gurobipy Course

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Latest update of the course, slides and exercises



https://nodet.github.io

Gurobi can solve models with linear and quadratic (incl. nonconvex) constraints and objective.

minimize
$$x^TQx + c^Tx + d$$

subject to $Ax = b$

$$x^TQ_ix + c_i^Tx \le d_i \qquad \forall i \in I$$

$$l \le x \le u$$

$$x_j \in \mathbb{Z} \qquad \forall j \in J$$

And nonlinear constraints as well!

1. Introductory examples

Let us start with a simple example

maximize
$$x + y + 2z$$

subject to $x + 2y + 3z \le 4$
 $x + y \ge 1$
 $x, y, z \in \{0, 1\}$

Here is the Python code to solve this problem:

```
import gurobipy as gp
from gurobipy import GRB
with gp.Env() as env, gp.Model("simple-example", env=env) as model:
    x = model.addVar(vtype=GRB.BINARY, name="x")
    y = model.addVar(vtype=GRB.BINARY, name="y")
    z = model.addVar(vtype=GRB.BINARY, name="z")
    model.addConstr(x + 2 * y + 3 * z \le 4, name="c0")
                                                                        (5)
   model.addConstr(x + y >= 1, name="c1")
   model.setObjective(x + y + 2 * z, sense=GRB.MAXIMIZE)
                                                                        (6)
   model.write("example.lp")
```

- ¹⁰ Import gurobipy package as gp for convenience
- ² GRB is the list of all Gurobi constants
- ³ Create a Gurobi environment and a model object
- ⁴ Define decision variables
- ⁵ Define constraints
- [©] Define objective

- ^⑦ Save the model as an LP file
- [®]Optimize model
- [®]X attribute is the variable's value in the solution

Here is the log of the execution of this program:

```
Gurobi Optimizer version 12.0.0 build v12.0.0rc1 (mac64[arm] - Darwin 24.2.0
24C101)
CPU model: Apple M1 Pro
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 2 rows, 3 columns and 5 nonzeros
Model fingerprint: 0x98886187
Variable types: 0 continuous, 3 integer (3 binary)
Coefficient statistics:
 Matrix range [1e+00, 3e+00]
 Objective range [1e+00, 2e+00]
  Bounds range [1e+00, 1e+00]
  RHS range [1e+00, 4e+00]
```

```
Found heuristic solution: objective 2.0000000
Presolve removed 2 rows and 3 columns
Presolve time: 0.00s
Presolve: All rows and columns removed
Explored 0 nodes (0 simplex iterations) in 0.00 seconds (0.00 work units)
Thread count was 1 (of 8 available processors)
Solution count 2: 3 2
Optimal solution found (tolerance 1.00e-04)
Best objective 3.000000000000e+00, best bound 3.0000000000e+00, gap 0.0000%
***** Solution ******
x: 1.0
y: 0.0
z: 1.0
```

And here's the generated LP file:

```
\ Model simple-example
\ LP format - for model browsing. Use MPS format to capture full model detail.
\ Signature: 0xd6af213f17f735ae
Maximize
 x + y + 2z
Subject To
 c0: x + 2 y + 3 z <= 4
 c1: x + y >= 1
Bounds
Binaries
X Y Z
End
```

The same example, using the matrix API

```
import gurobipy as gp
from gurobipy import GRB
import numpy as np
import scipy.sparse as sp
with gp.Env() as env, gp.Model("matrix1", env=env) as m:
   # Create variables
    x = m.addMVar(shape=3, vtype=GRB.BINARY, name="x")
    # Set objective
    obj = np.array([1.0, 1.0, 2.0])
    m.setObjective(obj @ x, GRB.MAXIMIZE)
```

```
# Build (sparse) constraint matrix
row = np.array([0, 0, 0, 1, 1])
col = np.array([0, 1, 2, 0, 1])
val = np.array([1.0, 2.0, 3.0, -1.0, -1.0])
# A is such that A[row[k], col[k]] = val[k]
A = sp.csr matrix((val, (row, col)), shape=(2, 3))
# Build rhs vector
rhs = np.array([4.0, -1.0])
# Add constraints
m.addConstr(A @ x <= rhs, name="c")</pre>
# Write the model
m.write("matrix1.lp")
# Optimize model
```

```
m.optimize()

print(x.X)

print(f"Obj: {m.ObjVal:g}")
```

Here's the log of the execution:

```
Gurobi Optimizer version 12.0.0 build v12.0.0rc1 (mac64[arm] - Darwin 24.2.0
24C101)
CPU model: Apple M1 Pro
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 2 rows, 3 columns and 5 nonzeros
Model fingerprint: 0x8d4960d3
Variable types: 0 continuous, 3 integer (3 binary)
Coefficient statistics:
 Matrix range [1e+00, 3e+00]
 Objective range [1e+00, 2e+00]
  Bounds range [1e+00, 1e+00]
  RHS range [1e+00, 4e+00]
```

```
Found heuristic solution: objective 2.0000000
Presolve removed 2 rows and 3 columns
Presolve time: 0.00s
Presolve: All rows and columns removed
Explored 0 nodes (0 simplex iterations) in 0.00 seconds (0.00 work units)
Thread count was 1 (of 8 available processors)
Solution count 2: 3 2
Optimal solution found (tolerance 1.00e-04)
Best objective 3.00000000000e+00, best bound 3.0000000000e+00, gap 0.0000%
[1. 0. 1.]
Obj: 3
```

And here's the generated LP file:

```
\ Model matrix1
\ LP format - for model browsing. Use MPS format to capture full model detail.
\ Signature: 0xd6af213f17f735ae
Maximize
  x[0] + x[1] + 2 x[2]
Subject To
 c[0]: x[0] + 2 x[1] + 3 x[2] <= 4
 c[1]: -x[0] - x[1] <= -1
Bounds
Binaries
 x[0] x[1] x[2]
End
```

2. Python Data Structures

• tuple: An ordered, compound grouping that cannot be modified once it is created and is ideal for representing multi dimensional subscripts.

```
("city_0", "city_1")
```

• list: An ordered group, so each item is indexed. Lists can be modified by adding, deleting or sorting elements.

```
["city_0", "city_1", "city_2"]
```

• set: An unordered group of unique elements. Sets can only be modified by adding or deleting.

```
{"city_0", "city_1", "city_2"}
```

• dict: A key-value pair mapping that is ideal for representing indexed data such as cost, demand, capacity.

```
demand = {"city_0": 100, "city_1": 50, "city_2": 40}
```

3. Extended Data Structures in gurobipy

3.1. tuplelist

tuplelist: a sub-class of Python list, used to build sub-lists efficiently. See in particular tuplelist.select(pattern).

```
<gurobi.tuplelist (2 tuples, 3 values each):
  ( A , B , C )</pre>
```

(A , E , C)

3.2. tupledict

tupledict: a sub-class of Python dict, where the values are usually Gurobi variables, to efficiently retrieve those whose key match a specified tuple pattern.

