Economic Analysis of Resource Market in Cloud Computing Environment

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***Abstract*—Cloud computing has been emerged as the flexible, efficient, and economical distributed computing platform to meet the dynamic and random demand from the users. In this paper, we consider cloud computing environment with resource market between private clouds (i.e., buyers) and service providers (i.e., sellers) in public cloud. Economic analysis is proposed for different types of resource markets, i.e., monopoly (single service provider), competitive and cooperative oligopolies (few service providers). We study the optimal strategy for service provider in monopoly market, the Nash equilibria in competitive oligopoly market, and bargaining solution in cooperative oligopoly market. In addition, the decision and condition for service providers to to establish collusion in the oligopoly market are also investigated.**

I. Introduction

Cloud computing has been introduced as the new large-scale distributed computing paradigm [1]. Cloud computing can be considered as the pool of resource (i.e., processing, infrastructure, storage, etc.). These resources can be dynamically and scalably provided as the accessible and reliable services to a number of users across Internet in an on-demand basis. One typical usage scenario of cloud computing will be based on workload outsourcing in which the spilled over workload (e.g., due to instantaneously peak demand) from in-house computing resource (e.g., small server farm or cluster) can be outsourced to the third-party service provider. This is referred to as Infrastructure-as-a-Services (IaaS) [2]. In cloud computing, workload outsourcing can be based on virtualization technology [3] in which the applications are assembled into a package (i.e., image). This package can be sent over the network and run on the virtual machine (VM) regardless of geographical location. With this cloud computing, the performance of job processing to the users can be improved while the service provider can generate more revenue.

This paper focus on the cloud computing environment with private clouds (i.e., in-house computing resources) and VM hosting service providers in public cloud. In this cloud computing environment, resource market is established for private clouds to buy VM hosting service from service providers. For private cloud, virtual machine manager (VMM) is used to control job processing on VM to maximize utility. With utility maximization, demand and willingness to pay (i.e., inverse demand function) for VM by private clouds can be determined. Then, given the inverse demand function of private clouds, service provider can optimize the VM supply strategy such that the profit is maximized. Three types of resource markets in cloud computing environment are considered, i.e., monopoly market with a single service provider, competitive and cooperative oligopoly markets with a few service providers. For monopoly market, optimization model is formulated to obtain optimal strategy. Noncooperative game and bargaining game models are formulated for competitive and cooperative oligopoly markets, respectively. Nash equilibrium and bargaining solution are obtained for supply strategies of service providers. In addition, the repeated game model is formulated for the event that service providers interact repeatedly. This repeated game model can determine the condition such that the cooperation strategy is credible for the service providers to achieve efficient and fair solution in the long term.

The main contribution of this paper is the tractable mathematical models for cloud computing environment in which the interaction between private and public clouds can be analytically investigated. The proposed mathematical models of this economic analysis will be useful for the planning and optimizing market decision in cloud computing environment.

II. Related Work

In cloud computing, the concepts of grid, cluster, and utility computing [1], [4] are integrated as a unified architecture. In [5], the ontology of cloud computing was introduced. The architecture with five layers, i.e., firmware/hardware, software kernel, software infrastructure, software environment, and cloud applications, was defined. In [6], the algorithm for mapping logical and physical networks for cloud computing was proposed. The objective of this graph-based algorithm is to find an optimal mapping strategy such that the demand of logical network is met subject to the physical network constraint (e.g., bandwidth). The problem of virtual machine (VM) management was studied [7], [8]. In [7], dynamic VM placement algorithm for server migration and consolidation was proposed. In the algorithm, the processing demand is predicted so that the placement can be optimized dynamically. In [8], the architecture and algorithm to support VM migration were introduced. The broker is used to place VM over physical resource.

Pricing and economic model are important for cloud computing. These problems were studied in the context of utility computing [9], [10], [11]. In [9], the architectural framework for the utility-driven cluster with resource management system (RMS) was presented. In this architecture, the job submission specification from the users is used by RMS to improve the

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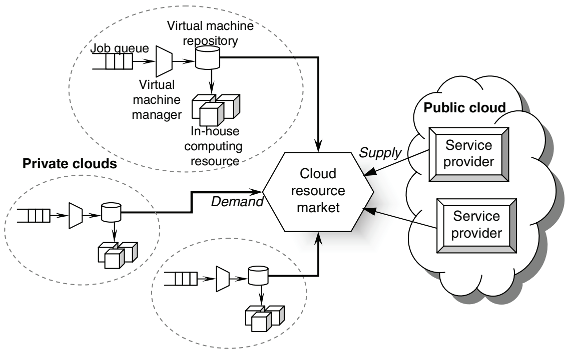

Fig. 1. Cloud computing environment.

resource allocation decision. Also, dynamic pricing function for resource owner to determine the level of sharing in cluster was introduced. In [10], the resource management of computational grids based on economic decision was presented. The computational grid was modeled as a commodity market whose price to be traded is determined by the supply-and-demand equilibrium. In [11], an economic model for self-tuned cloud caching of scientific data processing was introduced. However, all works in the literature ignored the competition and cooperation among service providers.

