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MHD turbulence

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ABSTRACT

Quick historical review on MHD turbulence theory.

Key words. Plasmas – Turbulence

1. Theory of turbulence

1.1. Kolmogorovian theory

Energy is pumped into a homogenous conducting medium with a fixed rate ϵ. Dimensionless analysis gives for the energy spectrum (?)

$$E(k) \sim \epsilon^{2/3} k^{-5/3}. \tag{1}$$

Same can be written in terms of average velocity increments

$$\delta u_\lambda \sim (\epsilon \lambda)^{1/3}. \tag{2}$$

1.2. Iroshnikov-Kraichan theory

If the $\boldsymbol{B}$ field has an important role in energy transfer, than similar dimensionless analysis gives Iroshnikov Kraichnan 1965

$$E(k) \sim (\epsilon v_A)^{1/2} k^{-3/2} \tag{3}$$

and

$$\delta u_\lambda \sim (\epsilon v_A \lambda)^{1/4}, \tag{4}$$

for the Alfv́en speed with density ρ

$$v_A = \frac{B}{\sqrt{4\pi\rho}}. \tag{5}$$

The reasoning is based on the fact that Alfv́en time, $\tau_A \sim 1/k v_A$ is the time which interactions occur so the energy must come with a combination ϵv_A.

The theory is incorrect because it assumes a uniform scale k whereas in reality in a presence of a strong guide field $\boldsymbol{B}_0$ the scales split into $k_\parallel$ and $k_\perp$.

1.3. Goldreich-Sridhar critical balance

In a strong magnetic field $k_\parallel \ll k_\perp$. Parallel direction variation propagation velocity corresponds to Alfv́en waves with

$$\tau_A = \frac{l_\parallel}{v_A}, \tag{6}$$

whereas perpendicular variation is governed by nonlinear intearctions with characteristic times

$$\tau_{\rm nl} \sim \frac{l_\perp}{\delta u_\lambda}. \tag{7}$$

For Alfv́enic perturbations $\delta u_\lambda \sim \delta b_\lambda$. The two times, τ_A and $\tau_{\rm nl}$, are assumed to be equal. The natural "cascade time" must also be of same order, $\tau_c \sim \tau_A \sim \tau_{\rm nl}$ This gives

$$\frac{\delta u_\lambda^2}{\tau_c} \sim \epsilon \quad \text{and} \quad \tau_c \sim \tau_{\rm nl} \sim \frac{\lambda}{\delta u_\lambda}, \tag{8}$$

so that

$$\delta u_\lambda \sim (\epsilon \lambda)^{1/3} \tag{9}$$

and equally (??)

$$E(k_\perp) \sim \epsilon^{2/3} k_\perp^{-5/3}, \tag{10}$$

yielding a Kolmogov scaling for the perpendicular scales. Simultaneously, along the field the velocity increment satisfy

$$\frac{\delta u_\parallel^2}{\tau_c} \sim \epsilon \quad \text{and} \quad \tau_c \sim \tau_A \sim \frac{l_\parallel}{v_A} \tag{11}$$

so that

$$\delta u_{l\parallel} \sim \left(\frac{\epsilon l_\parallel}{v_A} \right)^{1/2}. \tag{12}$$

From here it follows

$$l_\parallel \sim v_A \epsilon^{-1/3} \lambda^{2/3}. \tag{13}$$

Physically $l_\parallel$ is the distance an Alfvénic pulse travels along the field at speed v_A over time $\tau_{\rm nl}$.

1.4. Reduced MHD and Elsasser fields

Elsasser fields are given as $\boldsymbol{Z} = \boldsymbol{u} \pm \boldsymbol{b}$. Here $b = B_0/\sqrt{4\pi\rho}$.

