Assignment 2

1. **a.**

   For $f(n) = \lceil lg(n) \rceil !$:

   We know that $(lg(n!)) = \Theta(nlg(n))$

   We must show that $log(f(n)) = O(log(n))$

   $log(f(n)) = lg(n) * lg(lg(n))$

   $lg(n) * lg(lg(n))$ is not $O(lg(n))$, so $f(n)$ is not polynomially bounded.

   b.

   For $f(n) = \lceil lg(lg(n)) \rceil !$:

   We must show that $log(f(n)) = O(log(n))$

   $log(f(n)) = log(lg(lg(n))!) = lg(lg(n)) * lg(lg(lg(n)))$

   $lg(lg(n)) * lg(lg(lg(n)))$ can be said to be $o(log(n))$, so this function is polynomially bounded.

2. **Base Case:**

   For F_0, we have $\frac{\phi^0 - \hat{\phi 0}}{\sqrt{5}}$ = $\frac{1-1}{\sqrt{5}}$ = 0

   For F_1, we have $\frac{\phi^1 - \hat{\phi 1}}{\sqrt{5}}$ = $\frac{\sqrt{5}}{\sqrt{5}}$ = 1

   Thus, we have shown that this holds for both base cases

   Inductive Step:

   $F_{k+1} = F_k + F_{k-1}$

   This can be written as: $\frac{\phi^k - \hat{\phi k}}{\sqrt{5}}$ + $\frac{\phi^{k+1} - \phi \hat{k+1}}{\sqrt{5}}$

   This can also be written as: $\frac{\phi^{k-1}(\phi+1) + \hat{\phi}^{k-1}(\hat{\phi}+1)}{\sqrt{5}}$

   Now, $\phi + 1$ is equal to $\frac{3+\sqrt{5}}{2}$ which is equal to ϕ^2. This also applies to $\hat{\phi}$

   So, we can simplify the equation as: $\frac{\phi^{k-1}(\phi^2) + \hat{\phi}^{k-1}(\hat{\phi}^2)}{\sqrt{5}}$

   This can be simplified into: $\frac{\phi^{k+1} + \hat{\phi}^{k+1}}{\sqrt{5}}$

So, this holds for $k+1$.

3. We know that is $k * ln(k) = \Theta(n)$, then $n = \Theta(k * lnk)$ given that Θ is symmetric.

   Next, we'll take log_2 of $n = \Theta(k * lnk)$ to get $ln(n) = \Theta(ln(k * ln(k))) = \Theta(ln(k) + lnln(k)) = \Theta(ln(k))$

   If we divide n by $ln(n)$, we get $\frac{\Theta(klnk)}{\Theta(knk)}$ which can be simplified into $\Theta(\frac{k*ln(k)}{ln(k)})$ which is equal to $\Theta(k)$

   Since Θ is symmetric we can say, $k = \Theta(\frac{n}{ln(n)})$

4. **a.**

   Let's use the definition of $f(n) \leq Cg(n)$ where $f(n) = 2^{n+1}$ and $g(n) = 2^n$

   $2^{n+1} \leq C * 2^n$

   Divide both sides by 2^n

   $2^1 \leq C$

   Since $C > 0$, $2^{n+1} = O(2^n)$

   b.

   Let's use the definition of $f(n) \leq Cg(n)$ where $f(n) = 2^{2n}$ and $g(n) = 2^n$

   $2^{2n} \leq C * 2^n$

   Divide both sides by 2^n

   $2^n \leq C$

   This is never possible because there is no constant greater than 2^n for all n. Thus, $2^{2n} \neq O(2^n)$

5.

	A	B	O	o	Ω	ω	Θ
a.	$log(n)^k$	$n^{\in}$	Yes	Yes	No	No	No
b.	n^k	c^n	Yes	Yes	No	No	No
c.	$\sqrt{n}$	$n^{sin(n)}$	No	No	No	No	No
d.	2^n	$2^{n/2}$	No	No	Yes	Yes	No
e.	$n^{log(c)}$	$c^{log(n)}$	Yes	No	Yes	No	Yes
f.	$log(n!)$	$log(n^n)$	Yes	No	Yes	No	Yes

6. $$2^{2n+1} > 2^{2n} > (n+1)! > n! > e^n > n \cdot 2^n > (\frac{3}{2})^n > n^{lg(lg(n))} \text{ and } lg(n)^{lg(n)} >$$

$$lg(n)! > n^3 > n^2 \text{ and } 4^{lg(n)} > nlg(n) \text{ and } lg(n!) > n \text{ and } 2^{lg(n)} >$$

$$(\sqrt{2})^{lg(n)} > 2^{\sqrt{2lg(n)}} > (lg(n))^2 > ln(n) > \sqrt{lg(n)} > ln(ln(n)) >$$

$$(\sqrt{2})^{lg(n)} > 2^{\sqrt{2lg(n)}} > (lg(n))^2 > ln(n) > \sqrt{lg(n)} > ln(ln(n)) >$$

$$2^{lg^*(n)} > lg^*(n) \text{ and } lg^*(lg(n)) > lg(lg^*(n)) > 1 \text{ and } n^{1/lg(n)}$$