III. Cloud Computing Environment

We consider cloud computing environment with private and public clouds (Fig. 1). The private cloud corresponds to the in-house computer system (e.g., server farm or cluster). Job from users is stored in a queue with finite size of $J_{\max}$ jobs. The application running on the virtual machine (VM) can process R jobs concurrently (e.g., R CPU cores). Without loss of generality, one VM is assumed to run a single application. In private cloud, virtual machine manager (VMM) is used to control job processing. VMM assigns VMs from repository to the available in-house physical machines. The number of physical machines in private cloud is denoted by Y. Alternatively, if physical machines are all occupied, VMM will outsource VMs to service provider in public cloud. The number of service providers in public cloud is denoted by O. While running VMs on in-house physical machines incurs cost C_{phy} (e.g., power cost), the cost of outsourcing VMs to service provider is denoted by $p > 0$ per VM per unit of time.

The cloud resource market is established for the VM selling and buying between the private clouds and service providers in public cloud [12]. In this market, private cloud as a buyer determines the willingness to pay (i.e., price) given the supplied VMs from all service providers (i.e., inverse demand function). Given the willingness to pay, service provider as a seller determines the supply strategy (i.e., number of supplied VMs) to the market. In this case, the supply strategy will affect capacity planning and consequently profit of service providers. Without loss of generality, the VMs from all service provider are assumed to be identical, i.e., offering identical CPU speed, memory capacity, and network bandwidth.

IV. Private Cloud

A. Utility of Private Cloud

Private cloud is operated to maximize the utility (i.e., satisfaction) of users. Utility is a function of job processing throughput τ (i.e., the number of jobs finished being processed per unit of time), waiting time $\overline{D}$, and loss probability L (i.e., probability that a submitted job is refused by private cloud due to no space in queue). This utility function is defined as follows:

$$\mathscr{U}(p,m) = \omega_1 \tau - \left(\omega_2 \overline{D} + \omega_3 L + p \cdot (m - Y) + C_{\text{phy}} \cdot Y\right)$$

where ω_1, ω_2, and ω_3 are constants of this utility function. m is the number of VMs, and p is the price of VM charged by service provider. Given p and m, this utility can be obtained from analytical model, simulation, or experiment.

In this paper, we use analytical model to obtain the utility. Let us consider M/M/s/J queueing system. Job arrival is assumed to be Possion process with mean Λ job per minute, and job processing time is exponentially distributed with mean $1/\mu$ minutes. Traffic intensity is $\rho = \frac{\Lambda}{\mu}$, and J is queue size. The number of servers s of this queueing system is obtained from $s = m \cdot R$. Job waiting time can be obtained from Little's law as follows $\overline{D} = \frac{\sum_{j=1}^{J} j \cdot \pi_j}{\Lambda(1-\pi_J)}$ where π_j is the probability of having j jobs in private cloud. This probability is obtained from

$$\pi_j = \begin{cases} \frac{\rho^j}{j!}, & j \le s \\ \frac{\rho^s}{s!}\left(\frac{\rho}{s}\right)^{j-s}, & s < j \le s + J \end{cases} \tag{1}$$

and $\pi_0 = 1 - \sum_{j=1}^{J} \pi_j$. $L = \pi_J$ is the probability that queue is full (i.e., loss probability).

B. Demand and Willingness to Pay of Private Clouds

Demand (i.e., the number of VMs to be outsourced) of private cloud c is a function of price p [13]. This demand function can be obtained from utility maximization of private cloud, i.e.,

$$m^{\wr} = \arg\max_{m} \mathscr{U}(p,m). \tag{2}$$

This demand of private cloud c is observed to be non-increasing staircase (step) function of price p. Naturally, as the price of outsourcing VM increases, the cost to private cloud increases. Therefore, private cloud decreases the number of outsourced VMs. Also, since the number of VMs is an integer, the demand is clearly a step function (i.e., $\mathscr{V}_c(p) \in {}^{+}$). The detailed discussion on this demand function is presented in Section VI, specifically in Fig. 3(b).

