Homework for Math 351-003

Individual Homework: Due Wednesday, April 10

1. (Note: for this problem, you may use the fact that between any two real numbers there exists both a rational number and an irrational number. The former is proved in the "Additional Reading" section at the end of the course notes. The latter you may use without proof.)

   Define $f(x) = \begin{cases} 1 & \text{if } x \text{ is irrational} \\ 0 & \text{if } x \text{ is rational.} \end{cases}$

   For a partition P of the interval $[a, b]$ find $U(f, P)$ and $L(f, P)$, and then find $U(f)$ and $L(f)$. Is f integrable?

2. Prove that if f is bounded and monotone increasing on $[a, b]$ and $[t_{k-1}, t_k] \subset [a, b]$, then

$$\sup\{f([t_{k-1}, t_k])\} - \inf\{f([t_{k-1}, t_k])\} = f(t_k) - f(t_{k-1}).$$