ELEC 372 - Homework 3

(To be submitted on Moodle by July 31, 11:59PM)

Note:

- By the given deadline, upload a report (in pdf format) containing the solutions to problems below. Late reports will not be accepted.
- For problems solved by hand, include in the report all the steps of the solution.
- For problems requiring the use of Matlab/Simulink, include in the report the obtained results. Upload also the used/developed Matlab and Simlulink files, collecting them in a single zip folder. The uploaded Matlab/Simulink files must be working.
  Plots and figures (e.g., generated in Matlab/Simulink) can be included in the report to better describe/comment the obtained solutions.

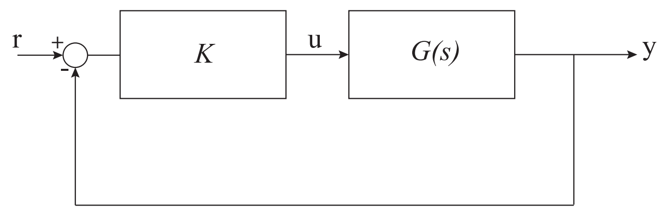
Problem 1 (5 points)**.** Consider the following third order system:

$$G_1(s) = \frac{50}{s(s+5)(s+50)}$$

a) Find the two dominant poles of $G_1(s)$ and approximate $G_1(s)$ with a second order transfer function $G_2(s)$. Verify, in Matlab, by plotting the step responses, that $G_2(s)$ is a good approximation of $G_1(s)$.

b) Are $G_1(s)$ and $G_2(s)$ stable systems? Are $G_1(s)$ and $G_2(s)$ BIBO stable systems?

c) Consider the feedback control system shown in the figure below, where the controller's transfer function is a static positive gain $K > 0$. Assuming $G(s) = G_2(s)$, compute the closed-loop transfer function $W_2(s) = \frac{Y(s)}{R(s)}$



d) Consider $W_2(s)$ and assume $R(s) = 1/s$. Find for which values (range) of $K \in [K_a,\, K_b]$, $K_a, K_b > 0$, $y(t)$ meets the following specifications:

- Damping ratio: $\delta \leq 0.7$
- Overshoot: $\%OS \leq 7\%$

e) Consider $K = K_a$ (i.e, the smallest admissible value of K found in the previous item). Discuss the stability and BIBO stability of $W_2(s)$.

f) In Matlab, consider $K = K_a$ and compare the closed-loop step responses of the systems $W_1(s)$ and $W_2(s)$ obtained using $G(s) = G_1(s)$ and $G(s) = G_2(s)$, respectively. Using Matlab, verify if the controller $K = K_a$, tuned on $W_2(s)$, allows $W_1(s)$ to fulfill the step-response specification $\%OS \leq 7$. Comment on why the specification is or is not verified.

g) Observe the stability of $G_1(s)$ in open-loop (i.e., in the absence of a controller and feedback loop), and observe the stability of the controlled closed-loop system $W_1(s) = \frac{Y(s)}{R(s)}$, with $G(s) = G_1(s)$ and $K = K_a$. Comment on the positive or negative effect of the joint use of a controller and feedback loop.

Problem 2 (5 points)**.** Consider the following fourth order system:

$$G(s) = \frac{(s+1)^2}{s^4 + 4s^3 + s^2 - 6s}$$

a) By using the Routh-Hurwitz criterion, analyze the stability and BIBO-stability of $G(s)$. Double-check your result in Matlab, by numerically computing the poles of $G(s)$ using the command *roots*.

b) Consider the feedback control system shown in the figure below, where the controller's transfer function is a static positive gain $K > 0$. Using the Routh-Hurwitz criterion determine for which values of K the closed-loop system is BIBO stable.

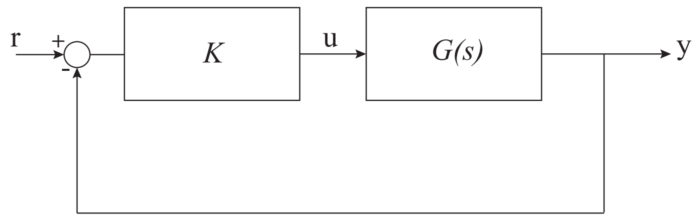


Figure 1: Block Diagram

c) By using the Routh-Hurwitz criterion, discuss the stability of the closed-loop system when $K = 5$

d) In Simulink, implement and simulate the closed-loop system, considering $K = 1$, $K = 5$, $K = 30$. Assume $r(t) = 1$, $t \geq 0$ and a simulation time of 10 second. Send the simulation data to the Matlab workspace and plot the output $y(t)$ for all the considered values of K. Comment if and why the obtained outputs are consistent with the stability results found in the previous steps.

e) In Simulink, implement and simulate the closed-loop system considering $K = 30$ and a ramp reference signal (i.e., $r(t) = t$, $t \geq 0$). Comment if and why the obtained outputs are consistent with the stability results found in the previous steps.

f) Consider the closed-loop system. Determine the tracking error for a step reference signal when $K = 1$ and $K = 10$