Mathematically, the individual non-increasing staircase demand function of private cloud c can be expressed as follows:

$$\mathscr{V}_c(p) = \sum_{i=0}^{I_c} m_{c,i} \chi_{\Theta_{c,i}}(p) \tag{3}$$

where I_c is the total number of steps, $m_{c,i}$ is the amount of demand at step i which can be obtained from (2). Without loss of generality, we assume that $m_{c,i-1} > m_{c,i}$ for $i = \{1, \ldots, I_c\}$ (i.e., decreasing step). $\Theta_{c,i}$ is the price interval

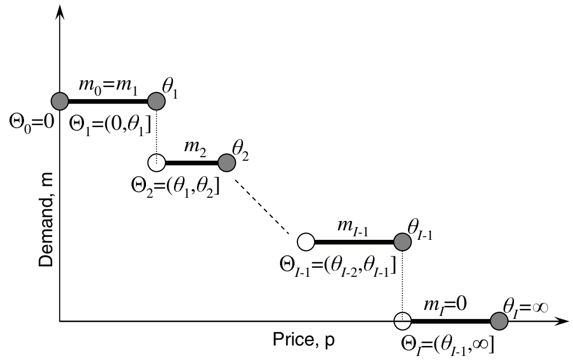

Fig. 2. An example of non-increasing staircase demand function. m_i is the demand, while Θ_i is the price interval of level i.

which is defined as $\Theta_{c,i} = (\theta_{c,i-1}, \theta_{c,i}]$ where $\theta_{c,i}$ is the boundary of step i. We can define $\Theta_{c,0} = 0$, $\Theta_{c,1} = (0, \theta_{c,1}]$, and $\Theta_{c,I_c} = (\theta_{c,I_c-1}, \theta_{c,I_c} = \infty]$ for $c = \{1, \ldots, C\}$, where $m_{c,I_c} = 0$ and C is the total number of private clouds. We assume that all intervals are non-overlapping, i.e., $\Theta_{c,i} \cap \Theta_{c,i'} = \emptyset$ for $i \neq i'$. $\chi_\Theta(p)$ is the indicator function which can be defined as follows:

$$\chi_\Theta(p) = \begin{cases} 1, & p \in \Theta \\ 0, & \text{otherwise} \end{cases}. \quad (4)$$

For aggregate demand of all private clouds, the parameters of staircase function, i.e., m_i and θ_i, can be obtained from

$$\theta_i = \theta_{\hat{c},\hat{i}}, \quad m_i = \sum_{c=1}^{C} \mathscr{V}_c(\theta_i) \quad (5)$$

$$(\hat{c}, \hat{i}) = \arg \min_{\theta_{c,i'} \in \mathbb{O}_i} \theta_{c,i'} \quad (6)$$

$$\mathbb{O}_i = \{(\theta_{c,i'}) | \theta_{c,i'} > \theta_{i-1}\} \quad (7)$$

where $\theta_0 = 0$. Therefore, aggregate demand function of all private clouds can be defined similar to that in (3) as follows:

$$\mathscr{V}(p) = \sum_{i=0}^{I} m_i \chi_{\Theta_i}(p) \quad (8)$$

where $m_I = 0$, $\Theta_0 = 0$, $\Theta_i = (\theta_{i-1}, \theta_i]$, and $\Theta_I = (\theta_{I-1}, \infty]$. Example of staircase aggregate demand function is shown in Fig. 2.

Inverse demand function maps the total number of supplied VMs n to price p (i.e., willingness to pay) [13]. This inverse demand is also non-increasing staircase function expressed as follows:

$$\mathscr{P}(n) = \sum_{i=0}^{I} \theta_{I-i} \chi_{\Xi_i}(n) \quad (9)$$

where $\Xi_0 = 0$, $\Xi_i = (\xi_{i-1}, \xi_i]$, $\Xi_I = (\xi_{I-1}, \xi_I = \infty]$, $\xi_{i-1} = m_{I-(i-1)}$, and $\xi_i = m_{I-i}$.

V. Optimal Supply Strategies of Service Providers in Public Cloud

With different types of resource markets in cloud computing environment, service providers in public cloud will optimize VM supply strategies according to inverse demand function from private clouds.

A. Monopoly Market

Monopoly is referred to as a resource market with a single service provider in public cloud. This service provider has a full control of the market to maximize profit [14]. Note that this monopoly resource market can exist if the virtual machine (VM) outsourcing by private cloud requires a special or proprietary feature from a particular service provider. For example, the private clouds in financial institutions can outsource their VMs which processes confidential customer data only to the security-certified service provider.

Let C_{fix} and C_{var} denote the fixed cost and variable cost of VM provided by service provider. With the inverse demand function, an optimization problem can be formulated to maximize the profit of monopolistic service provider as follows:

$$n^* = \arg \max_n \left(n \cdot \mathscr{P}(n) - C_{\text{fix}} - n \cdot C_{\text{var}} \right) \quad (10)$$

where n is the supply strategy (i.e., number of supplied VMs) of monopolistic service provider. $\mathscr{P}(n)$ is inverse demand function. Due to the non-increasing staircase property of VM inverse demand function, the optimal supply strategy which is the solution of optimization formulation in (10) can be obtained from $n^* = m_{i^*}$ where

$$i^* = \arg \max_{i \in \{1,\ldots,I\}} \left(\theta_i m_i - C_{\text{fix}} - m_i C_{\text{var}} \right) \quad (11)$$

Due to the finite number of steps I, this optimal supply strategy n^* can be obtained by enumeration. Profit of monopolistic service provider can be obtained from $F^*_{\text{mon}} = n^* \left(\mathscr{P}(n^*) - C_{\text{var}} \right) - C_{\text{fix}}$.

