

Numerical Check Report

Total	Failures	Missing	Status
26	0	0	✓ PASS

Notation

Diagnostics reported in the table.

e_∞ Supremum norm error $\max |LHS - RHS|$ on the evaluation grid. Values near machine precision indicate excellent agreement, whereas larger values usually reflect grid or Monte Carlo noise.

r_∞ Residual $\max |E|$ for an equation $E = 0$. This should be essentially 0 for a successful root/fixed-point verification.

m_{\min}/e_{\max} Extremal values for inequality checks; the sign condition $m_{\min} \geq 0$ (equivalently $e_{\max} \leq 0$) certifies the bound. If the extremum is close to 0, re-running on a finer grid is recommended.

Extrema/ratios Quantities such as $\min(\dots)$, $\max(\dots)$, y_{\max} , Δ , or LHS/RHS summarize bound, monotonicity, or comparison tests. Each reported extremum must have the correct sign/magnitude relative to the claim in `main.tex`, and any Δ should be small relative to the underlying scale.

Point tags Phrases like $(t = \dots)$, $(u = \dots, \kappa = \dots)$, or $(x = \dots, y = \dots)$ identify where the worst-case diagnostic occurred so the same location can be probed analytically if desired.

Symbols imported from `main.tex`. Let $Z, X \sim \mathcal{N}(0, 1)$ and $(x)_+ := \max\{x, 0\}$. Define

$$\begin{aligned}
\phi(u) &:= \frac{1}{\sqrt{2\pi}} e^{-u^2/2}, & \Phi(u) &:= \int_{-\infty}^u \phi(s) ds, & \bar{\Phi}(u) &:= 1 - \Phi(u), \\
E(u) &:= \frac{\phi(u)}{\bar{\Phi}(u)}, & F_q(x) &:= \frac{1}{\sqrt{1-q}} E\left(\frac{\kappa-x}{\sqrt{1-q}}\right), \\
P(r) &:= \mathbb{E}[\tanh^2(\sqrt{r}Z)], \\
R_\kappa(q, \alpha) &:= \alpha \mathbb{E}[F_q(\sqrt{q}Z)^2], \\
B(q) &:= (1-q) \mathbb{E}\left[E\left(\frac{\kappa-\sqrt{q}Z}{\sqrt{1-q}}\right)^2\right], & A(r) &:= r(1-P(r))^2 = r(\mathbb{E}[\operatorname{sech}^2(\sqrt{r}Z)])^2, \\
C_\kappa &:= \mathbb{E}[(\kappa-Z)_+^2] = (\kappa^2 + 1)\Phi(\kappa) + \kappa\phi(\kappa), \\
U_s &:= \frac{\kappa - \sqrt{s}Z}{\sqrt{1-s}} \quad (s \in [0, 1)), & d(u) &:= E(u) - u, \\
\mu_k(u) &:= \mathbb{E}[(X-u)^k \mid X \geq u], & g(u) &:= E'(u)^2 - 2(1-E'(u))E(u)^2, \\
H(u) &:= u^2d(u) + 6ud(u)^2 + 6d(u)^3 - u - 4d(u), \\
x &:= u d(u), & y &:= d(u)^2, & F(x, y) &:= x^2 + 6xy + 6y^2 - x - 4y.
\end{aligned}$$

