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Abstract

This paper investigates the effect of elitism on multi-objective ant colony optimization algorithms (MACOs). We use a straightforward and systematic approach in this investigation with elitism implemented through the use of local, global and mixed non-dominated solutions. Experimental work is conducted using a suite of multi-objective traveling salesman problems (mTSP), each with two objectives. The experimental results indicate that elitism is essential to the success of MACOs in solving multi-objective optimization problems. Further, global elitism is shown to play a particularly important role in refining the pheromone information for MACOs during the search process.

Inspired by these results, we also propose an adaptation strategy to control the effect of elitism. With this strategy, the solutions most recently added to the global non-dominated archive are given a higher priority in defining the pheromone information. The obtained results on the tested mTSPs indicate improved performance in the elitist MACO when using the adaptive strategy compared to the original version.

I. Introduction

Ant colony optimization [1] (ACO) is a popular paradigm in evolutionary computation. It has been applied to solve various types of problems such as network routing [2] and path planning [3], [4]. Apart from its original domain of combinatorial optimization, ACO is also now used to solve continuous optimization problems [5]. A number of ACO designs have been proposed in the literature such as Ant System (AS) [6], Ant Colony System (ACS) [7], and the MAX-MIN Ant System (MMAS) [8].

Recently there has been a number of studies extending ACO to the area of multi-objective optimization [9]. There are several aspects that should be considered when adapting ACO to this problem domain including issues related to pheromone information, heuristic information, and the use of elitism. For the case of pheromone and heuristic information, a number of implementations are possible such as the use of a single matrix or multiple matrices for one or both types of information.

A thorough investigation related to pheromone and heuristic information has recently been carried out in [9] to observe the effect of these variations in ACO design. Another important issue to consider is Elitism in ACO which involves using the best solutions to update the pheromone matrix. The best solutions within a multi-objective context can refer to either the non-dominated solutions within an iteration (the local best) or the non-dominated solutions found so far (the global best). Although several studies have implemented elitism in MACOs, it is not clear if there is a preferred strategy for selecting elitist solutions and to our knowledge, an investigation of the effect of elitism has yet to take place within this context. Furthermore, most of the proposed MACOs appear to have been designed in an "*ad hoc*" fashion with a number of design modifications taking place. Although these designs may very well be effective, it is difficult to determine how the proposed changes to the elitism process have impacted algorithm behavior and performance.

This paper aims to address this issue by systematically investigating the behaviour of ACO algorithms using elitism to solve multi-objective optimization problems (MOPs). Our experimental setup considers two sets of non-dominated solutions: the local-best for a single iteration, and the global-best which is also typically referred to as the external archive. A third set considered in this work is the *local-set* which refers to all solutions (dominated and non-dominated) that are generated in a single iteration. One of these three sets or a combination of the sets can be used to update the pheromone information in the ACO with possible options including: *local-set*, *local-best*, *local-set* + *global-best*, *global-best*, and *local-best* + *global-best*. Notice that using only the local-set represents the non-elitist default case while the remaining options represent the range of elitist possibilities.

We also introduce an adaptive technique to control the use of the external archive in updating pheromone matrices in order to discourage premature convergence and encourage the participation of newly discovered solutions. For this adaptive technique, each solution in the archive is assigned an age to indicate how long it has existed in the archive. This value is used to adjust the amount of pheromone that an aging ant will deposit.

To validate the proposed method, five instances of the multi-objective traveling salesman problem (mTSP) have been selected for experimental studies. The obtained results indicate that elitism plays an important role in the convergence of MACOs and that strong performance requires an elitism technique that uses the global-best set. Furthermore, our adaptive technique produces promising results on the tested problems in comparison with the non-adaptive ACOs.

The rest of the paper is organized as follows: A brief review of MOEAs and ACO is provided next followed by a description of the elitism methodology in Section III. A comparative study is then carried out in section IV. The last section concludes the paper.

II. Background

A. Evolutionary Multi-objective Optimization

Multi-objective evolutionary algorithms (MOEAs) are stochastic optimization techniques, which normally use a population-based approach to find Pareto optimal solutions. The majority of existing MOEAs today employ the concept of dominance during selection (but see VEGA [10] for a counterexample); as a result, we focus here on the class of dominance-based MOEAs only.