Some important methods to build linear expressions efficiently:

- tupledict.select(pattern) → list
- tupledict.sum(pattern) → gp.LinExpr
- tupledict.prod(coeff, pattern) → gp.LinExpr

```
import gurobipy as gp
m = gp.Model()
```

```
x = m.addVars([(1,2), (1,3), (2,3)], name="x")  # x is a tupledict
m.update()  # Process all model updates
expr = x.sum('*', 3)
print(expr)
```

$$x[1,3] + x[2,3]$$

3.3. multidict()

multidict() is a convenience function to split a dict of lists.

```
import gurobipy as gp
keys, dict1, dict2 = gp.multidict( {
   'key1': [1, 2],
   'key2': [1, 3],
   'key3': [1, 4] } )
print(keys)
print(dict1)
print(dict2)
```

```
['key1', 'key2', 'key3']
{'key1': 1, 'key2': 1, 'key3': 1}
```

{'key1': 2, 'key2': 3, 'key3': 4}

3.4. Example of extended structures

```
import gurobipy as gp
from qurobipy import GRB
data = gp.tupledict([
        (("a", "b", "c"), 3),
        (("a", "c", "b"), 4),
        (("b", "a", "c"), 5),
        (("b", "c", "a"), 6),
        (("c", "a", "b"), 7),
       (("c", "b", "a"), 3)
print(f"data: {data}")
print("\nTuplelist:")
```

```
keys = qp.tuplelist(data.keys())
print(f"\tselect: {keys.select('a', '*', '*')}")
print("\nTupledict:")
print(f"\tselect : {data.select('a', '*', '*')}")
print(f"\tsum : {data.sum('*', '*', '*')}")
coeff = {("a", "c", "b"): 6, ("b", "c", "a"): -4}
print(f"\tprod : {data.prod(coeff, '*', 'c', '*')}")
arcs, capacity, cost = qp.multidict({
        ("Detroit ", "Boston "): [100, 7],
        ("Detroit ", "New York "): [80, 5],
        ("Detroit ", "Seattle "): [120, 4],
        ("Denver ", "Boston "): [120, 8],
        ("Denver ", "New York "): [120, 11],
        ("Denver ", "Seattle "): [120, 4],
    })
```

```
print("\nMultidict:")
print(f"\tcost: {cost}")
print("\n")
print(f"\tcapacity: {capacity}")
```

```
data: {('a', 'b', 'c'): 3, ('a', 'c', 'b'): 4, ('b', 'a', 'c'): 5, ('b', 'c',
'a'): 6, ('c', 'a', 'b'): 7, ('c', 'b', 'a'): 3}
Tuplelist:
    select: <gurobi.tuplelist (2 tuples, 3 values each):</pre>
(a,b,c)
(a,c,b)
>
Tupledict:
    select : [3, 4]
```

```
sum : 28.0
    prod : 0.0
Multidict:
    cost: {('Detroit ', 'Boston '): 7, ('Detroit ', 'New York '): 5, ('Detroit
', 'Seattle '): 4, ('Denver ', 'Boston '): 8, ('Denver ', 'New York '): 11,
('Denver', 'Seattle'): 4}
    capacity: {('Detroit ', 'Boston '): 100, ('Detroit ', 'New York '): 80,
('Detroit ', 'Seattle '): 120, ('Denver ', 'Boston '): 120, ('Denver ', 'New
York '): 120, ('Denver ', 'Seattle '): 120}
```

4. Environments

Environments hold data that is global to one or more models.

- They hold a Gurobi license.
- They capture sets of parameter settings.
- They delineate a (single-threaded) Gurobi session.

The basic usage pattern is the following:

```
import gurobipy as gp
from gurobipy import GRB

with gp.Env() as env, gp.Model("name", env=env) as m:
    # Use the model
...
```

A more advanced usage pattern is:

```
import gurobipy as gp
from gurobipy import GRB
with gp.Env(empty=True) as env:
    # Set licensing parameters
    env.setParam("CloudAccessID", "...")
    env.setParam("CloudSecretKey", "...")
    env.setParam("LicenseID", ...)
    # Start the environment before creating a model
    env.start()
    with gp.Model("name", env=env) as m:
        # Use the model
        . . .
```

5. Models

A model holds:

- variables
- constraints
- parameters, that define the behavior of the solver

```
with gp.Env() as env, gp.Model("simple-example", env=env) as model:
    x = model.addVar(vtype=GRB.BINARY, name="x")
    y = model.addVar(vtype=GRB.BINARY, name="y")
    c1 = model.addConstr(x + y >= 1, name="c1")
   model.setObjective(x + y + 2 * z, sense=GRB.MAXIMIZE)
   model.params.MipFocus=1
    model.params.TimeLimit = 3600
   model.optimize()
```

- [©] Focus on finding the best possible solutions
- ² Stop after one hour

It has methods to create and edit variables and constraints, to set parameters, to solve the model, to retrieve information, and more.

```
# ...
model.write("model.mps")
model.chgCoeff(c1, x, 2)
model.computeIIS()

①
②
```

- ^① Store the model in an MPS file
- ² Change the coefficient of variable x in constraint c1 to 2
- ³ Compute an Irreducible Inconsistent Set

5.1. Decision Variables, Model.addVar()

A decision variable is necessarily associated to exactly one instance of Model, and gets created using methods such as Model.addVar() to create a single variable, Model.addVars() to create multiple variables at once and Model.addMVars() to create a matrix of variables.

