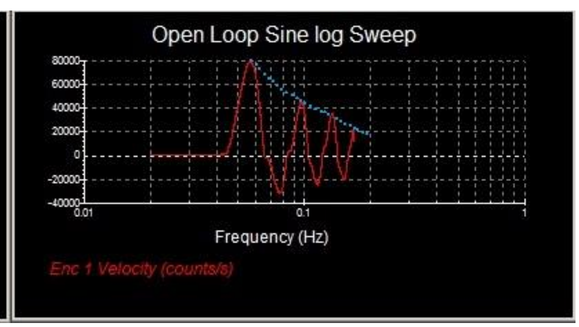


   Obtain records for each of the above mentioned frequencies, and 'full screen' displays and obtain the required data as accurately as possible by visual observation.

   <u>Tabulate</u> the values of radian frequency ω(radians/sec), Ω_{pp} (counts/sec.) and ϕ^o.

5) Sweep frequency display: From the COMMAND menu, select 'Trajectory', Sine Sweep. Then click on 'Set Up'. Then select OPEN LOOP, Amplitude 0.5 volts (Amplitude 0.5 volts is fine for some systems; however, if there is a problem showing the plots, you can use Amplitude 0.4 volts or smaller), Frequency range from 0.02 Hz to 0.09 Hz, Sweep Time (Linear Sweep) 50 sec. Under 'Set Up Plot' select 'linear frequency' sweep. Go to 'Execute' and RUN the test. After the data has been acquired, plot the output and minimize the plot. Then re-plot the data using 'logarithmic frequency' sweep. Typical linear and log frequency plots are shown in the screen photograph below.





In these plots, the upper envelope of the plot (as shown by the dotted lines) gives the approximate 'shape' of the magnitude frequency response as explained in Expt#1.

RESULTS:

1) Using the data tabulated in Step 4 above, use the conversion factor K_e to convert the values of Ω_{pp} (counts/sec.) to Ω_{pp} (radians/sec.). Plot Ω_{pp} (radians/sec.) against the radian frequency ω using a linear scale.
2) Assuming that the input remained constant, the output magnitude should also be a constant (say, Y) at low frequencies approaching 0 Hz. Using the magnitude equation it can be shown that the

output magnitude will drop to (1/ $\sqrt{2}$)Y or 0.707Y when $\omega_c = 1/\tau$, where ω_c is called the 'cut-off frequency' or 'break frequency'. Assuming Y is the magnitude corresponding to the frequency of 0.04 Hz, determine ω_c and hence find τ.

How does this value compare with the value found in EXPR#2(Sect4.3.2)? Comment on any difference observed.

(The value of $\tau = 1/\omega_c$ can also be estimated by finding $f_c = 1/2\pi\omega_c$ where the phase shift is -45^o.)

