

## Homework for Math 351-003

**Individual Homework:** Due Wednesday, March 20.

1. Prove, using the  $\epsilon$ - $\delta$  definition of continuity, that  $h(x) = 2x^3$  is continuous at 1.
2. Suppose that  $f : D \rightarrow \mathbb{R}$  is continuous at  $x_0 \in D$  in the  $\epsilon$ - $\delta$  definition of continuity. Prove that if  $(x_n)$  is a sequence in  $D$  converging to  $x_0$ , then  $(f(x_n))$  converges to  $f(x_0)$ .
3. Prove that if a function  $f : D \rightarrow \mathbb{R}$  does *not* satisfy the  $\epsilon - \delta$  definition of continuity at some  $x_0 \in D$ , then it does not satisfy the sequence definition of continuity at  $x_0$ .
4. Prove that if  $\sum_{n=m}^{\infty} a_n = s$  for some real number  $s$ , then  $\lim_{n \rightarrow \infty} a_n = 0$ .  
(Hint: If  $s_k = a_m + \cdots + a_k$ , then  $a_k = s_k - s_{k-1}$ .)