B. Competitive Oligopoly Market

In competitive oligopoly resource market, O service providers in public cloud compete to supply VM to private clouds. This market is similar to that of Cournot competition [15]. In this market, service providers are economically rational and act strategically to maximize their profits. Price (i.e., willingness to pay) from private cloud in this market can be observed by all service providers. The fixed and variable costs for oligopolistic service provider o are denoted by $C^{(o)}_{\text{fix}}$ and $C^{(o)}_{\text{var}}$, respectively.

Noncooperative game [16] is formulated for this competitive oligopoly market. *Players* of this game are the service providers. *Strategy* of service provider is the number of supplied VMs n_o. *Payoff* of service provider is the profit. In this game, Nash equilibrium is considered as the solution. To obtain Nash equilibrium, the best response is used. Best response is the optimal supply strategy of service provider given the strategies of other service providers. The best response of service provider $o \in \{1, \ldots, O\}$ is defined as follows:

$$n^\star_o = \mathscr{B}_o(\vec{\mathbf{n}}_{-o}) = \arg \max_{n_o} \mathscr{F}_o(n_o, \vec{\mathbf{n}}_{-o}) \quad (12)$$

where $\vec{\mathbf{n}}_{-o}$ is a vector of strategies of all service providers in the market except service provider o. $\mathscr{F}_o(n_o, \vec{\mathbf{n}}_{-o})$ is a profit function which is defined as follows:

$$\mathscr{F}_o(n_o, \vec{\mathbf{n}}_{-o}) = n_o \cdot \mathscr{P}(n_o + n_{-o}) - C_{\text{fix}} - n_o \cdot C_{\text{var}} \quad (13)$$

where $n_{-o} = \vec{\mathbf{n}}_{-o}^T \vec{\mathbf{1}}$ is the total number of VMs supplied by all service providers except service provider o, and $\vec{\mathbf{1}}$ is a vector of ones.

Again, due to the non-increasing staircase property of inverse demand function, the best response $n_o^\star$ can be obtained from

$$n_o^\star = \mathscr{B}_o(\vec{\mathbf{n}}_{-o}) = m_{i^\star} - n_{-o} \quad (14)$$

$$i^\star = \arg \max_{i \in \{1,\ldots,\tilde{i}\}} \Big(\theta_i (m_i - n_{-o}) - C_{\text{fix}}^{(o)} - (m_i - n_{-o}) C_{\text{var}}^{(o)} \Big) \quad (15)$$

where $\tilde{i} = \arg\min_{m_i \in \mathbb{M}} m_i$ and $\mathbb{M} = \{(m_i) | m_i > n_{-o}\}$. The Nash equilibrium is a set of strategies with the property that no service provider can improve its profit by changing supply strategy while the other service providers keep their strategies unchanged. With the best response, Nash equilibrium is defined as $n_o^\star = \mathscr{B}_o(\vec{\mathbf{n}}_{-o}^\star)$, $\forall o$ where $\vec{\mathbf{n}}_{-o}^\star = \left[\begin{array}{cccccc} n_1^\star & \cdots & n_{o-1}^\star & n_{o+1}^\star & \cdots & n_O^\star \end{array} \right]^T$ is a vector of best responses. The profit of service provider o at Nash equilibrium $n_o^\star$ can be obtained from (13), i.e., noncooperative profit $F_{\text{noc}}^{(o)\star} = \mathscr{F}_o(n_o^\star, \vec{\mathbf{n}}_{-o}^\star)$.

To reach Nash equilibrium, iterative myopic algorithm as shown in Algorithm 1 can be applied. In this algorithm, $n_o(k)$ is the supply strategy of oligopolistic service provider o at iteration k, and ϵ is the small number used to terminate algorithm (e.g., $\epsilon = 10^{-6}$). $\vec{\mathbf{n}}_{-o}(k)$ is a vector of strategies of all service providers except service provider o at iteration k. $K_{\max}$ is the maximum number of iterations.

Algorithm 1 Iterative algorithm to obtain Nash equilibrium of competitive oligopoly market

```
Initialize n_o(k), ∀o for k = 0
repeat
    k ← k + 1
    for o = {1, ..., O} do
        n_o(k) ← B_o(n_{-o}(k))
    end for
until (max_o |n_o(k) − n_o(k − 1)| < ε) OR (k > K_max)
return n_o* ← n_o(k)
```

C. Cooperative Oligopoly Market

Alternatively, all service providers in oligopoly resource market can cooperate to improve their profits. This cooperative market can be modeled using the bargaining game [17] in which the service providers can negotiate with each other to reach efficient and fair solution in terms of VM supply. Specifically, service providers are group rational to maximize total profits from the market, and at the same time individual profit of each service provider is fair. Solution of this bargaining game is the bargaining equilibrium [17].