1.5. Weak turbulence

Weak turbulence theory stems from a perturbation in a (assumedly) small ratio $\tau_A/\tau_{\rm nl}$. WT scaling originates from

$$\delta Z_\lambda \sim \left(\frac{\epsilon}{\tau_A}\right)^{1/4} \lambda^{1/2} \tag{14}$$

where δZ_λ is perturbed Elsasser field. This gives a scaling

$$E(k_\perp) \sim \left(\frac{\epsilon}{\tau_A}\right)^{1/2} k_\perp^{-2}. \tag{15}$$

Eventually weak turbulence will transition to strong turbulence. For balanced turbulence this happens when the perturbation parameter becomes of order unity,

$$\frac{\tau_A}{\tau_{\rm nl}} \sim \frac{\tau_A^{3/4}\epsilon^{1/4}}{\lambda^{1/2}} \sim 1 \tag{16}$$

corresponding to a scale (assuming critical balance)

$$\lambda_{\rm CB} \sim \epsilon^{1/2}\tau_A^{3/2}. \tag{17}$$

2. MHD etc notes

(Sobacchi & Lyubarsky 2019) paragraphs:

The most important property of MHD turbulence is its strong anisotropy, with the turbulent eddies becoming strongly elongated in the direction of the background magnetic field at small scales. As first proposed by Goldreich & Sridhar (1995), in the course of the turbulent cascade the ratio of the longitudinal scale of the eddies, $\lambda_\parallel$, to the Alfven velocity remains equal to the ratio of the, perpendicular scale, $\lambda_\perp$,to the turbulent velocity This condition is known as critical balance. From this condition one finds

$$\frac{\lambda_\perp}{\lambda_\parallel} \sim \sqrt{\frac{\lambda_\parallel}{R}} \tag{18}$$

The statement that in collisionless plasmas the anisotropic MHD turbulence decays by heating/accelerating particles along the background magnetic field is general. Indeed, the dissipation occurs at the wave-particle resonances

$$\omega - \boldsymbol{k}\cdot\boldsymbol{v} = n\omega_B \tag{19}$$

where ω and $\boldsymbol{k}$ are the frequency and the wavenumber, $\boldsymbol{v}$ is the particle velocity, ω_B is the Larmor frequency, and n is an integer. The cyclotron resonance condition $n \neq 0$ is satisfied when the longitudinal scale of the wave packet, $\lambda_\parallel$ is of the order of the particle Larmor radius, $r_L = c/\omega_B$. Since $r_L \ll R$, due to the strong anisotropy of the MHD turbulence a particle crosses many wave packerts during one Larmor orbit. It was first noticed by Gruzinov (1998), Quataert (1998), and Quataert & Gruzinov (1999) that in this case the dissipation occurs at the Landau resonance $n = 0$. Since the two physical mechanisms of wave-particle interaction at the $n = 0$ resonance are due to i) the longitudinal electric field of the wave, and ii) the interaction between the effective particle's magnetic moment and the longitudinal magnetic perturbation, one is led to the conclusion that the turbulent energy is primarily dissipated on to the longitudinal particle motion.

It has also been found that the turbulent fluctuations tend to align with one another forming small-scale current sheets (e.g. Beresnyak & Lazarian 2006; Boldyrev 2006;Mason, Cattaneo & Boldyrev 2006), which could be disrupted via magnetic reconnection thus providing an additional dissipation mechanism (e.g. Boldyrev & Loureiro 2017; Loureiro & Boldyrev 2017; Mallet, Schekochihin & Chandran 2017a,b). Note that the background magnetic field, which is much larger than the reconnecting field and lies in the same plane of the current sheet, plays the role of a guide field. Since the magnetic energy is transferred to the plasma particles at the Landau resonance between the particles and the tearing mode that disrupts the current sheet, also in this case one would expect the particles to be heated in the longitudinal direction.

Even if the perpendicular heating is negligible, in a weakly magnetized plasma the fire-hose instability quickly erases any momentum anisotropy (e.g. Parker 1958; Lerche 1966). However, since the fire-hose instability develops once $P_\parallel - P_\perp > B^2/4\pi$, one immediately sees that this instability is not effective if the magnetic-to-plasma energy ratio exceeds $\frac{1}{2}$.

