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Particle acceleration

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ABSTRACT

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(Birdsall & Langdon 1985)

Standard Fermi-like acceleration is

$$\langle \frac{\Delta p^2}{2\Delta t} \rangle \sim \beta_u^2 \frac{p^2}{t_{\rm int}}, \quad (1)$$

where acceleration time is then

$$t_{\rm acc} \sim \frac{t_{\rm int}}{\beta_u^2}. \quad (2)$$

Quasi-linear theory here says that $t_{\rm int} = t_{\rm scatt}$.

Generally, the particle interacts with a broad spectrum of compressive, shearing, accelerating and vortical motions, leading to stochastic acceleration.

Lemoine 2019 then predicts that

$$D_{pp}^{\rm non-res} \approx \langle \delta u^2 \rangle \frac{p^2}{l_0 (\frac{t_{\rm scatt}}{l_0})^{2-q_u}} \propto p^{2-\epsilon}, \quad (3)$$

where q_u is the index of 4-velocity spectrum, l_0 is the stirring scale, and $\langle \delta u^2 \rangle$ is the combination of 4-velocity perturbation modes. Then

$$t_{\rm acc}^{\rm non-res} \sim \frac{l_0}{\langle \delta u^2 \rangle}. \quad (4)$$

1. Theory

Macroscopic turbulence: large-scale mass motion, long-period large amplitude hydromagnetic waves or shock waves.

Microscopic turbulence: intensities of the various wave modes which propagate in the medium are above thermal equilibrium value

(Fermi 1949) (Fermi 1954) (Lemoine 2019)

Scattering depends on the length of the weakly damped turbulent wave along the mean magnetic field direction, $\lambda_\parallel$, and energetic particle scattering mean-free path, $\lambda_{\rm mfp}$. In the weak scattering limit $\lambda_\parallel \ll \lambda_{\rm mfp}$. In this case, particle's velocity along the mean magnetic field, $v_\parallel$ is almost constant. Interaction is of resonant type and gyrofrequency is an integer multiple of the wave length. In the opposing case of strong scattering, $\lambda_\parallel \gg \lambda_{\rm mfp}$ (Ptuskin 1988). Particle interacts non-resonantly with the wave traveling diffusively accross a wave.

Basics (Melrose 1968) (**?**)

effect of plasma waves on charged particles are studied by expanding Vlasov equation to second order phase space density perturbations, and ensebmle averaging over the statistical properties of the plasma waves. This is the quasi-linear theory, QLT.

In a very general way, the diffusion coefficient is given as (**?**)

$$D_{p_\parallel} = \frac{p_\perp^2 v_\perp^2}{4B_0^2} \int d^3k k_\parallel^2 I_B(\boldsymbol{k}) R(\boldsymbol{k}), \quad (5)$$

where $I_B(\boldsymbol{k})$ is the three-dimensional power spectrum of magnetic field fluctuations, and

$$R(\boldsymbol{k}) \propto \int_0^\infty dt \exp[i(\omega(\boldsymbol{k}) - v_\parallel k_\parallel)t] f(t), \quad (6)$$

is a resonance function that describes the interaction of the particle and the wave; $f(t)$ is the time correlation function for wave-particle interactions.

1.1. Slow stochastic diffusion

Standard models assume exponential or Gaussian wave decay in time due to nonlinear interactions. In this case, $f(t) = \exp[-\omega_{\rm nl}^2 t^2]$. One such a selection is, for example, (**??**)

$$R(\boldsymbol{k}) = \frac{\sqrt{\pi}}{2\omega_{\rm nl}} \exp\left[-\frac{k_\parallel^2 (v_\parallel - v_p)^2}{4\omega_{\rm nl}^2} \right], \quad (7)$$

where a dispersion relation of $\omega = k_\parallel v_p$ for the waves is assumed. Physically this means that particle couples to a spectrum of perturbations as it passes a wave,i.e., $k_\parallel(v_\parallel - v_p) \lesssim \omega_{\rm nl}$.

