Homework for Math 351-003

Team Homework: Due Monday, April 1

1. Prove that if $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are differentiable at $x_0 \in \mathbb{R}$, then fg is differentiable at x_0 and $(fg)'(x_0) = f'(x_0)g(x_0) + g'(x_o)f(x_0)$.

2. a) Prove that the function $f : \mathbb{R} \setminus \{\frac{1}{2}\} \to \mathbb{R}$ given by $f(x) = \frac{3x+4}{2x-1}$ is differentiable at $x_0 = 1$ and evaluate $f'(1)$.
   b) Prove that the function $g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = x^{\frac{1}{3}}$ is not differentiable at $x_0 = 0$.

3. Prove that if $f : D \to \mathbb{R}$ is differentiable at a point $a \in D$, then f is continuous at a. (Hint: Problem 4 from last week's team homework is helpful here.)

4. a) Suppose $f : D \to \mathbb{R}$ is a differentiable function, that D contains an open interval (a, b) for some $a < b$, and that $f'(x) > 0$ for all $x \in (a, b)$. Prove that f is strictly increasing on (a, b). That is, prove that if $a < x < y < b$, then $f(x) < f(y)$.

   b) Suppose $f : D \to \mathbb{R}$ is a differentiable function, that D contains an open interval (a, b) for some $a < b$, and that $f'(x) < 0$ for all $x \in (a, b)$. Prove that f is strictly decreasing on $(a.b)$.