The available variable types in Gurobi are:

- Continuous: GRB.CONTINUOUS
- General integer: GRB.INTEGER
- Binary: GRB.BINARY
- Semi-continuous: GRB.SEMICONT
- Semi-integer: GRB.SEMIINT

A semi-continuous variable has the property that it takes a value of 0, or a value between the specified lower and upper bounds. A semi-integer variable adds the additional restriction that the variable should take an integral value.

```
# Define a binary decision variable with (default) lb=0
x = model.addVar(vtype=GRB.BINARY, name="x")
# Define an integer variable with lb=-1, ub=100
y = model.addVar(lb=-1, ub=100, vtype=GRB.INTEGER, name="y")
```

5.2. Model.addVars()

To add multiple decision variables to the model, use the Model.addVars() method which returns a Gurobi tupledict object containing the newly created variables:

The first argument is an iterable giving indices for accessing the variables:

- several integers (specifying the dimensions of the matrix)
- several lists of scalars (each list specifies indices across one dimension of the matrix)
- one list of tuples, or a tuplelist

When the given name is a single string, it is subscripted by the index of the generator expression. The names are stored as ASCII strings, you should not use non-ASCII characters or spaces.

```
import gurobipy as gp
from gurobipy import GRB

with gp.Model(name="model") as model:
    # 3D array of binary variables
    x = model.addVars(2, 3, 4, vtype=GRB.BINARY, name="x")
    model.update()
    print(model.getAttr("VarName", model.getVars()))
```

```
['x[0,0,0]', 'x[0,0,1]', 'x[0,0,2]', 'x[0,0,3]', 'x[0,1,0]', 'x[0,1,1]', 'x[0,1,2]', 'x[0,1,3]', 'x[0,2,0]', 'x[0,2,1]', 'x[0,2,2]', 'x[0,2,3]', 'x[1,0,0]', 'x[1,0,1]', 'x[1,0,2]', 'x[1,0,3]', 'x[1,1,0]', 'x[1,1,1]', 'x[1,1,2]', 'x[1,1,3]', 'x[1,2,0]', 'x[1,2,1]', 'x[1,2,2]', 'x[1,2,3]']
```

```
import gurobipy as gp
from gurobipy import GRB
with qp.Model(name="model") as model:
   # Use arbitrary lists of immutable objects -> tupledict
    y = model.addVars([1, 5], [7, 3, 2], ub=range(6),
                      name=[f"y_{i}" for i in range(6)])
   model.update()
    print("\nVariables names, upper bounds, and indices:")
    for index, var in y.items():
        print(f"name: {var.VarName}, ub: {var.UB}, index: {index}")
```

```
Variables names, upper bounds, and indices:
name: y_0, ub: 0.0, index: (1, 7)
name: y_1, ub: 1.0, index: (1, 3)
name: y_2, ub: 2.0, index: (1, 2)
```

name: y_3, ub: 3.0, index: (5, 7) name: y_4, ub: 4.0, index: (5, 3) name: y_5, ub: 5.0, index: (5, 2)

```
import gurobipy as gp
from gurobipy import GRB
with gp.Model(name="model") as model:
    # Use arbitrary list of tuples as indices
    z = model.addVars(
        [(3, "a"), (3, "b"), (7, "b"), (7, "c")], name="z",
   model.update()
    print("\nVariables names and lower and upper bounds:")
    for index, var in z.items():
        print(f"name: {var.VarName}, lb: {var.LB}, ub: {var.UB}")
```

```
Variables names and lower and upper bounds:
name: z[3,a], lb: 0.0, ub: inf
name: z[3,b], lb: 0.0, ub: inf
```

name: z[7,b], lb: 0.0, ub: inf name: z[7,c], lb: 0.0, ub: inf

5.3. Constraints, Model.addConstr()

Like variables, constraints are also associated with a model. Use the method Model.addConstr() to add a constraint to a model.

```
Model.addConstr(constr, name="")
```

constr is a TempConstr object that can take different types:

- Linear Constraint: x + y <= 1
- Ranged Linear Constraint: x + y == [1, 3]
- Quadratic Constraint: x*x + y*y + x*y <= 1
- Linear Matrix Constraint: A @ x <= 1
- Quadratic Matrix Constraint: x @ Q @ y <= 2
- Absolute Value Constraint: x == abs_(y)
- Logical Constraint: x == and_(y, z)
- Min or Max Constraint: z == max_(x, y, constant=9)
- Indicator Constraint: $(x == 1) \gg (y + z <= 5)$

```
import gurobipy as gp
from gurobipy import GRB
# Add constraint "\sum_{i=0}^{n-1} x_i \le b" for any given n and b.
n, b = 10, 4
with gp.Model("model") as model:
    x = model.addVars(n, vtype=GRB.BINARY, name="x")
    c1 = model.addConstr(x.sum() <= b, name="c1")</pre>
    model.update()
    print(f"RHS, sense = {c1.RHS}, {c1.Sense}")
    print(f"row: {model.getRow(c1)}")
```

```
RHS, sense = 4.0, < row: x[0] + x[1] + x[2] + x[3] + x[4] + x[5] + x[6] + x[7] + x[8] + x[9]
```

```
import gurobipy as gp
from gurobipy import GRB
# Add constraints x_i + y_j - x_i * y_j >= 3.
n, m = 3, 2
with gp.Model("model") as model:
    x = model.addVars(n, name="x")
    y = model.addVars(m, name="y")
    for i in range(n):
        for j in range(m):
            model.addConstr(x[i] + y[j] - x[i] * y[j] >= 3, name=f"c_{i}{j}")
    model.update()
    for c in model.getQConstrs():
        print(f"Name: {c.QCName}, RHS: {c.QCRHS}, sense: {c.QCSense}")
        print(f"\trow: {model.getQCRow(c)}")
```

```
Name: c 00, RHS: 3.0, sense: >
    row: x[0] + y[0] + [-1.0 x[0] * y[0]]
Name: c_01, RHS: 3.0, sense: >
    row: x[0] + y[1] + [-1.0 x[0] * y[1]]
Name: c 10, RHS: 3.0, sense: >
    row: x[1] + y[0] + [-1.0 x[1] * y[0]]
Name: c_11, RHS: 3.0, sense: >
    row: x[1] + y[1] + [-1.0 x[1] * y[1]]
Name: c 20, RHS: 3.0, sense: >
    row: x[2] + y[0] + [-1.0 x[2] * y[0]]
Name: c 21, RHS: 3.0, sense: >
    row: x[2] + y[1] + [-1.0 x[2] * y[1]]
```

