Homework for Math 351-003

Individual Homework: Due Wednesday, April 3

1. (a) Prove that a constant function $f(x) = c$ for some $c \in \mathbb{R}$ is differentiable at all $a \in \mathbb{R}$ and $f'(a) = 0$.

   (b) Prove that a linear function $f(x) = mx + b$, where $m, b \in \mathbb{R}$, is differentiable at all $a \in \mathbb{R}$ and $f'(a) = m$.

2. Suppose $g : D \to \mathbb{R}$ is differentiable at a and that $g(a) \neq 0$. Prove that the function $\frac{1}{g}(x) = \frac{1}{g(x)}$ is differentiable at a and that

$$\left(\frac{1}{g}\right)'(a) = \frac{-g'(a)}{g^2(a)}.$$

   (Notice that it follows from the fact that g is continuous at a with $g(a) \neq 0$ that g is non-zero on some open interval about a, so that $\frac{1}{g}$ is defined on an open interval about a.)

3. Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = x^2$ for $x \geq 0$ and $f(x) = x$ for $x < 0$. Prove that f is not differentiable at 0.

4. Suppose that $f : D \to \mathbb{R}$ and $g : D \to \mathbb{R}$ are both uniformly continuous on D. Prove that the function $f + g : D \to \mathbb{R}$ is uniformly continuous on D.