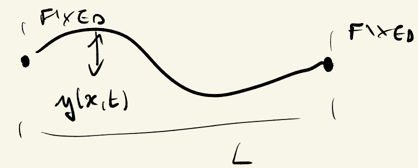
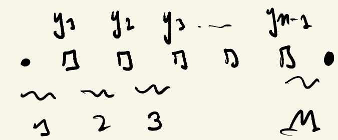
SPACE-TIME DISCRETISATION OF THE WAVE EQUATION

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \qquad \text{PDE}$$



$y = y(x,t) \qquad x \in [0, L]$

$\hookrightarrow \; y(x,t) \longmapsto \vec{y}(t) \in \mathbb{R}^{M-1}$



$$\frac{\partial^2}{\partial x^2} \longmapsto \underbrace{\underline{D}_{xx}}_{\text{LAPLACIAN}} \in \mathbb{R}^{M-1 \times M-1}$$

DISCRETISATION OF LAPLACIAN!

$$\delta_{xx} y_m = \frac{y_{m-1} - 2y_m + y_{m+1}}{\Delta x^2} \Rightarrow \text{INTERNAL POINTS}$$

$$\left(\delta_{xxxx} y_m = \frac{y_{m-2} - 4y_{m-1} + 6y_m - 4y_{m+1} + y_{m+2}}{\Delta x^4}\right)$$

(DISCRETISATION OF BIHARMONIC OPERATOR!)

UNDEFINED!

$$\cancel{\delta_{xx} y_0} = \frac{y_{-1} - 2y_0 + y_1}{\Delta x^2} \qquad \text{DIRICHLET CONDITION}$$

$$\delta_{xx} y_1 = \frac{\cancel{y_0} - 2y_1 + y_2}{\Delta x^2}$$

FROM THE NUMERICAL BOUNDARY COND.

$y_0 = 0$

FOR NEUMANN?

$$\delta_{xx} y_0 = \frac{y_{-1} - 2y_0 + y_1}{\Delta x^2} = \frac{-2y_0 + 2y_1}{\Delta x^2}$$

$$\delta_{x\cdot} y_0 = 0 \to y_{-1} = y_1$$

IN YOUR EXAM

$y_0 = 0$
$\delta_{xx} y_0 = 0$ } SIMPLY-SUPPORTED

$$\underbrace{\left(c^2 \delta_{xx} - c_L^2 \kappa^2 \delta_{xxxx}\right)}_{\text{STIFF STRING!}} y_m \quad , m=1$$

AFTER SPATIAL DISCRETISATION, PROCEED WITH TIME INTEGRATION

$$\ddot{\vec{y}} = c^2 \underline{\underline{D}}_{xx} \vec{y}$$

↳ $\vec{y}(t) \to \vec{y}^n$

↳ $\delta_{tt} \vec{y}^n = c^2 \underline{\underline{D}}_{xx} \vec{y}^n$

$$\frac{\vec{y}^{n+1} - 2\vec{y}^n + \vec{y}^{n-1}}{k^2} = c^2 \underline{\underline{D}}_{xx} \vec{y}^n$$

$$\vec{y}^{n+1} = \left[2\underline{\underline{I}} + c^2 k^2 \underline{\underline{D}}_{xx} \right] \vec{y}^n - \vec{y}^{n-1}$$

UPDATE EQN

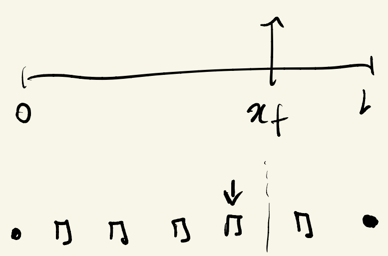
⇒ F.D. SCHEME UPDATE!

- EXPLICIT
- CONDITIONALLY STABLE! (WHAT IS THE STABILITY CONDITION HERE?)
- REQUIRES TWO PREVIOUS STEPS TO BE STORED IN MEMORY, NO MORE!

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2}{\partial x^2} y + \delta(x - x_f) f(t)$$

$$\ddot{\vec{y}}(t) = c^2 \underline{\underline{D}}_{xx} \vec{y} + \vec{J} f(t)$$

SPREADING OPERATOR!



$$\vec{J} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$