5.4. Model.addConstrs()

To add multiple constraints to the model, use the Model.addConstrs() method which returns a Gurobi tupledict that contains the newly created constraints:

```
Model.addConstrs(generator, name="")
```

```
import gurobipy as gp
from gurobipy import GRB
I = range(2)
J = ["a", "b", "c"]
with gp.Model("model") as model:
    x = model.addVars(I, name="x")
    y = model.addVars(J, name="y")
    # Add constraints x_i + y_j <= 1 for all (i, j)
    model.addConstrs((x[i] + y[j] <= 1 for i in I for j in J), name="c")
    model.update()
    print(model.getAttr("ConstrName", model.getConstrs()))
```

```
['c[0,a]', 'c[0,b]', 'c[0,c]', 'c[1,a]', 'c[1,b]', 'c[1,c]']
```

5.5. Objective Function

To set the model objective equal to a linear or a quadratic expression, use the Model.setObjective() method:

```
Model.setObjective(expr, sense=GRB.MINIMIZE)
```

expr can be:

- LinExpr, a linear expression
- QuadExpr, a quadratic expression

sense is either GRB.MINIMIZE (the default) or GRB.MAXIMIZE.

```
import gurobipy as gp
from gurobipy import GRB
import numpy as np
# Add linear objectives c^Tx
n = 5
c = np.random.rand(n)
with gp.Model("model") as model:
    x = model.addVars(n, name="x")
    linexpr = gp.quicksum(c_i * x_i for c_i, x_i in zip(c, x.values()))
    model.setObjective(linexpr)
    model.update()
    print(f"obj: {model.getObjective()}")
```

```
obj: 0.7718485205029192 \times [0] + 0.14662643832254052 \times [1] + 0.22459297798832945 \times [2] + 0.503496718082293 \times [3] + 0.743870372088844 \times [4]
```

```
import gurobipy as gp
from gurobipy import GRB
import numpy as np
n = 5
Q = np.random.rand(n, n)
with gp.Model("model") as model:
    x = model.addVars(n, name="x")
    quadexpr = 0
    # Add quadratic objective in the form x^T Q x
    for i in range(n):
        for j in range(n):
            quadexpr += x[i] * Q[i, j] * x[j]
    model.setObjective(quadexpr)
```

```
model.update()

# Print objective expression
obj = model.getObjective()
print(f"\nobj: {obj}")
```

```
obj: 0.0 + [ 0.577773246188264 x[0] ^ 2 + 0.22815028400798743 x[0] * x[1] + 0.7057926009856951 x[0] * x[2] + 0.5668766160796567 x[0] * x[3] + 1.4647081669806783 x[0] * x[4] + 0.8558057179316467 x[1] ^ 2 + 0.8517145625303452 x[1] * x[2] + 1.0168639584110595 x[1] * x[3] + 0.7445533188159639 x[1] * x[4] + 0.8917813894437818 x[2] ^ 2 + 0.9074512745571801 x[2] * x[3] + 0.37284204762772444 x[2] * x[4] + 0.5109223586747896 x[3] ^ 2 + 1.1242190867842021 x[3] * x[4] + 0.47962043877973015 x[4] ^ 2 ]
```

5.6. Optimizing for Multiple Objectives

Gurobi supports two ways to combine multiple linear objectives:

- Blended objectives.
- Hierarchical objectives.

Objectives have:

- priorities
- weights
- absolute and relative tolerances

```
setObjectiveN(expr, index, priority=0, weight=1, abstol=1e-6, reltol=0, name='')
```

```
# Primary objective: x + 2 y
model.setObjectiveN(x + 2*y, 0, priority=0)

# Alternative, lower priority objectives: 3 y + z and x + z
model.setObjectiveN(3*y + z, 1, priority=-1)
model.setObjectiveN(x + z, 2, priority=-2)
```

5.7. SOS Constraints

A Special-Ordered Set, or SOS constraint, is a highly specialized constraint that places restrictions on the values that variables in a given list can take.

- SOS constraint of type 1 (SOS1): at most one variable is allowed to take a non-zero value.
- SOS constraint of type 2 (SOS2): at most two variables are allowed to take non-zero values, and those non-zero variables must be contiguous.

Use Model.addSOS() to add such constraints:

```
Model.addSOS(type, vars)
```

With:

- type: the type of SOS constraint. Can be either GRB.SOS_TYPE1 or GRB.SOS_TYPE2.
- vars: list of variables that participate in the consstraint.

For example, the MIP formulation of

$$z = max(x, y, 3)$$

using SOS1 constraints, is:

$$z = x + s_{1}$$
 (1)

$$z = y + s_{2}$$
 (2)

$$z = 3 + s_{3}$$
 (3)

$$v_{1} + v_{2} + v_{3} = 1$$
 (4)

$$SOS1(s_{1}, v_{1})$$
 (5)

$$SOS1(s_{2}, v_{2})$$
 (6)

$$SOS1(s_{3}, v_{3})$$
 (7)

$$s_{1}, s_{2}, s_{3} \in \mathbb{R}^{+}$$
 (8)

$$v_{1}, v_{2}, v_{3} \in \{0, 1\}$$
 (9)

5.8. General Constraints

General constraints allow you to directly model complex relationships between variables.

