DS3001 EDA Assignment

1. $m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i) = \frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bx_i = a + b\cdot\frac{1}{N}\sum_{i=1}^{N} x_i = a + bm(X)$

2. $\text{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))[(a+by_i) - m(a+bY)] = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(b(y_i - m(Y))) = b\cdot\frac{1}{N}\sum_{i=1}^{N}$
$(x_i - m(X))(y_i - m(Y)) = b\,\text{cov}(X,Y)$

3. Let $Y = X$

$\text{cov}(a+bX, a+bX) = b\,\text{cov}(a+bX, X) = b\,\text{cov}(X,X)$

$\text{cov}(a+bX, a+bX) = b^2\,\text{cov}(X,X)$ $\quad \text{Cov}(X,X) = s^2$

4. $\text{median}(g(X)) = g(\text{median}(X))$; $Q_p(g(X)) = g(Q_p(X))$

IQR: $g(Q_{0.75}(X)) - g(Q_{0.25}(X))$ $\quad$ Range: $g(\max(X)) - g(\min(X))$

Non-decreasing transformations preserve the median & all quantiles, but may distort distances like IQR & range unless the transformation is linear

5. No, except when $g(x)$ is linear

Ex. $X = \{0, 2\}$

$m(X) = \frac{0+2}{2} = 1$

$g(x) = x^2$

$m(g(X)) = \frac{0^2 + 2^2}{2} = 2$, $g(m(X)) = (1)^2 = 1$

$g(m(X)) \neq m(g(X))$