In general, bargaining game can be defined as follows $\Upsilon(\mathbb{F}, \Gamma) = F_{\text{bar}}^\dagger$ where $\mathbb{F}$ is a set of feasible payoffs (i.e., profit as obtained in (13)) defined as $\mathbb{F} = \left\{ \left(F_{\text{bar}}^{(1)}, \ldots, F_{\text{bar}}^{(o)}, \ldots, F_{\text{bar}}^{(O)} \right) | F_{\text{bar}}^{(o)} \geq 0 \right\}$, and Γ is a threat point. This threat point is the payoffs of service providers if the agreement cannot be reached. In other word, if the bargaining fails, service providers will act noncooperatively to choose Nash equilibrium supply strategy. Therefore, threat point is defined as $\Gamma = \left(F_{\text{noc}}^{(1)\star}, \ldots, F_{\text{noc}}^{(o)\star}, \ldots, F_{\text{noc}}^{(O)\star} \right)$. $F^\dagger$ is the bargaining solution which is defined as $F^\dagger = \left(F_{\text{bar}}^{(1)\dagger}, \ldots, F_{\text{bar}}^{(o)\dagger}, \ldots, F_{\text{bar}}^{(O)\dagger} \right)$. The efficiency of the bargaining solution is characterized by the Pareto optimality. This Pareto optimality is a set of payoffs such that one service provider cannot increase its profit without decreasing the profit of the other service providers. Specifically, the point $\hat{F}_{\text{bar}} = \left(\hat{F}_{\text{bar}}^{(1)}, \ldots, \hat{F}_{\text{bar}}^{(o)}, \ldots, \hat{F}_{\text{bar}}^{(O)} \right)$ is Pareto optimal if there exists the point $\tilde{F}_{\text{bar}} = \left(\tilde{F}_{\text{bar}}^{(1)}, \ldots, \tilde{F}_{\text{bar}}^{(o)}, \ldots, \tilde{F}_{\text{bar}}^{(O)} \right)$ such that $\tilde{F}_{\text{bar}}^{(o)} \geq F_{\text{bar}}^{(o)}$, $\forall o$, then $F_{\text{bar}} = \tilde{F}_{\text{bar}}$. The bargaining solution of the game is the defined for the Pareto optimality that

$$F_{\text{bar}}^\dagger = \max_{\hat{F}_{\text{bar}} = \left(\hat{F}_{\text{bar}}^{(1)}, \ldots, \hat{F}_{\text{bar}}^{(o)}, \ldots, \hat{F}_{\text{bar}}^{(O)} \right)} \prod_{o=1}^{O} \left(\hat{F}_{\text{bar}}^{(o)} - F_{\text{noc}}^{(o)\star} \right). \quad (16)$$

The supply strategy $n_o^\dagger$ which corresponds to the bargaining solution $F_{\text{bar}}^\dagger$ defined in (16) is referred to as the bargaining solution strategy.

D. Collusion among Service Providers

In general, profit of service provider in cooperative oligopoly market is not less than that in competitive market. This is due to the fact that service providers can make agreement to achieve better profit. However, bargaining solution strategy of the cooperative service provider may not be the best response of other service provider. Therefore, if game is played only once, some service provider can deviate from bargaining solution to the best response. On the other hand, if game is played repeatedly, the deviation of service provider will be concerned by future profit. For instance, initially all service providers cooperate to use bargaining solution strategy to gain efficient and fair profit. However, some service providers can deviate from bargaining solution to gain higher profit since bargaining solution is not its best response. However, since the bargaining solution is Pareto optimal, the profit of other service providers will be decreased. Therefore, the affected service providers will switch to compete with deviating service provider (i.e., punishment). As a result, the solution of supply strategy becomes Nash equilibrium, and all service providers receive worse profit than that of bargaining solution. In this case, the cooperation could be maintained if the service providers are aware of such a punishment (i.e., less profit in the future).

The above situation in oligopoly cloud resource market can be modeled as a repeated game for service providers to maintain cooperation (i.e., collusion). This repeated game among service providers choosing between "cooperate" and "deviate" strategies is played periodically (e.g., every quater). Each play is referred to as the stage. At current stage g, service providers can observe strategies and outcomes of previous stage $g-1$. The payoff in the next stage is weighted by the discount factor $0 \leq \delta_o < 1$ in which the future profit has smaller value than the present profit. This discount factor can be determined from the interest rate r as follows $\delta_o = \frac{1}{1+r}$. Suppose that the game will be played forever, payoff in terms of present-value profit (i.e., the current profit plus weighted future profit) of the service provider can be obtained from

$$\begin{aligned} F_{\text{pre}}^{(o)} &= F^{(o)}(1) + \delta_o F^{(o)}(2) + \delta_o^2 F^{(o)}(3) + \cdots \\ &= \sum_{g=1}^{\infty} \delta_o^{g-1} F^{(o)}(g) \end{aligned} \tag{17}$$