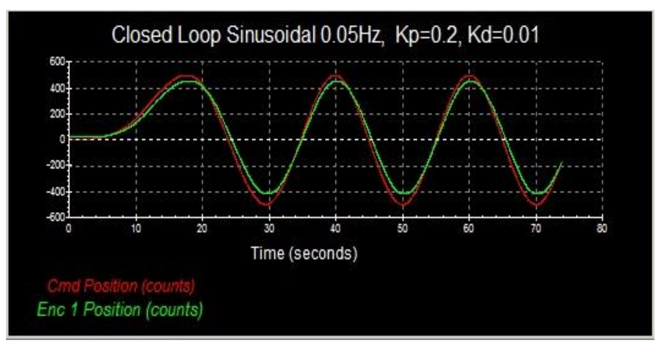
6.5 CLOSED-LOOP FREQUENCY RESPONSE

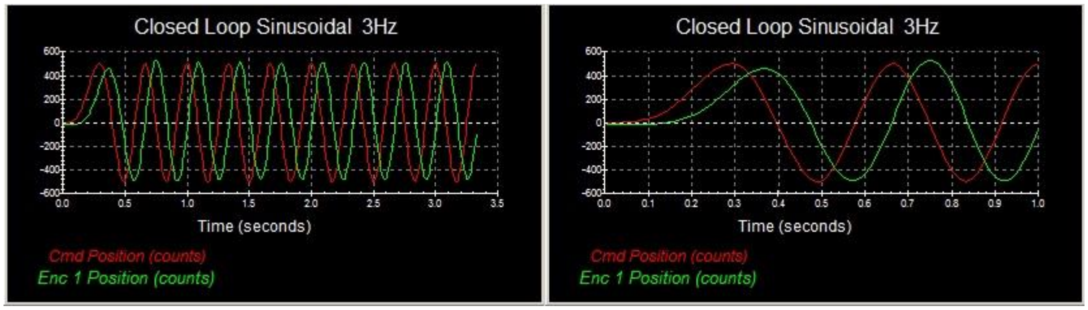
In this experiment, frequency-response measurements are made on the 'optimum' closed-loop control system that was set up at the end of the experiment 5.3.2. The magnitude response is also obtained by a 'swept-frequency' measurement procedure (available in the ECP Model 220 equipment) in which the frequency is increased either logarithmically or linearly over a given range while automatically acquiring and plotting output magnitude data.

6.5.1 SPOT-FREQUENCY MEASUREMENT

1) From the Utility menu, reset the controller. Implement the PID configuration of #1, with the controller values that you selected for 'best performance' from last experiment 5.3.2, PID-Control. Using a step size of 4000 Counts, dwell time of 4000ms, number of repetitions 1. Verify that the 'desirable' response that you had selected is obtained.
2) Next, from the 'Command' > 'Trajectory' > 'Sinusoidal' menu, set up a sinewave input of 500 Counts amplitude, Frequency **0.5** Hz, number of repetitions 10. From 'setup plot' choose both 'commanded position' and 'Encoder #1 Position' on the Left axis. Then, RUN the test. The very slow sinusoidal motion of the mechanism will be observed. A typical display is shown below.

   Repeat the test at another five well-spaced 'spot' frequencies in the range of 1 to 8 Hz. In each case, use 'Axis Scaling' to 'expand' and display the region beyond the initial (transient) part where the input shows a constant value of ± 500 Counts (1000 Cts, peak-to-peak). Use the scaled display to determine the magnitude ratio M and the phase shift ϕ: M is found by calculating the ratio between output and input peak-to-peak amplitudes. Phase shift ϕ is found by finding the time-difference t_{phi} between adjacent peaks of the output and input waveforms and then using the equation $\phi = -360\, f\, t_{phi}$ (degrees), (*Note: Since the output lags inputs, a negative sign is added*) where f is the frequency in Hz. Typical plots at 3 Hz, for both original and axis-scaled versions are shown below.





3) Make a three-column table of values of frequency f (Hz), Peak-Peak Output (Counts) and t_{phi} (sec) and enter the six sets of data in the table.

RESULTS:

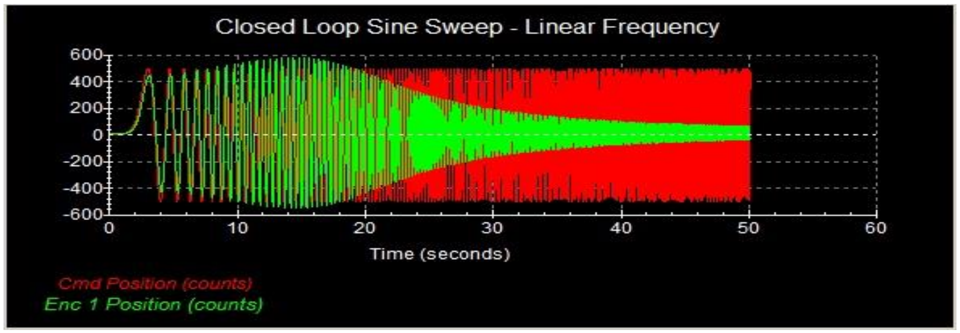
1) Use the data table obtained in Step 3 to calculate and tabulate M(dB) and ϕ(degrees) against ω=2πf (radians/sec). Obtain Bode magnitude and phase plots for the frequency range 0.05 to 8 Hz (~0.3 to 50.3 radians/sec). Use 3-cycle semi-logarithmic graph paper. Also draw the corresponding polar plot for your data range.
2) Attempt to fit the Bode magnitude data to an asymptotic plot and hence obtain an approximate transfer function.