Let us assume a strong anisotropic magnetic turbulence spectrum

$$I(\boldsymbol{k}) = \frac{\delta B^2 l_0^3}{12\pi} (k_\perp l_0)^{-10/3} g\left(\frac{k_\parallel}{k_\perp^{2/3}} l_0^{1/3} \right) \quad (8)$$

where a sharp cutoff function of $g(x) \propto \exp(-x)$ is favored by simulations (?). The cutoff means that there is not much power outside the anisotropic Goldreich-Sridhar cone (?) in a weakly compressible turbulence. Nonlinear frequency in this case is $\omega_{\rm nl} \approx (v_A/l_0)(k_\perp l_0)^{2/3}$ that is about one eddy turnover time.

Under these assumptions, the total momentum diffusion coefficient becomes

$$D_p = \frac{\sqrt{\pi} p_\perp^2 v_\perp^2}{48 v_A} \frac{\delta B^2}{B_0^2} \left(\frac{v_\parallel}{v_\perp}\right)^2 \int dk_\parallel dk_\perp \frac{k_\parallel^2}{k_\perp^3} g\left(\frac{k_\parallel}{k_\perp^{2/3}}\right) \exp\left(-\frac{k_\parallel^2 l_0^2 v_\parallel^2}{4(k_\perp l_0)^4 v_A^2}\right), \tag{9}$$

Both the exponential and $g(x)$ tend to limit the high $k_\parallel$ perturbation modes for a given $k_\perp$. Then,

$$D_p = p^2 \frac{\pi}{24} \frac{v_A^2}{c l_0} \frac{\delta B^2}{B_0^2} \frac{(1-\mu^2)^2}{\mu^3} \log(k_{\max} l_0), \tag{10}$$

where $\mu = v_\parallel / v_\perp$.

Acceleration time is then

$$t_{\rm acc} \sim \left(\frac{\delta B^2}{B_0^2}\right)^{-2} \frac{l_0}{v_A} \frac{c}{v_A}, \tag{11}$$

that importantly is independent of the particle energy. Physically this corresponds to about one eddy turnover time divided by the fraction of the magnteic energy at low $k_\parallel$ that satisfy $k_\parallel \lesssim \omega_{\rm nl}/c$.

1.2. Fast ballistic diffusion

If the particle trajectory is ballistic it can scatter via linear resonances. The resonance function is then a δ-function like,

$$R(\boldsymbol{k}) = \pi \delta(k v_p \pm k_\parallel v_\parallel), \tag{12}$$

where v_p is the isotropic phase velocity of fast modes; v_A for $\beta \ll 1$ and c_s for $\beta \gg 1$. Performing the integral over the δ-function and $(\delta B/B)^{-q}$ spectrum of perturbation amplitudes then gives

$$D_p = p^2 \frac{v_p^2}{c l_0} \left(\frac{\delta B^2}{B_0^2}\right)^{-q} \tag{13}$$

1.3. Main particle acceleration questions

1. How efficient is the acceleration?
2. What is the slope of the power-law high-energy tail?
3. What is the maximum attainable energy?
4. Which physicala mechanisms govern the injection of particles from thermal pool to higher energies?
5. What timescales are particles accelerated?

Fully-kinetic simulations allow to study the acceleration process and the possible subsequent back-reaction self-consistently. Albeit computationally heavy, very useful.

(Fermi 1949) (Fermi 1954) (Lemoine 2019)

Fermi scheme can apply to any flow for which to electromagnetic 4−scalers fulfil $\boldsymbol{E}\cdot\boldsymbol{B} = 0$ and $E^2 - B^2 < 0$. Under these conditions one can always boost to a frame for which $\boldsymbol{E}$ vanishes, by

$$\boldsymbol{\beta_B} = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}. \tag{14}$$

It is important to note here that this only occurs when ideal magnetohydrodynamics does not apply, since in that case the aforementioned velocity coincides with the local plasma bulk velocity.

Mean 4-momentum change $\langle \Delta p^\alpha / \Delta t \langle$. Diffusion tensor $\langle \Delta p^\alpha \Delta p^\beta / \Delta t \langle$ for $\alpha, \beta = 0, \ldots, 4$.

