MODIFIED EQUATION TECHNIQUES

$$\delta_{tt} u(t) = \frac{u(t+h) - 2u(t) + u(t-h)}{h^2}$$

$$u(t+h) \approx u(t_n) + h \left.\frac{du}{dt}\right|_{t=t_n} + \frac{h^2}{2} \left.\frac{d^2u}{dt^2}\right|_{t=t_n} + \dots$$

$$u(t-h) \approx \dots$$

CORRECTION FACTOR

$$\delta_{tt} = \frac{d^2}{dt^2} + \frac{h^2}{12}\frac{d^4}{dt^4} + \frac{h^4}{360}\frac{d^6}{dt^6} + \dots = \sum_{l=1}^{+\infty} \frac{2h^{2(l-1)}}{(2l)!} \frac{d^{2l}}{dt^{2l}}$$

CORRECTION FACTOR

$$\delta_{tt} u^n = -\omega_0^2 u^n$$

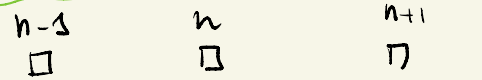
$\delta_{tt} \to -\omega_0^2$

$\Rightarrow \delta_{tt}\delta_{tt} \to \omega_0^4$

MODIFY:

$$\left(\delta_{tt} - \frac{h^2}{12}\delta_{tt}\delta_{tt}\right) u^n = -\omega_0^2 u^n$$

$$\left(\delta_{tt} - \frac{h^2}{12}\omega_0^4\right) u^n = -\omega_0^2 u^n$$



$\varepsilon = O(h^4)$

FOURTH-ORDER ACCURATE...

REWRITE AS

$$\delta_{tt} u^n = \frac{1}{h^2}\left[-\omega_0^2 h^2 + \frac{\omega_0^4 h^4}{12}\right] u^n$$

ADDING AN EXTRA CORRECTION FACTOR

$$\delta_{tt} u^n = \frac{1}{h^2}\left(-\omega_0^2 h^2 + \frac{\omega_0^4 h^4}{12} - \frac{\omega_0^6 h^6}{320}\right) u^n$$

$\varepsilon = O(h^6)$

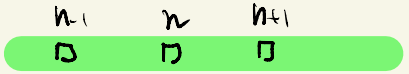
SIXTH-ORDER ACCURATE

$$\frac{1}{h^2}\left(-\omega_0^2 h^2 + \frac{\omega_0^4 h^4}{12} - \frac{\omega_0^6 h^6}{320} + \dots\right)$$

$$= \frac{2}{h^2}\left(-1 + 1 - \frac{\omega_0^2 h^2}{2} + \frac{\omega_0^4 h^4}{4!} - \frac{\omega_0^6 h^6}{6!} + \dots\right)$$

$\cos(\omega_0 h)$

$$\delta_{tt} u^n = \frac{2}{h^2}\left(-1 + \cos(\omega_0 h)\right) u^n$$



EXACT SCHEME !!