• Simple constraints: min, max, abs, OR, etc.

```
m.addConstr(z == gp.and_(x, y))
m.addConstr(z == gp.max_(x, y, 3))
```

• Nonlinear constraints: polynomial, exponential, logistic, trigonometric, etc

```
model.addGenConstrNL(y, nlfunc.sin(2.5 * x1) + x2)
```

```
import gurobipy as gp
from gurobipy import nlfunc
# Minimize sin(2.5 x1) + x2
# s.t. -1 <= x1, x2 <= 1
with gp.Env() as env, gp.Model(env=env) as model:
    x1 = model.addVar(lb=-1, ub=1, name="x1")
    x2 = model.addVar(lb=-1, ub=1, name="x2")
    y = model.addVar(lb=-float("inf"), name="y")
    model.addGenConstrNL(y, nlfunc.sin(2.5 * x1) + x2)
    model.setObjective(y)
    model.optimize()
    print(f"x1={x1.X} x2={x2.X} obj={y.X}")
```

```
Gurobi Optimizer version 12.0.0 build v12.0.0rc1 (mac64[arm] - Darwin 24.2.0
24C101)
CPU model: Apple M1 Pro
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 0 rows, 3 columns and 0 nonzeros
Model fingerprint: 0x00339c10
Model has 1 general nonlinear constraint (1 nonlinear terms)
Variable types: 3 continuous, 0 integer (0 binary)
Coefficient statistics:
 Matrix range [0e+00, 0e+00]
 Objective range [1e+00, 1e+00]
  Bounds range [1e+00, 1e+00]
  RHS range [0e+00, 0e+00]
Presolve model has 1 nlconstr
```

Added 2 variables to disaggregate expressions. Presolve time: 0.00s Presolved: 10 rows, 6 columns, 21 nonzeros Presolved model has 1 nonlinear constraint(s) Solving non-convex MINLP Variable types: 6 continuous, 0 integer (0 binary) Found heuristic solution: objective -2.0000000 Explored 1 nodes (0 simplex iterations) in 0.00 seconds (0.00 work units) Thread count was 8 (of 8 available processors) Solution count 1: -2 Optimal solution found (tolerance 1.00e-04) Best objective -1.999999998349e+00, best bound -2.00000000000e+00, gap 0.0000% x1=-0.6282022274684884 x2=-1.0 obj=-1.9999999983490295

6. Matrix-based API

- Term-based modeling can be slow
- Matrix-friendly API leans on Numpy concepts (vectorization, broadcasting)
- Model.addMVar(): Add an MVar object to a model. An MVar acts like a NumPy ndarray of Gurobi decision variables. An MVar can have an arbitrary number of dimensions, defined by the shape argument.

```
addMVar(shape, lb=0.0, ub=float('inf'), obj=0.0, vtype=GRB.CONTINUOUS, name='')

# A vector of 3 continuous variables
v = model.addMVar(3)

# A 5x10 matrix of binary variables
x = model.addMVar((5,10), vtype=GRB.BINARY)
```

Here's the example from the beginning of the course:

```
x = m.addMVar(shape=3, vtype=GRB.BINARY, name="x")
val = np.array([1.0, 2.0, 3.0, -1.0, -1.0])
row = np.array([0, 0, 0, 1, 1])
col = np.array([0, 1, 2, 0, 1])
A = sp.csr_matrix((val, (row, col)), shape=(2, 3))
                                                      (2)
rhs = np.array([4.0, -1.0])
m.addConstr(A @ x <= rhs, name="c")</pre>
obj = np.array([1.0, 1.0, 2.0])
m.setObjective(obj @ x, GRB.MAXIMIZE)
                                                      (5)
```

¹ Create variables

- ² Build a (sparse) matrix
- ³ Build an rhs vector
- ⁴ Add constraints
- ^⑤ Set objective

Here is another with timing of the difference.

```
import gurobipy as gp
import numpy as np
from timeit import default_timer
n = 1000
Q = np.random.rand(n, n)
def term_based():
    with gp.Model("term-based") as model:
        x = model.addVars(n, name="x")
        model.addConstr(
            gp.quicksum(x[i] * Q[i, j] * x[j] for j in range(n)
                                                for i in range(n)) <= 10</pre>
```

```
def matrix api():
    with gp.Model("matrix-based") as model:
        x = model.addMVar(n, name="x")
        model.addConstr(x.T @ Q @ x <= 10)
matrix api() # To create the default env
for f in [term_based, matrix_api]:
    start = default timer()
    f()
    end = default timer()
    print(f"Running {f. name } took {end - start} seconds")
```

Running term_based took 4.9921418750309385 seconds Running matrix_api took 0.1097855000407435 seconds

7. Interacting with the Model

7.1. Attributes

The primary mechanism for querying and modifying properties of a Gurobi object is through the attribute interface. Attributes exist on instances of Model, Variable, all types of constraints, and more.

Model:

- number of modeling elements of each type (NumConstrs, NumVars, etc.)
- information about the type of model (IsMip, IsMultiObj, etc.), its statistics (SolCount, NodeCount, etc.)
- information about the solutions found (SolCount), the best known bound (ObjBound), the gap (MipGap), etc.

• ...

Variables:

- lower (LB) and upper (UB) bounds
- value in a MIP start vector (Start)
- value in the best solution (X)

•

Constraints:

- right-hand side value (RHS)
- dual value in the best solution (Pi)

• ..

```
import gurobipy as gp
from gurobipy import GRB
with gp.read("data/glass4.mps.bz2") as model:
   model.optimize()
   print("*********** SOLUTION ************")
   print(f"\tStatus : {model.Status}")
   print(f"\t0bj : {model.0bjVal}")
   print(f"\tSolutionCount: {model.SolCount}")
   print(f"\tRuntime : {model.Runtime}")
   print(f"\tMIPGap : {model.MIPGap}")
   print("\n")
   for var in model.getVars()[:20]:
       print(f"\t{var.VarName} = {var.X}")
```

```
Read MPS format model from file data/glass4.mps.bz2
Reading time = 0.01 seconds
glass4: 396 rows, 322 columns, 1815 nonzeros
Gurobi Optimizer version 12.0.0 build v12.0.0rc1 (mac64[arm] - Darwin 24.2.0
24C101)
CPU model: Apple M1 Pro
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 396 rows, 322 columns and 1815 nonzeros
Model fingerprint: 0x18b19fdf
Variable types: 20 continuous, 302 integer (0 binary)
Coefficient statistics:
 Matrix range [1e+00, 8e+06]
 Objective range [1e+00, 1e+06]
  Bounds range [1e+00, 8e+02]
```

```
RHS range [1e+00, 8e+06]
Presolve removed 6 rows and 6 columns
Presolve time: 0.00s
Presolved: 390 rows, 316 columns, 1803 nonzeros
Variable types: 19 continuous, 297 integer (297 binary)
Found heuristic solution: objective 3.133356e+09
Root relaxation: objective 8.000024e+08, 72 iterations, 0.00 seconds (0.00 work
units)
   Nodes | Current Node | Objective Bounds
                                                              Work
Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time
                        0 72 3.1334e+09 8.0000e+08 74.5%
    0
          0 8.0000e+08
                                                                  05
                             2.600019e+09 8.0000e+08 69.2% -
Н
                                                                  0s
Н
                             2.366684e+09 8.0000e+08 66.2% -
                                                                  0s
          0 8.0000e+08 0 72 2.3667e+09 8.0000e+08 66.2%
                                                                  0s
```

