

## Homework for Math 351-003

Individual Homework: Due 2/14

1. Write up a careful proof of this problem from clas: prove that every sequence has a monotone subsequence. (Hint: We say that a term  $s_N$  in a sequence  $(s_n)$  is *dominant* if  $s_n < s_N$  for all  $n > N$ . Divide your proof into two cases depending on whether or not there are infinitely many dominant terms in your sequence.)
2. Let  $k \in \mathbb{R}$ .
  - (a) Prove that the function  $f(x) = kx$  is continuous at  $x = 5$ .
  - (b) Prove that the function  $f(x) = kx$  is continuous.
3. A problem to think about, but you do not need to submit a solution: Prove that if a Cauchy sequence  $(x_n)$  has a subsequence  $(x_{n_k})$  that converges to a real number  $x$ , then  $(x_n)$  converges to  $x$  as well.