Homework for Math 351-003
Individual Homework: Due 2/7.

1. Prove or disprove the following claim: *There exists a subset of $\mathbb{R}$ that is closed and not bounded.*

2. Prove that if S is a non-empty subset of a bounded set T, then

$$\inf T \leq \inf S \leq \sup S \leq \sup T$$

3. Let S be a nonempty subset of $\mathbb{R}$. Prove that $-\sup S = \inf -S$, where $-S = \{-s \mid s \in S\}$. (Your solution should not cite Problem 25.)

4. Use results we have already established to prove that if (s_n) and (t_n) are convergent sequences, $s = \lim s_n$, $t = \lim t_n \neq 0$, and t_n is non-zero for all n, then $\left(\frac{s_n}{t_n}\right)$ converges to $\frac{s}{t}$. (This is called the Division Limit Law.)