		0	0	8.0000e+08	0	72 2.3667e+09	8.0000e+08	66.2%	-	0s	
		0	0	8.0000e+08	0	77 2.3667e+09	8.0000e+08	66.2%	-	0s	
		0	0	8.0000e+08	0	76 2.3667e+09	8.0000e+08	66.2%	-	0s	
		0	2	8.0000e+08	0	75 2.3667e+09	8.0000e+08	66.2%	-	0s	
	Н	26	80			2.116683e+09	8.0000e+08	62.2%	38.4	0 s	
	Н	36	80			2.016683e+09	8.0000e+08	60.3%	30.6	0 s	
	Н	65	80			2.000015e+09	8.0000e+08	60.0%	20.1	0s	
	Н	251	274			1.991127e+09	8.0000e+08	59.8%	10.8	0s	
	*	1128	1027		112	1.920014e+09	8.0000e+08	58.3%	6.3	0s	
	Н	1695	1546			1.812517e+09	8.0000e+08	55.9%	6.2	0s	
	Н	1788	1729			1.800017e+09	8.0000e+08	55.6%	6.1	0s	
	Н	2010	1645			1.766684e+09	8.0000e+08	54.7%	6.0	0s	
	Н	2353	1796			1.700017e+09	8.0000e+08	52.9%	5.6	0s	
	Н	2399	1782			1.700017e+09	8.0000e+08	52.9%	5.6	0s	
	Н	3109	1938			1.700016e+09	8.0000e+08	52.9%	5.4	0s	
	*	9626	5603		77	1.700016e+09	8.0000e+08	52.9%	3.9	0s	
	*	9627	5603		77	1.700016e+09	8.0000e+08	52.9%	3.9	0s	
1											

H10514 6175		1.700016e+09 8.0000e+08	52.9%	3.9	0s
H11625 7019		1.666683e+09 8.0000e+08	52.0%	3.8	0s
H11975 6974		1.650016e+09 8.0000e+08	51.5%	3.7	0s
H13781 7848		1.650016e+09 8.0000e+08	51.5%	3.6	0s
H13830 7848		1.650016e+09 8.0000e+08	51.5%	3.6	0s
H14108 8324		1.600015e+09 8.0000e+08	50.0%	3.6	0s
H16117 9426		1.600015e+09 8.0000e+08	50.0%	3.5	0s
H18938 10840		1.600014e+09 8.0000e+08	50.0%	3.5	1s
H19146 9570		1.500013e+09 8.0000e+08	46.7%	3.5	1s
H20592 10593		1.500013e+09 8.0000e+08	46.7%	3.5	1s
H30377 13890		1.500012e+09 8.3576e+08	44.3%	3.7	2s
30568 14024 1.1000e+09	66	89 1.5000e+09 8.8699e+08	40.9%	3.9	5s
30880 14242 1.0000e+09	27	114 1.5000e+09 8.9978e+08	40.0%	4.3	10s
39957 16368 1.0000e+09	177	54 1.5000e+09 9.2005e+08	38.7%	6.4	15s
*135241 22495	219	1.400016e+09 1.1000e+09	21.4%	7.0	19s
*136789 22885	210	1.400014e+09 1.1000e+09	21.4%	7.0	19s
*142250 22031	209	1.400013e+09 1.1098e+09	20.7%	7.0	19s
C					

```
148470 22002 1.4000e+09
                        186
                             35 1.4000e+09 1.1750e+09 16.1%
                                                                     20s
                                                               7.0
*186816 33080
                         211
                                                               7.1
                                                                     21s
                             1.400013e+09 1.2000e+09 14.3%
*228748 44827
                        206
                               1.400013e+09 1.2000e+09 14.3%
                                                               6.9
                                                                     23s
266386 52292 infeasible
                         203
                                                               6.7
                                                                     25s
                                 1.4000e+09 1.2000e+09 14.3%
*284522 5732
                         204
                               1.200013e+09 1.2000e+09 0.00%
                                                               6.6
                                                                     25s
Cutting planes:
 Learned: 1
 Gomory: 7
 Implied bound: 12
 Projected implied bound: 1
 MIR: 42
 Flow cover: 17
 RLT: 5
 Relax-and-lift: 16
Explored 284537 nodes (1881113 simplex iterations) in 25.78 seconds (29.59 work
```

```
units)
Thread count was 8 (of 8 available processors)
Solution count 10: 1.20001e+09 1.40001e+09 1.40001e+09 ... 1.60001e+09
Optimal solution found (tolerance 1.00e-04)
Best objective 1.200012600000e+09, best bound 1.200009038285e+09, gap 0.0003%
Status : 2
   Obj : 1200012600.0
   SolutionCount: 10
   Runtime : 25.77737784385681
   MIPGap : 2.968064893289939e-06
   x1 = 0.0
   x2 = 700.0000000000001
```

```
x3 = 1000.0
x4 = 1000.0
x5 = 400.0
x6 = 200.0
x7 = 200.0
x8 = 500.0
x9 = 700.0
x10 = 1200.0
x11 = 0.0
x12 = 0.0
x13 = 200.0
x14 = 800.0
x15 = 800.0
x16 = 600.0
x17 = 300.0
x18 = 300.0
x19 = 199.9999999999991
```

x20 = 1.0

7.2. Parameters

Parameters control the mechanics of the Gurobi Optimizer.

```
import gurobipy as gp
from gurobipy import GRB

with gp.read("data/glass4.mps.bz2") as model:
    model.params.Threads = 1
    model.params.TimeLimit = 10
    model.optimize()
```

```
Read MPS format model from file data/glass4.mps.bz2
Reading time = 0.02 seconds
glass4: 396 rows, 322 columns, 1815 nonzeros
Set parameter Threads to value 1
Set parameter TimeLimit to value 10
Gurobi Optimizer version 12.0.0 build v12.0.0rc1 (mac64[arm] - Darwin 24.2.0 24C101)
```