Simulation Results

Main.pdf	Status	Message
Lemma 4	✓ PASS	$P(r)$ increases on $r \in [0, 200]$ with $P(0) = 0$ and $1 - P(200) = 5.70 \times 10^{-2}$; $\min \Delta P = 3.58 \times 10^{-4}$.
Lemma 5	✓ PASS	$A(r)$ is increasing on $r \in [0, 400]$ with $A(0) = 0$ and $ A(400) - 2/\pi = 1.97 \times 10^{-2}$; $\min \Delta A = 9.63 \times 10^{-5}$.
Lemma 6	✓ PASS	On $u \in [-5, 5]$ the inequalities $E(u) \leq u + 1/u$ and $E(u) \geq u$ hold. $C_{\text{est}} = 0.793$, max upper slack -1.35×10^{-2} , max lower slack -1.87×10^{-1} .
Lemma 7	✓ PASS	$B(q)$ over $q \in [0, 0.99]$ stays in $[1.933, 2.326]$ and decreases. $\max \Delta B = 8.99 \times 10^{-3}$, $\max \Delta B = -3.56 \times 10^{-3}$.
Lemma 8	✓ PASS	Endpoint limits hold: $ B(0) - E(\kappa)^2 = 4.44 \times 10^{-16}$, $\max B(1 - \varepsilon) - C_\kappa = 8.21 \times 10^{-4}$.
Lemma 9	✓ PASS	With $\alpha = 0.231$ ($0.7\alpha_c$) and $\kappa = 1$, $f(r)$ is increasing. $\min \Delta f = 3.39 \times 10^{-4}$ on $r \in [0, 50]$.
Eq. (14)	✓ PASS	$B'(t) = \mathbb{E}[g(U_t)]$, $U_t = \frac{\kappa - \sqrt{t} Z}{\sqrt{1-t}}$ at $\kappa = 1$. $e_\infty = 9.90 \times 10^{-4}$ on $t \in [0.05, 0.90]$.
Eq. (18)	✓ PASS	$\mu_1(u) = d(u)$. $e_\infty = 3.16 \times 10^{-15}$ for $u \in [-2, 3]$.
Lemma 12	✓ PASS	$F(x, y) = x^2 + 6xy + 6y^2 - x - 4y < 0$ on the feasible set. $\max F = -7.86 \times 10^{-5}$ across 1.40×10^4 samples.
Eq. (38)	✓ PASS	$F(x, y) = x^2 + 6xy + 6y^2 - x - 4y < 0$ on the feasible set. $\max F = -7.86 \times 10^{-5}$ across 1.40×10^4 samples.
Eq. (22)	✓ PASS	$\det M_1(u) = u d(u) + 2d(u)^2 - 1 \geq 0$. $e_\infty = 4.30 \times 10^{-14}$, $\min \det M_1 = 4.23 \times 10^{-3}$.
Eq. (23)	✓ PASS	$\det \widetilde{M}_2(u) = u^2 d(u)^2 + u d(u)^3 - 3d(u)^2 - 3u d(u) + 2 \geq 0$. $e_\infty = 4.85 \times 10^{-14}$, $\min \det \widetilde{M}_2 = 3.28 \times 10^{-4}$.
Eq. (24)	✓ PASS	$\text{Var}(Y_u) = 1 - u d(u) - d(u)^2 > 0$. $e_\infty = 3.84 \times 10^{-14}$, $\min \text{Var}(Y_u) = 4.67 \times 10^{-2}$.
Eq. (25)	✓ PASS	Constraints $x + 2y \geq 1$, $x^2 + xy - 3x - 3y + 2 \geq 0$, $x + y < 1$. $\min(x + 2y - 1) = 4.23 \times 10^{-3}$, $\min(1 - x - y) = 4.67 \times 10^{-2}$.
Eq. (26)	✓ PASS	$0 < y = d(u)^2 \leq 2/\pi < 2/3$. $y_{\max} = 2/\pi$, $y_{\min} = 5.09 \times 10^{-2}$.
Eq. (27)	✓ PASS	$d(u)H(u) = x^2 + 6xy + 6y^2 - x - 4y$. $e_\infty = 4.03 \times 10^{-16}$.
Eq. (28)	✓ PASS	$g(0) = \frac{12}{\pi^2} - \frac{4}{\pi}$. $ \Delta = 5.55 \times 10^{-17}$.
Lemma 13	✓ PASS	$\sup_{u \leq 0} g(u) = 3.30 \times 10^{-2} \leq 1/18$.
Eq. (29)	✓ PASS	$g(u_*) = \frac{r_*(4-r_*)(1-r_*)^2}{(r_*^2 - 6r_* + 6)^2}$ for critical points. $e_\infty = 7.52 \times 10^{-15}$.
Eq. (31)	✓ PASS	$-\frac{4(\pi-3)}{\pi^2} < -\frac{1}{18}$. Slack = 1.83×10^{-3} .
Eq. (32)	✓ PASS	$\mathbb{E}[\log \cosh(\sqrt{r} Z)] \leq r \mathbb{E}[\operatorname{sech}^2(\sqrt{r} Z)]$. Margin = 2.33×10^{-2} on $r \in [0.05, 20]$.

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Eq. (33)	✓ PASS	$-\frac{r\varepsilon}{2} + \mathbb{E}[\log \cosh(\sqrt{r} Z)] \leq \frac{\alpha}{2\varepsilon} B(q)$ at $\kappa = 1$. $\max(\text{lhs} - \text{rhs}) = -2.33 \times 10^{-2}$.
Eq. (34)	✓ PASS	$\mathbb{E}[\log \bar{\Phi}(U_n)] \leq -\frac{A_n}{2\varepsilon_n} + \frac{\Phi(\kappa-\delta)}{2} \log \varepsilon_n + C(\delta)$ with $\delta = 0.5$. $\max(\text{lhs} - \text{rhs}) = -2.50$.
Eq. (35)	✓ PASS	$0 \leq B(q_n) - A_n \leq C_0 \varepsilon_n$ with $C_0 = 3$. $\max(B - A - 3\varepsilon) = -1.34 \times 10^{-3}$, $\min(B - A) = 1.66 \times 10^{-3}$.
Lemma 14	✓ PASS	$\frac{r(4-r)(1-r)^2}{(r^2 - 6r + 6)^2} \leq \frac{1}{18}$. Slack = 1.27×10^{-2} .