where $F^{(o)}(g)$ is the profit of service provider o at stage g. The service providers adopt trigger strategy. That is, the service provider cooperates if the other service providers cooperate in stage $g-1$. Otherwise, the service provider applies best response (i.e., Nash equilibrium) strategy forever after stage $g-1$. This is referred to as the "punishment". For this repeated game, the decision of service provider to use "cooperate" strategy or "deviate" strategy and then be "punished" at stage g will depend on the present-value profit. Let $F_{\text{pre}}^{(o)}(C)$ and $F_{\text{pre}}^{(o)}(D)$ are present-value profits of "cooperate (C)" and "deviate (D)" strategies, respectively. These profits can be obtained from

$$\begin{aligned} F_{\text{pre}}^{(o)}(C) &= F_{\text{bar}}^{(o)\dagger} + \delta_o F_{\text{bar}}^{(o)\dagger} + \delta_o^2 F_{\text{bar}}^{(o)\dagger} + \cdots \\ &= \frac{F_{\text{bar}}^{(o)\dagger}}{1-\delta_o} \end{aligned} \tag{18}$$

$$\begin{aligned} F_{\text{pre}}^{(o)}(D) &= \mathscr{F}_o(\mathscr{B}_o(\vec{\mathbf{n}}_{-o}^{\dagger}), \vec{\mathbf{n}}_{-o}^{\dagger}) + \delta_o F_{\text{noc}}^{(o)\star} + \\ &\quad \delta_o^2 F_{\text{noc}}^{(o)\star} + \cdots \end{aligned} \tag{19}$$

$$= \mathscr{F}_o(\mathscr{B}_o(\vec{\mathbf{n}}_{-o}^{\dagger}), \vec{\mathbf{n}}_{-o}^{\dagger}) + \frac{F_{\text{noc}}^{(o)\star}\delta_o}{1-\delta_o} \tag{20}$$

$\mathscr{F}_o(\mathscr{B}_o(\vec{\mathbf{n}}_{-o}^{\dagger}), \vec{\mathbf{n}}_{-o}^{\dagger})$ is the immediate profit gained from deviating from "cooperate" strategy. That is, the other service provider uses "cooperate" strategy, while deviating service provider uses best response $\mathscr{B}_o(\vec{\mathbf{n}}_{-o}^{\dagger})$ strategy. Recall that $F_{\text{bar}}^{(o)\dagger}$ and $F_{\text{noc}}^{(o)\star}$ are the profits of service provider at bargaining solution and Nash equilibrium in cooperative and competitive markets, respectively.

The "cooperate" strategy is credible for any service provider, if its present-value profit obtained from (18) is higher or equal to that of "deviate" strategy which is obtained from (20). That is,

$$\frac{F_{\text{bar}}^{(o)\dagger}}{1-\delta_o} \geq \mathscr{F}_o(\mathscr{B}_o(\vec{\mathbf{n}}_{-o}^{\dagger}), \vec{\mathbf{n}}_{-o}^{\dagger}) + \frac{F_{\text{noc}}^{(o)\star}\delta_o}{1-\delta_o} \tag{21}$$

where $\vec{\mathbf{n}}_{-o}^{\dagger}$ is a vector of bargaining solution strategies of all service providers except service provider o. In other word, "cooperate" strategy is credible if the service provider has large enough weight to the future profit. That is, the discount factor δ_o satisfies follwoing condition

$$\frac{\mathscr{F}_o(\mathscr{B}_o(\vec{\mathbf{n}}_{-o}^{\dagger}), \vec{\mathbf{n}}_{-o}^{\dagger}) - F_{\text{bar}}^{(o)\dagger}}{\mathscr{F}_o(\mathscr{B}_o(\vec{\mathbf{n}}_{-o}^{\dagger}), \vec{\mathbf{n}}_{-o}^{\dagger}) - F_{\text{noc}}^{(o)\star}} \leq \delta_o. \tag{22}$$

VI. Numerical Studies

A. Performances of Private Cloud

We first consider a single private cloud. The number of physical machines is $Y = 2$. Each physical machine is capable of running one VM, and one VM processes $R = 1$ job at a time. The average job processing time is $1/\mu = 30$ seconds. Constants in utility function are as follows $\omega_1 = 15$, $\omega_2 = 0.5$ and $\omega_3 = 5$, respectively. Note that values of these constants depend on types of applications and users (e.g., web server). The price per VM is $p = 1.0$ cent per unit of time. Maximum queue size for jobs is $J = 100$.