CPU model: Apple M1 Pro
Thread count: 8 physical cores, 8 logical processors, using up to 1 threads

Non-default parameters:

TimeLimit 10

Threads 1

Optimize a model with 396 rows, 322 columns and 1815 nonzeros

```
Model fingerprint: 0x18b19fdf
Variable types: 20 continuous, 302 integer (0 binary)
Coefficient statistics:
 Matrix range [1e+00, 8e+06]
 Objective range [1e+00, 1e+06]
 Bounds range [1e+00, 8e+02]
 RHS range [1e+00, 8e+06]
Presolve removed 6 rows and 6 columns
Presolve time: 0.00s
Presolved: 390 rows, 316 columns, 1803 nonzeros
Variable types: 19 continuous, 297 integer (297 binary)
Found heuristic solution: objective 3.133356e+09
Root relaxation: objective 8.000024e+08, 72 iterations, 0.00 seconds (0.00 work
units)
   Nodes
           Current Node | Objective Bounds |
                                                                Work
```

	Expl I	Jnexpl	Obj	Depth	IntI	nf	Incumbent	BestBd	Gap	It/Node	Time
	0	0	8.0000e	+08	0	72	3.1334e+09	8.0000e+08	74.5%	-	0s
Н	0	0				2.	600019e+09	8.0000e+08	69.2%	-	0s
	0	0	8.0000e	+08	0	72	2.6000e+09	8.0000e+08	69.2%	-	0s
	0	0	8.0000e	+08	0	72	2.6000e+09	8.0000e+08	69.2%	-	0s
	0	0	8.0000e	+08	0	76	2.6000e+09	8.0000e+08	69.2%	-	0s
	0	0	8.0000e	+08	0	76	2.6000e+09	8.0000e+08	69.2%	-	0s
Н	0	0				2.	500018e+09	8.0000e+08	68.0%	-	0s
Н	0	0				2.	400019e+09	8.0000e+08	66.7%	-	0s
	0	2	8.0000e	+08	0	76	2.4000e+09	8.0000e+08	66.7%	-	0s
Н	52	52				2.	288909e+09	8.0000e+08	65.0%	4.1	0s
Н	52	52				2.	200018e+09	8.0000e+08	63.6%	4.1	0s
Н	52	52				2.	150019e+09	8.0000e+08	62.8%	4.1	0s
Н	52	52				2.	133352e+09	8.0000e+08	62.5%	4.1	0s
Н	54	54				2.	000018e+09	8.0000e+08	60.0%	4.1	0s
Н	461	419				2.	000018e+09	8.0000e+08	60.0%	4.1	0 s

	164	440		2 000047 00	0.0000	60.00	4.4		
Н	461	419		2.000017e+09	8.0000e+08	60.0%	4.1	0s	
Н	461	411		1.950017e+09	8.0000e+08	59.0%	4.1	0s	
Н	461	411		1.933350e+09	8.0000e+08	58.6%	4.1	0s	
Н	461	411		1.900017e+09	8.0000e+08	57.9%	4.1	0s	
Н	547	479		1.900017e+09	8.0000e+08	57.9%	4.3	0 s	
Н	688	536		1.900017e+09	8.0000e+08	57.9%	4.4	0 s	
Н	708	526		1.900016e+09	8.0000e+08	57.9%	4.4	0s	
Н	708	501		1.866683e+09	8.0000e+08	57.1%	4.4	0s	
*	735	483	126	1.833351e+09	8.0000e+08	56.4%	4.5	0s	
Н	931	559		1.833351e+09	8.0000e+08	56.4%	4.4	0s	
Н	931	538		1.800017e+09	8.0000e+08	55.6%	4.4	0s	
Н	931	520		1.800017e+09	8.0000e+08	55.6%	4.4	0 s	
Н	958	518		1.800017e+09	8.0000e+08	55.6%	4.4	0s	
Н	958	502		1.800017e+09	8.0000e+08	55.6%	4.4	0 s	
*	1108	537	83	1.800015e+09	8.0000e+08	55.6%	4.6	0 s	
Н	2106	1138		1.775015e+09	8.0000e+08	54.9%	4.1	0 s	
Н	2555	1485		1.775015e+09	8.0000e+08	54.9%	4.0	0s	

* 4189	2637	57	1.758351e+09	8.0000e+08	54.5%	3.9	1s
H 4665	2932		1.725017e+09	8.0000e+08	53.6%	3.9	1s
H 5665	3637		1.725017e+09	8.0000e+08	53.6%	3.9	1s
H 5665	3621		1.700016e+09	8.0000e+08	52.9%	3.9	1s
H 6130	3954		1.700016e+09	8.0000e+08	52.9%	3.9	1s
10353	6940 1.2200e+09	56	123 1.7000e+09	8.4590e+08	50.2%	4.3	5s
H10418	6636		1.675016e+09	8.4985e+08	49.3%	4.4	5s
H10435	6314		1.650016e+09	8.5187e+08	48.4%	4.4	6s
H10435	5998		1.650016e+09	8.5187e+08	48.4%	4.4	6s
H10760	5883		1.600016e+09	8.8011e+08	45.0%	5.1	8s
H10801	5616		1.600016e+09	8.8011e+08	45.0%	5.2	8s
H10801	5339		1.600016e+09	8.8011e+08	45.0%	5.2	8s
H11173	5253		1.600016e+09	9.0000e+08	43.8%	5.7	9s
Cutting	planes:						
Learn	ed: 1						
Gomor	y: 11						

```
Cover: 1
  Implied bound: 5
  Projected implied bound: 2
  Clique: 1
 MIR: 34
  Flow cover: 20
 RIT: 5
  Relax-and-lift: 13
Explored 12753 nodes (79852 simplex iterations) in 10.00 seconds (11.07 work
units)
Thread count was 1 (of 8 available processors)
Solution count 10: 1.60002e+09 1.60002e+09 1.60002e+09 ... 1.72502e+09
Time limit reached
```

Best objective 1.600015500000e+09, best bound 9.000054810120e+08, gap 43.7502%

7.3. Callbacks

A callback is a user-defined function invoked by Gurobi while the optimization process is going on. They enable more control over the optimization. They can be used to:

- customize the termination of the solve
- add user cuts and lazy constraints
- add custom feasible solutions
- monitor the progress of optimization
- customize the optimization progress display

A callback must be a function (actually, any callable object) that accepts two arguments:

- model: the model that is being solved
- where: from where is the Gurobi Optimizer is the callback invoked (presolve, simplex, barrier, MIP, at a node, etc.)

