RA: An Exploratory Assignment on Minimum Spanning Trees

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1 Introduction

Let the weight of a tree be the sum of the squares of its edges lengths. Given a set of points P in the unit square $I \times I$ let $W(P)$ be the weight of the *minimum spanning tree (MST)* of P, where an edge length is the *Euclidean distance* between its endpoints. If P consist of the four corners of the square, then $W(P) = 3$. Gilbert and Pollack [1] proved that $W(P)$ is $\mathcal{O}(1)$ and this was extended to an arbitrary number of dimensions by Bern and Eppstein [2]. While more recent *divide-and-conquer* approaches have show that $W(P) \leq 4$, no point set is known with $W(P) > 3$, and hence it has been widely conjectured that $W(P) \leq 3$. In 2013, it was proven that $W(P) < 3.41$ [3]. Here we show an empirical experiment to check whether $W(P) < 3.41$ holds for any $MST(P)$.

2 Experiment

In order to check the previous theorem, we uniformaly at random generate points in the unite square P and compute the weight of the MST. We do this with an increasing number of points in order to explore the solution space. It is important to note that the exploration is not exhaustive since exploring the whole solution space would require a large amount of computational power. Also, the implementation is naïve since it does not explicitly aims for the degenerate instances where $W(P) \sim 3.41$ may happen.

The it uses *Prim's Algorihm* to search for the MST on the unit square.

Algorithm 1: Minimum Spanning Tree Prim's Algorithm

Input: An undirected weighted graph $G = (V, E)$
Output: The minimum spanning tree of the input graph G

foreach $v \in V$ **do**
  $key(v) = \infty$
  $parent(v) = NIL$
end
$key(r) = 0$ `// Pick` ***u.a.r.*** `the initial vertex` $r \in V$
$Q = V$
while $Q \neq \emptyset$ **do**
  $u = EXTRACT - MIN(Q)$
  foreach $v \in ADJ(u)$ **do**
    if $v \in Q$ *and* $w(u, v) < key(v)$ **then**
      $key(v) = w(u, v)$
      $parent(v) = u$
    end
  end
end

$\|V\|$	Time	R^2	μ	σ
128	8.1 ms	0.996	8.358 ms	327.8 μ
512	406.0 ms	1.0	391.9 ms	10.33 ms
1024	153.6 ms	1.0	150.3 ms	3.178 ms
2048	943.1 ms	1.0	934.4 ms	14.24 ms

Table 1: Prim's Algorithm benchmark.

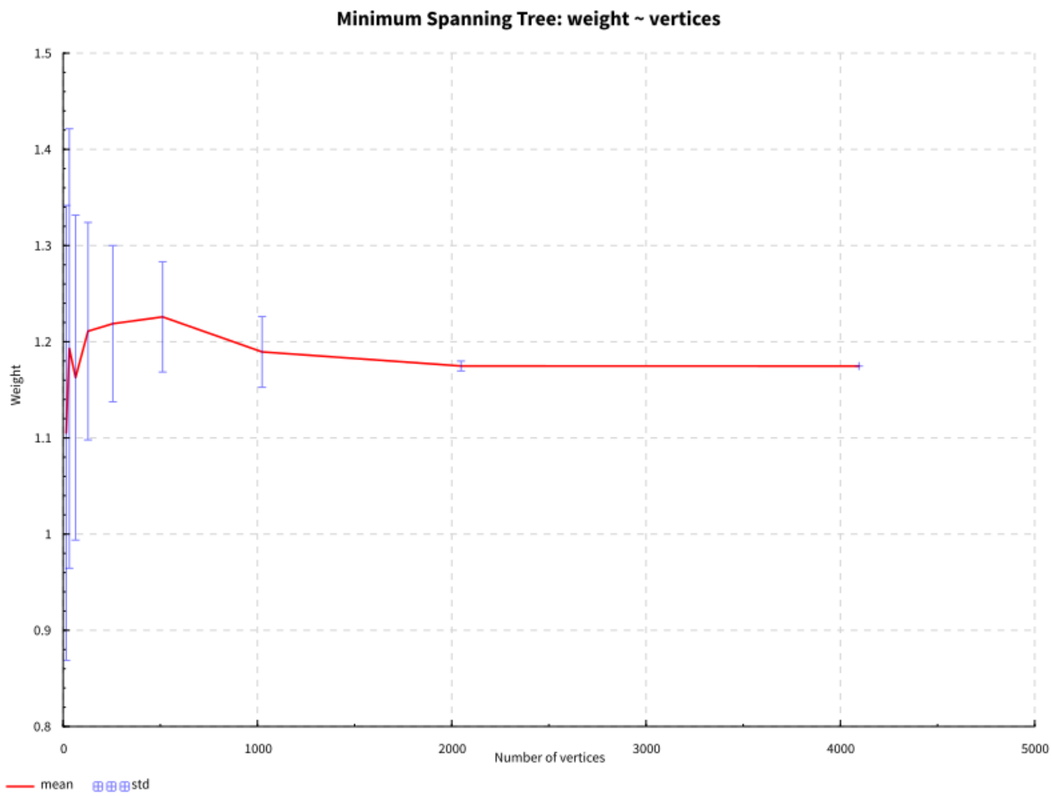

Figure 1: Experiment results.

3 Results

References

[1] E. N. Gilbert and H. O. Pollack. Steiner minimal trees. *SIAM J. App. Math.*, 16(1):1–29, 1968.

[2] M. W. Bern and D. Eppstein. Worst-case bounds for subadditive geometric graphs. *In Proc. 9th Symp. on Comp. Geom.*, pages 183–188, 1993.

[3] Oswin Aichholzer, S Allen, Greg Aloupis, Luis Barba, Prosenjit Bose, JL de Varufel, John Iacono, Stefan Langerman, D Souvaine, Perouz Taslakian, et al. Sum of squared edges for mst of a point set in a unit square. In *Japanese Conference on Discrete and Computational Geometry (JCDCG)*, 2013.