Homework for Math 351-003

Team Homework: Due 2/5. Be sure to assign the role of scribe to a new person this week.

1. Prove that if (x_n) converges to $x \neq 0$ and x_n is non-zero for all n, then $(\frac{1}{x_n})$ converges to $\frac{1}{x}$. (Hint: $\left|\frac{1}{x_n} - \frac{1}{x}\right| = \frac{|x_n - x|}{|x_n \cdot x|}$.)

2. (a) Prove that if (s_n) is monotone increasing and unbounded, then $\lim s_n = +\infty$.
   (b) Give an example of a sequence that is unbounded but does not diverge to infinity. (Remember that sequences can be "piecewise defined"!)

3. Note: you might find it useful to look over all the things we have proved so far about sequences in order to help with this one (especially part (a)).
   (a) Prove that $[a, b]$ is a closed subset of $\mathbb{R}$.
   (b) Prove that $[a, b)$ is not a closed subset of $\mathbb{R}$.

4. Find the supremum and infimum of $\{1 - \frac{1}{n} \mid n \in \mathbb{N}\}$, and prove that your answer is correct.

5. Suppose that S and T are non-empty subsets of $\mathbb{R}$ and that for any $s \in S$ and $t \in T$, $s \leq t$. Prove that $\sup S \leq \inf T$.