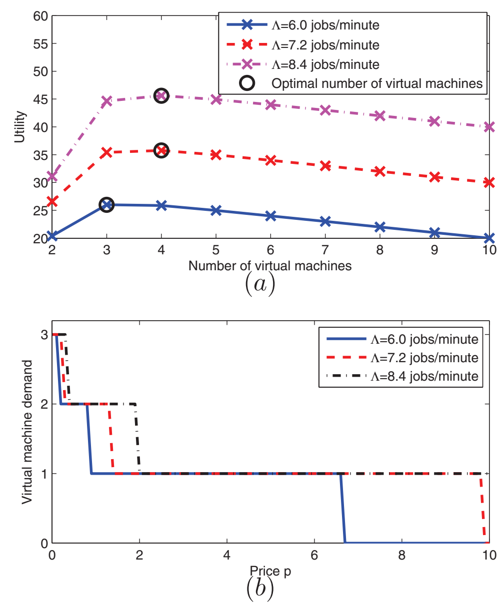

Fig. 3. (a) Utility of private cloud under different number of outsourced VMs and (b) demand under different prices.

Fig. 3(a) shows the utility of private cloud. With fixed job arrival rate, as the number of VMs increases, the utility first increases, and then decreases. At small number of VMs the waiting time and loss are high, cost due to performance degradation is high. At large number of VMs, cost of outsourcing is high. In this case, there is an optimal number of VMs which maximizes utility of private cloud. Fig. 3(b) shows the optimal number of VMs to be outsourced given the price p (i.e., demand of private cloud). For high job arrival rate (i.e., $\Lambda = 8.4$), VMM requires more number of VMs to minimize the cost of performance degradation. However, if the price is high, the cost of outsourcing VM increases. Therefore, the number of outsourced VMs decreases. Clearly, the demand is observed to be non-increasing staircase function of price.

B. Analyses of Cloud Resource Market

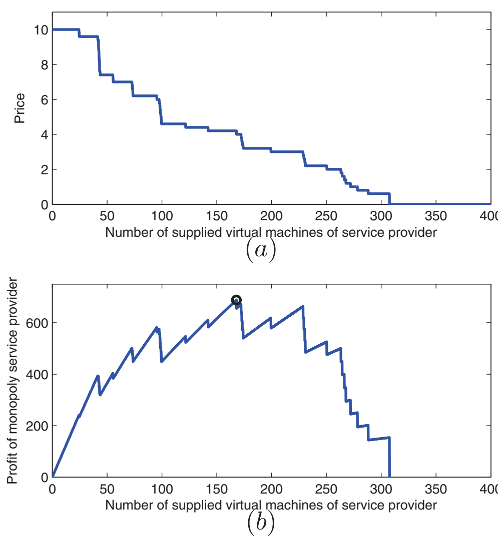

Fig. 4. (a) Willingness to pay (i.e., price) and (b) profit of monopoly service provider under different number of supplied VMs.

1) Monopoly Market: Next, we consider the monopoly cloud resource market (i.e., a single service provider). The inverse demand (i.e., the willingness to pay or price given the number of supplied VMs) from $C = 120$ private clouds with various job arrival rates is shown in Fig. 4(a). Again, this inverse demand is non-increasing staircase function. Given inverse demand function, service provider in public cloud can seek for the number of supplied VMs, so that the profit is maximized (Fig. 4(b)).

2) Competitive Duopoly Market: Then, duopoly cloud resource market with two service providers in public cloud is considered. In this market, service providers can adjust supply strategies strategically. Figs. 5 shows the best response of both service providers. As observed, there are many Nash equilibria lying on the overlapping best responses of both service providers. At the Nash equilibria, the service provider cannot increase its profit by changing the number of supplied VMs given that the other service provider does not change from Nash equilibria.

Fig. 5 also shows strategy update from iterative algorithm (Algorithm 1). In this algorithm, service providers change their strategies iteratively to reach Nash equilibria. Given initial point (i.e., $n_1(0)$ and $n_2(0)$ at point 1), service providers 1 and 3 change strategies to reach Nash equilibria (i.e., from point $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$).

Note that the similar result is observed for the case of multiple service providers in the oligopoly market.

3) Cooperative Duopoly Market: Fig. 6 shows the bargaining strategies of two service providers. The Pareto optimality can be defined as a boundary of the set of bargaining strategies.

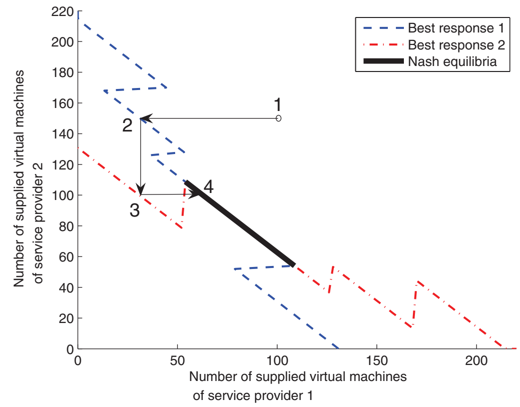

Fig. 5. Nash equilibria of competitive duopoly market.

