Force-free magnetohydrodynamics

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ABSTRACT

Force-free MHD equations.

Key words. turbulence — plasmas — non-thermal particles

1. Theory

Blandford gives force-free equation set as Maxwell's equations

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \frac{4\pi}{c}\boldsymbol{J} \tag{1}$$

and

$$\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} \tag{2}$$

in addition to current term

$$\boldsymbol{J} = \frac{c}{4\pi}(\nabla \cdot \boldsymbol{E})\frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2} + \frac{c}{4\pi}\boldsymbol{B}\frac{\boldsymbol{B} \cdot \nabla \times \boldsymbol{B} - \boldsymbol{E} \cdot \nabla \times \boldsymbol{E}}{B^2} \tag{3}$$

The current can be represented as

$$\boldsymbol{J} = \boldsymbol{J}_\parallel + \boldsymbol{J}_{\mathrm{drift}}, \tag{4}$$

Here the first term

$$\boldsymbol{J}_\parallel = \boldsymbol{B}\frac{\boldsymbol{B} \cdot \nabla \times \boldsymbol{B} - \boldsymbol{E} \cdot \nabla \times \boldsymbol{E}}{B^2} \tag{5}$$

is the parallel conduction current that maintains the force-free condition $\boldsymbol{E} \cdot \boldsymbol{B} = 0$ reducing to $(\nabla \times \boldsymbol{B})_\parallel$ in steady state. The second term

$$\boldsymbol{J}_{\mathrm{drift}} = \nabla \cdot \boldsymbol{E}\frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2} \tag{6}$$

is the drift current given by the $\boldsymbol{E} \times \boldsymbol{B}$ advection of the charge density.

Effect of parallel current can be mimicked with an even simpler system by reseting values of $\boldsymbol{E}$ such that $\boldsymbol{E}_\parallel = 0$. This is equivalent of replacing the parallel current with a resistive term $\sigma_\parallel E_\parallel$ where $\sigma_\parallel \rightarrow \infty$. Note that retention of the parallel current results in much lower diffusion error.

Force-free evolution can lead to configurations in which $B^2 - E^2 > 0$ is violated. Therefore, we set

$$\boldsymbol{E} \rightarrow \sqrt{\frac{B^2}{E^2}}\boldsymbol{E}. \tag{7}$$

This acts like a small, highly localized source of dissipation in current sheets. Other simpler option is to set $E \leq B$ after every time step by reseting the electric field to $E = B$ in the regions where $E > B$.

Physically these limiters can be thought of converting electromagnetic energy to thermal energy and radiation. This violation can happen during interaction of strong waves as the Alfén speed becomes imaginary and leads to exponential instability, which physically would be averted by dissipation. If the $\boldsymbol{E} \times \boldsymbol{B}$ drift approximation is violated, particles will be accelerated while decoupling form the magnetic field lines. This will restore the $E = B$ on plasma timescales (without taking any stance on the actual microphysics). This can be parameterized by tracking $J_\perp = \sigma_\perp E$ when $E > B$ and $J_\perp = 0$ otherwise where the perpendicular conductivity $\sigma_\perp \rightarrow \infty$.

Another example is the formation of current sheets. In this case the magnetic dominance is invalid (magnetic field reverses and goes through zero) and the plasma pressure must sustain the sheet. However, in FFE pressure is not included so we try to keep the current sheets unresolved on the grid with a jump over one cell.

Third-order Runge-Kutta time integration. We maintain time alignment of the fields. Linear interpolation of the fields to the same location on the grid.