6.5.2 SWEEP FREQUENCY MEASUREMENT

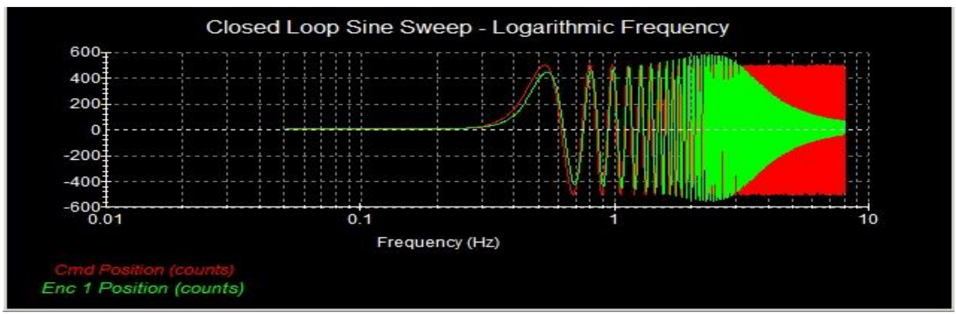
The Command/trajectory menu also offers the facility of displaying the nature of the magnitude-frequency response curve, by choosing the "sine sweep" trajectory. The input signal amplitude, the frequency sweep range, the sweep-time as well as the frequency-sweep mode (either linear or logarithmic) can be specified under Set Up. The vertical axis may be selected to show either linear or dB values. The positive side of the output envelope in the display will indicate the shape of the magnitude-frequency response and the frequency at which a peak occurs can be identified. It must be noted, however, that the displayed shape is only an approximation (especially dependent on the sweep-time) since the excitation time at each frequency may not be sufficient for the system to have reached steady state conditions.

PROCEDURE:

1) From the Utility menu, reset the controller. Implement the PID configuration of #1, with the controller values that you selected for 'best performance' from last experiment 5.3.2, PID-Control. Using a step size of 4000 Counts, dwell time of 4000ms, number of repetitions 1.
2) Use Set Up Trajectory and choose "Sinusoidal sweep". Setup an input of 500 Counts amplitude, Start frequency 0.05 Hz, End frequency 8Hz, linear sweep and a sweep time of 50 sec. From 'setup plot' choose both 'commanded position' and 'Encoder #1 Position' on the Left axis. Then RUN the test. After a short delay, the sinusoidal motion of the rotary mechanism starting at the low frequency and increasing will be observed. Allow sufficient time for data acquisition to be completed and then display the plot. Typical sweep linear-frequency and log-frequency displays are shown below:

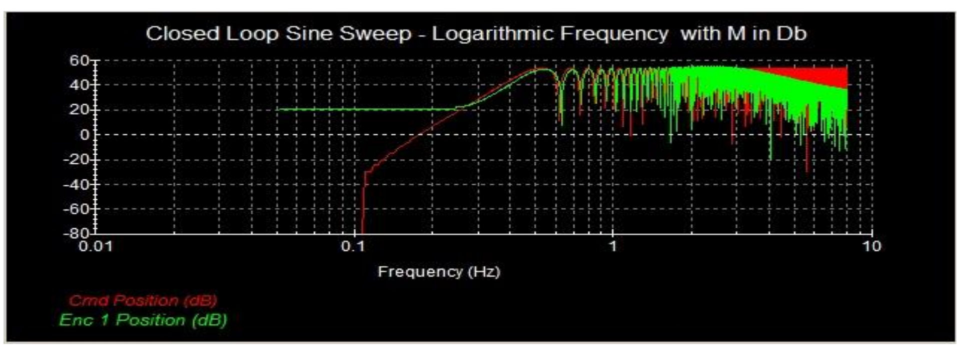


Linear frequency sweep



Logarithmic frequency sweep

3) Export raw data into a data file for further MATLAB analysis.
4) Try setting other display options for the horizontal and vertical axes. For example, a logarithmic plot with M in dB is shown below (for this part it is enough to change the logarithmic frequency to dB in the plot data setting).



Logarithmic frequency sweep with M (dB) values (upper envelope value only)

RESULT

1) Evaluate the amplitude and frequency and at a few points on the curve and compare the results with the spot-frequency data obtained earlier. Comment on the results.
2) *Use MATLAB LTI viewer to do system frequency-domain analysis. Refer to the code in the section 6.3 CLOSED-LOOP TRANSIENT RESPONSE .
3) Summarize the roles of engineering tool MATLAB/Simulink when you design a control system.

================================