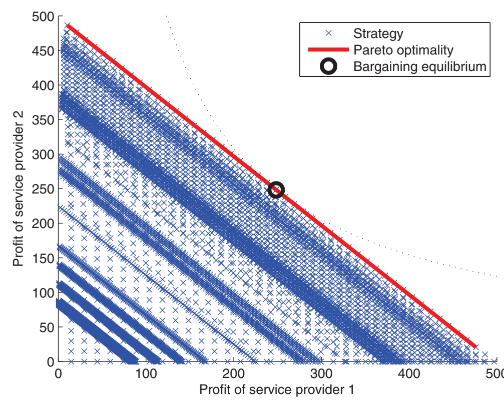

Fig. 6. Profit of two service providers, Pareto optimality, and bargaining solution.

That is, on Pareto optimality, one service provider cannot increase its profit without decreasing the profit of the other service provider. The bargaining solution is found to be on the Pareto optimality. The bargaining solution ensures that the product of the profits will be maximized and also the corresponding bargaining strategy is fair to all service providers. That is, the profit of both service providers are identical.

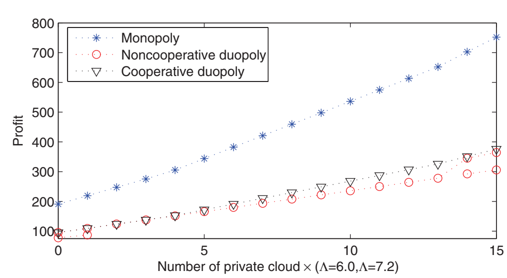

Fig. 7. Comparison of profits in different resource markets.

4) Comparison among Different Types of Markets: Figs. 7 shows the profits of the service providers in different types of markets. Clearly, the service provider in monopoly market can achieve the highest profit, since the service provider can fully control the market. In duopoly market, the cooperative service providers always achieve the profit higher or equal to that of competitive service providers. In cooperative market, service provider can reduce the number of supplied VMs. Therefore, the price and hence profit are higher. However, in competitive market, service providers compete with each other by supplying more VMs to gain market share. Negatively, the price is low, and hence the profit is not maximized.

TABLE I
MINIMUM DISCOUNTED FACTOR FOR SERVICE PROVIDERS TO MAINTAIN COLLUSION

Case	$F^{(o)\dagger}_{\mathrm{bar}}$	$F^{(o)\star}_{\mathrm{noc}}$	$\mathscr{F}_o(\mathscr{B}_o(\vec{\mathbf{n}}^{\dagger}_{-o}), \vec{\mathbf{n}}^{\dagger}_{-o})$	δ_o
1	95.555	95.555	95.555	-
2	152.725	151.670	195.460	0.97591
3	287.410	249.980	296.670	0.19833

5) Collusion of Service Providers: Table. I shows the present-value profits of cooperative, competitive, and deviated service provider. From Table. I, we observe that if competitive and cooperative strategies yield identical profit (cases 1), the deviated service provider will also yield the same profit. Therefore, there is no difference between cooperate and deviate strategies. However, if the competitive strategy yields lower profit than that of cooperative strategy (case 2,3), the profit receives from deviation is higher than the cooperative strategy. As a result, the service provider may deviate from cooperate strategy (i.e., collusion may not be credible). Two factors which affect the deviation from collusion of service provider are the gained profit $\mathscr{F}_o(\mathscr{B}_o(\vec{\mathbf{n}}^{\dagger}_{-o}), \vec{\mathbf{n}}^{\dagger}_{-o})$ (i.e., profit of deviated service provider) and the remaining profit after being punished $F^{(o)\star}_{\mathrm{noc}}$ (i.e., profit of noncooperative service provider). The minimum discount factor δ_o such that the collusion is credible is shown in Table. I. In case 2, the deviation yields high profit, while the remaining profit after being punished is only slightly smaller than that of cooperate strategy. Therefore, the motivation of any service provider to deviate is significant, and the collusion can be maintained only if the future is sufficiently weighted or equivalently the minimum discount factor is large enough. The opposite situation is observed in case 3.

VII. CONCLUSION

In this paper, we have presented an economic analysis of the resource market in cloud computing environment. With utility of users in private cloud, the inverse demand function (i.e., the willingness to pay or price given the number of supplied virtual machines) can be obtained. This inverse demand is observed to be non-increasing staircase (step) function. Based on this inverse demand function, the virtual machine hosing service providers in public cloud can optimize supply strategies to maximize profit. Three types of resource market between private clouds (i.e., consumers) and service providers (i.e., suppliers) have been considered, i.e., monopoly, competitive and cooperative oligopoly. While an optimization model has been applied for the monopoly market, noncooperative and bargaining game models have been formulated for the competitive and cooperative oligopoly markets, respectively. Also, repeated game model has been used to analyze the cooperation behavior (i.e., collusion) of the service providers in public cloud to reach efficient and fair profit.

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