3. Literature

Objects themselves:
PWN: Woosley 1993
jets from AGNs: Reynolds 1996
GRBs: Wardle 1998

3.1. Non-thermal particles from turbulence

Melrose 1980 Petrosian 2012 Lazarian 2012

3.2. Turbulence in astrophysics

turbulence in stellar coronae Matthaeus 1999, Cranmer 2007
ISM Armstrong 1995, Lithwick & Goldreich 2001
SNRs Weiler & Sramek 1988, Roy 2009
PWN Porth 2014, Lyutikov 2019
BH disks Balbus & Hawley 1998, Brandenburg & Subramanian 2005
jets from AGNs Marscher 2008, MacDonald & Marscher 2018
radio lobes Vogt & Ensslin 2005, O'Sullivan 2009
GRBs Wardle 1998 Piran 2004, Kumar & Narayan 2009
Galaxy clusters Zweibel & Heiles 1997, Subramanian 2006
Laser laboratory plasma Sarri 2015

3.3. Magnetically dominated turbulence

Sustained relativistic turbulence (force-free) (Thompson & Blaes 1998): extension of Goldreich & Sridhar 1995 to exterme relativistic limit (no plasma inertia; force-free MHD). Anisotropic cascade is formed, dissipation occurs at the scale of current starvation (when not enough charge carriers in plasma to maintain currents from Alfén waves).

(Cho 2005) Inoue 2011 (Cho & Lazarian 2014) (Zrake & East 2016)

Relativistic MHD (Zrake & MacFadyen 2012) (Zrake 2014)

3.4. Bright non-thermal synchrotron and inverse Compton signatures

pulsar magnetospheres and winds Buhler & Blandford 2014

jets from AGNs Begelman 1984

coronae of accretion disks Yuan & Narayan 2014

3.5. Kinetic turbulence

Kinetic turbulence: (Zhdankin et al. 2017b) Letter
(Zhdankin et al. 2017a) Paper
(Zhdankin et al. 2018) System size convergence
(Zhdankin et al. 2019a) electron-proton plasma
(Zhdankin et al. 2019b) radiative turbulence
(Comisso & Sironi 2018) acceleration
(Wong et al. 2019) acceleration
(Nättilä 2019) Runko and turbulence
(Comisso & Sironi 2019) acceleration

3.6. Radiative turbulence

Analytic work on radiative turbulence (Uzdensky 2018) (Zrake et al. 2018); GRBs (Sobacchi & Lyubarsky 2019)

PIC simulations: (Zhdankin et al. 2019b) radiative turbulence

Fokker-Planck equation in momentum space with radiative cooling term (Schlickeiser 1984, 1985); not in original list (Schlickeiser 1989) (Stawarz & Petrosian 2008)

3.7. radiative PIC simulations

Reconnection: (Jaroschek & Hoshino 2009) (Cerutti et al. 2013, 2014b,a) (Kagan et al. 2016b,a) (Hakobyan et al. 2019) (Werner et al. 2019) (Schoeffler et al. 2019)

decay of magnetostatic equilibria (Yuan et al. 2016) (Nalewajko et al. 2018)

pulsar wind (Cerutti & Philippov 2017)

pulsar magnetospheres (Cerutti et al. 2016) (Philippov & Spitkovsky 2018)

Synchrotron and jitter radiative signatures of collisionless shocks (Medvedev & Spitkovsky 2009) (Sironi & Spitkovsky 2009) (Kirk & Reville 2010) (Nishikawa et al. 2011)

Radiative turbulence (Zhdankin et al. 2019c)

