Homework for Math 351-003

Individual Homework: Due Wednesday, January 24.

1. Prove that the sequence $(\sin(\frac{n\pi}{3}))$ does not converge.

2. Prove that if (a_n) and (b_n) are two convergent sequences such that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then $(a_n b_n)$ converges to ab. (This is the Multiplication Limit Law for sequences.)

   (Hint: $a_n b_n - ab = a_n b_n - a_n b + a_n b - ab$)

> **Definition.** A sequence (x_n) of real numbers is called **Cauchy** if for all $\epsilon > 0$, there exists a real number N such that for all $m, n > N$, $|x_n - x_m| < \epsilon$.

3. Prove that every convergent sequence is Cauchy.