1.4. Fermi-like diffusion

Based on Pelletiere 2017

, relative velocity β_{rel} between upstream and downstream flows (velocity of the downstream plasma relative to the upstream frame). Pitch angle cosine μ_1 before scattering and scattered pitch angle cosine μ_2 when it is coming back (expressed in the upstream rest-frame). Its energy gain is

$$G = \frac{1 - \beta_{rel}\mu_1}{1 - \beta_{rel}\mu_2} \tag{15}$$

Choosing downstream reference frame, $\beta_{rel} \approx 1 - 1/\gamma_{sh}^2$. Pitch angle cosine of the crossing particle has $-1 < \mu_1 < \beta_{sh}$; A particle coming back from downstream to upstream has $\beta_{sh} < \mu_2 < 1$.

Average energy gain (assuming incorrectly) isotropic distribution functions and independent μ_1 and μ_2:

$$\langle G \rangle = \left(1 + \frac{2}{3}|\beta_{rel}|\right)^2. \tag{16}$$

Moreover, because the almost isotropic distirbution, probability of escapre through advection in the downstream plasma is $P_{\rm esc} = 4|\beta_d|$. The index of the powerlaw distribution $dN/dp \propto p^{-s}$ is then Bell 1978

$$s = 1 - \frac{\ln(1 - P_{esc})}{\ln\langle G \rangle} \approx 1 + 3|\frac{\beta_d}{\beta_{rel}}| \tag{17}$$

i.e., $s = 2$ for strong adiabatic shocks where $|\beta_{rel} = 3|\beta_d|$. In reality, the strong anisotropy of the distribution function complicates these calculations.

Note that this means that escapre probability P_{esc} controls the non-thermal slope.

Fermi acceleration is possible if scattering frequency in the turbulence exceeds gyrofrequency in the background field. Turbulent scattering frequency

$$\nu = \frac{e^2 \delta B_{|d}^2 \tau_{c|d}}{m^2 c^2 \gamma^2}. \tag{18}$$

This means that $\nu > \omega_p = e B_{|d} / m c \gamma$. Alternatively,

$$\sqrt{\sigma} < \zeta_B \tag{19}$$

Here ζ_B is the fraction of energy going to the turbulence energy as

$$W_{em} = \zeta_B \gamma_{sh}^2 n_u m c^2 \tag{20}$$

1.5. Diffusive shock acceleration (DSA)

Most widely accepted theory for non-thermal particle population generation Blandford & Eichler 1987 DSA ssumes presence of turbulence in the upstream. Energetics particles scattered by MHD waves gain energy by diffusively crossing the shock front back and forth many times. Central problem is the injection since DSA is only effective for energetic particles that can interact with MHD waves.

1.6. Shock-surfing acceleration

Thought to be a pre-acceleration mechanism that can feed particles to the later-stage diffusive processes.

Electrons can be accelerated by an interaction with electrostatic waves from Buneman instability. Shimada & Hoshino 2000 Hoshino & Shimada 2002

In turbulent ramp-overshoot region electrons undergo stochastic Fermi-like acceleration (Bohdan 2017) or stochastic shock drift acceleration in the quasi-perpendicular case (Matsumoto 2017)

Sound speed

$$c_s^2 = \frac{\Gamma_{\mathrm{AD}} k_{\mathrm{B}} T_i}{m_i} \tag{21}$$

and $k_{\mathrm{B}} T_i = \frac{1}{2} m_i v_{\mathrm{th},i}^2$ where $v_{\mathrm{th},i}$ is the most probable speed.

1.7. Shock mediators

low-magnetization: Weibel splits current and subsequently also charge into filaments

high-magnetization: magnetic reflection magnetic reflection of the compressed downstream magnetic field. Initially everything is moving following $\boldsymbol{E} \times \boldsymbol{B}$ motion. Subsequently, a reflected particle sees a wrong sign of $\boldsymbol{E}$ field and undergo Larmor gyration. This gyration causes positive and negative particles to go in opposite directions transverse to the flow and the associated transverse current increases the magnetic field. Incoming particles now see a jump in magnetic field and undergo gyration that deccelerates and so also increases the local density of the flow. In magnetized case, the transition therefore happens within a few Larmor radii in the compressed field.

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