References

Cerutti, B. & Philippov, A. A. 2017, A&A, 607, A134
Cerutti, B., Philippov, A. A., & Spitkovsky, A. 2016, MNRAS, 457, 2401
Cerutti, B., Werner, G. R., Uzdensky, D. A., & Begelman, M. C. 2013, ApJ, 770, 147
Cerutti, B., Werner, G. R., Uzdensky, D. A., & Begelman, M. C. 2014a, Physics of Plasmas, 21, 056501
Cerutti, B., Werner, G. R., Uzdensky, D. A., & Begelman, M. C. 2014b, ApJ, 782, 104
Cho, J. 2005, ApJ, 621, 324
Cho, J. & Lazarian, A. 2014, ApJ, 780, 30
Comisso, L. & Sironi, L. 2018, Physical Review Letters, 121
Comisso, L. & Sironi, L. 2019, arXiv e-prints, arXiv:1909.01420
Hakobyan, H., Philippov, A., & Spitkovsky, A. 2019, ApJ, 877, 53
Jaroschek, C. H. & Hoshino, M. 2009, Phys. Rev. Lett., 103, 075002
Kagan, D., Nakar, E., & Piran, T. 2016a, ApJ, 826, 221
Kagan, D., Nakar, E., & Piran, T. 2016b, ApJ, 833, 155
Kirk, J. G. & Reville, B. 2010, ApJ, 710, L16
Medvedev, M. V. & Spitkovsky, A. 2009, ApJ, 700, 956
Nalewajko, K., Yuan, Y., & Chruślińska, M. 2018, Journal of Plasma Physics, 84, 755840301
Nättilä, J. 2019, arXiv e-prints
Nishikawa, K. I., Niemiec, J., Medvedev, M., et al. 2011, Advances in Space Research, 47, 1434
Philippov, A. A. & Spitkovsky, A. 2018, ApJ, 855, 94
Schlickeiser, R. 1984, A&A, 136, 227
Schlickeiser, R. 1985, A&A, 143, 431
Schlickeiser, R. 1989, ApJ, 336, 243
Schoeffler, K. M., Grismayer, T., Uzdensky, D., Fonseca, R. A., & Silva, L. O. 2019, ApJ, 870, 49
Sironi, L. & Spitkovsky, A. 2009, ApJ, 707, L92
Sobacchi, E. & Lyubarsky, Y. E. 2019, MNRAS, 484, 1192
Stawarz, Ł. & Petrosian, V. 2008, ApJ, 681, 1725
Thompson, C. & Blaes, O. 1998, Phys. Rev. D, 57, 3219
Uzdensky, D. A. 2018, Monthly Notices of the Royal Astronomical Society, 477, 2849
Werner, G. R., Philippov, A. A., & Uzdensky, D. A. 2019, MNRAS, 482, L60
Wong, K., Zhdankin, V., Uzdensky, D. A., Werner, G. R., & Begelman, M. C. 2019
Yuan, Y., Nalewajko, K., Zrake, J., East, W. E., & Blandford, R. D. 2016, ApJ, 828, 92
Zhdankin, V., Uzdensky, D. A., Werner, G. R., & Begelman, M. C. 2017a, Monthly Notices of the Royal Astronomical Society, 474, 2514
Zhdankin, V., Uzdensky, D. A., Werner, G. R., & Begelman, M. C. 2018, The Astrophysical Journal, 867, L18
Zhdankin, V., Uzdensky, D. A., Werner, G. R., & Begelman, M. C. 2019a, Phys. Rev. Lett., 122, 055101
Zhdankin, V., Uzdensky, D. A., Werner, G. R., & Begelman, M. C. 2019b, arXiv e-prints, arXiv:1908.08032
Zhdankin, V., Uzdensky, D. A., Werner, G. R., & Begelman, M. C. 2019c, arXiv e-prints, arXiv:1908.08032
Zhdankin, V., Werner, G. R., Uzdensky, D. A., & Begelman, M. C. 2017b, Physical Review Letters, 118
Zrake, J. 2014, ApJ, 794, L26
Zrake, J., Beloborodov, A. M., & Lundman, C. 2018
Zrake, J. & East, W. E. 2016, ApJ, 817, 89
Zrake, J. & MacFadyen, A. I. 2012, ApJ, 744, 32