A callback can query information from the solver using Model.cbGet(). The type of information that can be queried depends on from where the callback is invoked. For example, when where == PRESOLVE, you can query the number of rows removed using what == PRE_ROWDEL. All the callback code values

that you can query are listed here.

```
import gurobipy as gp
from gurobipy import GRB
from functools import partial
# Used by the callback to store or retrieve information
class CallbackData:
    def __init__(self):
        self.invocations = 0
# The callback, the function that will be invoked
def mycallback(model, where, *, cbdata):
    # We're only interested in this case
    if where == GRB.Callback.MIP:
        chdata.invocations += 1
```

```
# Get information from the model
        nodecnt = model.cbGet(GRB.Callback.MIP NODCNT)
        if nodecnt > 100:
            # Terminate the search
            model.terminate()
# Create a model from an instance file
with gp.read("data/glass4.mps.bz2") as model:
    # Create the object to store/retrieve information
    cbdata = CallbackData()
    # Create a callable with two args: model and where
    callback_func = partial(mycallback, cbdata=cbdata)
    # Optimize, with the callback
    model.optimize(callback func)
```

```
# Confirm the callback was invoked
print(f"MIP Callback was invoked {cbdata.invocations} times.")
```

```
Read MPS format model from file data/glass4.mps.bz2
Reading time = 0.03 seconds
glass4: 396 rows, 322 columns, 1815 nonzeros
Gurobi Optimizer version 12.0.0 build v12.0.0rc1 (mac64[arm] - Darwin 24.2.0
24C101)
CPU model: Apple M1 Pro
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 396 rows, 322 columns and 1815 nonzeros
Model fingerprint: 0x18b19fdf
Variable types: 20 continuous, 302 integer (0 binary)
Coefficient statistics:
 Matrix range [1e+00, 8e+06]
 Objective range [1e+00, 1e+06]
  Bounds range [1e+00, 8e+02]
```

```
RHS range [1e+00, 8e+06]
Presolve removed 6 rows and 6 columns
Presolve time: 0.00s
Presolved: 390 rows, 316 columns, 1803 nonzeros
Variable types: 19 continuous, 297 integer (297 binary)
Found heuristic solution: objective 3.133356e+09
Root relaxation: objective 8.000024e+08, 72 iterations, 0.00 seconds (0.00 work
units)
   Nodes | Current Node | Objective Bounds
                                                              Work
Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time
                        0 72 3.1334e+09 8.0000e+08 74.5%
    0
          0 8.0000e+08
                                                                  05
                             2.600019e+09 8.0000e+08 69.2% -
Н
                                                                  0s
Н
                             2.366684e+09 8.0000e+08 66.2% -
                                                                  0s
          0 8.0000e+08 0 72 2.3667e+09 8.0000e+08 66.2%
                                                                  0s
```

```
0
          0 8.0000e+08
                              72 2.3667e+09 8.0000e+08 66.2%
                                                                       0s
          0 8.0000e+08
                              77 2.3667e+09 8.0000e+08 66.2%
                                                                       0s
          0 8.0000e+08
                              76 2.3667e+09 8.0000e+08 66.2%
                                                                       05
    0
          2 8.0000e+08
                              75 2.3667e+09 8.0000e+08 66.2%
                                                                       0s
   26
                               2.116683e+09 8.0000e+08 62.2% 38.4
Н
         80
                                                                       0s
Н
   36
         80
                               2.016683e+09 8.0000e+08 60.3%
                                                               30.6
                                                                       0s
Н
                               2.000015e+09 8.0000e+08 60.0% 20.1
   65
         80
                                                                       0s
Cutting planes:
 Gomory: 86
 Cover: 2
 Implied bound: 31
 MIR: 2
 RLT: 70
 Relax-and-lift: 40
 PSD: 7
```

Explored 161 nodes (2760 simplex iterations) in 0.09 seconds (0.08 work units) Thread count was 8 (of 8 available processors)

Solution count 6: 2.00002e+09 2.01668e+09 2.11668e+09 ... 3.13336e+09

Solve interrupted
Best objective 2.000015200000e+09, best bound 8.000033092112e+08, gap 60.0001%

User-callback calls 606, time in user-callback 0.00 sec MIP Callback was invoked 117 times.

Other methods of interest are:

- Model.cbGetNodeRel(): retrieve the values of the variables in the node relaxation solution at the current node.
- Model.cbGetSolution(): retrieve the values of the variables in the new MIP solution.
- Model.cbCut(): add a new cutting plane to the model.
- Model.cbLazy(): add a new lazy constraint to the model.
- Model.cbSetSolution(): import a user-constructed solution into Gurobi.

8. Requirements and installation

To work with Python:

- pyenv or an equivalent
- PIP_REQUIRE_VIRTUALENV=true

To work with gurobipy:

- pip install gurobipy
- It comes with a (limited) license
- As students, get your free Academic Web License Service (WLS)
 License

Here's a suggested setup. The instructions below were tested on macOS.

Create a local directory for the course.

```
$ mkdir gurobipy-course
$ cd gurobipy-course
$ git init
$ git commit --allow-empty -m "Initial commit"
```

• Create a repository on github.com to hold your content, and add that as a remote in your working copy:

```
$ git remote add origin git@github.com:nodet/test-course.git
$ git push -u origin main
```

• Get the latest Python version supported for gurobipy and set it as the default for this project.

```
$ pyenv install 3.12
$ pyenv local 3.12.7
$ git add .python-version
$ git commit -a -m "Set Python version"
```

• Setup a virtual environment using that python version

```
$ python -m venv venv --prompt "gurobipy-course"
$ source venv/bin/activate
$ pip install --upgrade pip
```

Install the latest version of gurobipy

```
$ pip install gurobipy
$ python -c "import gurobipy as gp; gp.Model()"
Restricted license - for non-production use only - expires 2026-11-23
```