

Homework for Math 351-003

Individual Homework: Due Wednesday, March 6

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \sin\left(\frac{1}{x}\right)$ if $x \neq 0$ and $f(0) = 0$. Prove that f is not continuous at 0.
2. Suppose that $f : D \rightarrow \mathbb{R}$, $g : D \rightarrow \mathbb{R}$ and $h : D \rightarrow \mathbb{R}$ are 3 functions and that

$$f(z) \leq g(z) \leq h(z)$$

for all $z \in D$. Show that if f and h are both continuous at $x \in D$ and $f(x) = h(x)$, then g is also continuous at x .

3. Exhibit a continuous function $f : (0, 1] \rightarrow \mathbf{R}$ so that $f((0, 1])$ is not bounded. Prove all your claims.
4. Prove that if $f : D \rightarrow \mathbf{R}$ is continuous and $C \subset D$ is compact, then $f(C)$ is bounded (i.e. there exists R so that $|f(x)| \leq R$ for all $x \in C$).