Mathematically, in a *k*-objective optimization problem, a vector function $\overrightarrow{f}(\overrightarrow{x})$ of *k* objectives is defined:

$$\overrightarrow{f}(\overrightarrow{x}) = \left[f_1(\overrightarrow{x}), f_2(\overrightarrow{x}), ..., f_k(\overrightarrow{x})\right] \tag{1}$$

in which $\overrightarrow{x}$ is a vector of decision variables in the *n*-dimensional space $\mathbb{R}^n$; *n* and *k* are not necessarily the same. Each individual is assigned a vector $\overrightarrow{x}$ which is used to evaluate the solution and compute the vector $\overrightarrow{f}$. An individual is preferentially selected based on the values given in the vector defined in Eq. 1. Here we assume that each of the objectives are in conflict with one another. Under these conditions, an MOEA will produce a set of solutions which extend over a tradeoff surface in objective space. The selection of a final solution (i.e. a solution to implement from among this set) requires a biased decision that is typically left to a human decision maker.

Many MOEAs have been developed over the years. These algorithms can be classified into two broad categories: non-elitist and elitist. With the elitism approach, MOEAs employ an external set (the archive) to store the non-dominated solutions after each generation. This set will then be included in the next generation. The best individuals in each generation are always preserved, which helps drive the search process towards the Pareto Optimal Front (POF). Algorithms such as SPEA2 [11], PDE [12], NSGA-II [13] are examples of this category. In contrast, the non-elitism approach does not employ the any concept of elitism during the selection process [14]. Examples of this category include VEGA [10] and NSGA [14]. Although MOEAs can differ from one other in several ways, the common steps of these algorithms can be summarized as shown below:

- **Step 1:** *Initialize a population* P
- **Step 2:** *(optional):Select elitist solutions from* **P** *to create an external set* FP *(For non-elitism algorithms,* FP *is empty)*
- **Step 3:** *Create mating pool from one or both of* P *and* FP
- **Step 4:** *Perform reproduction based on the pool to create the next generation P*
- **Step 5:** *Possibly combine* FP *into* P
- **Step 6:** *Go to step 2 if the termination condition is not satisfied*

B. Ant Colony Optimization

Ant colony optimization, as implied by its name, is strongly inspired from the behavior of colonies of ants. Ants frequently travel between their nest and food sources. The simple, self-organizing processes that ants use to gather food allow for coordinated behavior that is of great benefit to the colony. During travel, some ants deposit a special substance called pheromone on their paths which is used to communicate with other ants about the directions they should take to reach a food source. Based on the amount of pheromone, ants (probabilistically, perhaps) are driven towards shorter paths between food and nest locations. The communication between ants occurs indirectly with individual ants modifying the surrounding environment using pheromone deposits (sometimes referred to as *stigmergy*).

The original version of ACO was developed by Dorigo, Maniezzo, and Colorni [6], and is called Ant system (AS). The AS uses an iterative search process where for each iteration (or generation), a number of ants are generated to find paths. The problem solution is encoded in the paths that are taken by the ants. The pheromone information (represented by a data structure, usually a matrix) of paths is updated using two steps:

- Deposit: ants lay an amount of pheromone on each of the locations that they visit. For each ant *h* and each pair of nodes *i* and *j* which the ant visited:
$$\tau_{ij} = \tau_{ij} + \Delta\tau_{ij}^h \tag{2}$$
$\Delta\tau_{ij}^h$ is usually defined based on the quality of the solution (represented by the path). In TSP:
$$\Delta\tau_{ij}^h = \frac{1}{C^p} \tag{3}$$
with C^p is the cost associated with the path
- Evaporation: This process involves the reduction of pheromone density. Evaporation occurs in each iteration by decreasing the pheromone value by a predefined factor (called ρ). Given two nodes *i* and *j*, the pheromone value τ_{ij} is adjusted as follows:
$$\tau_{ij} = (1-\rho)\tau_{ij} \tag{4}$$

A solution (a path) is determined by an ant based on probabilistic decisions. For each move from node *i* to node *j*, a moving probability is defined as follows:

$$p_{ij}^h = \begin{cases} \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{u \in \mathcal{N}_i^h} [\tau_{ij}]^\alpha [\eta_{ij}]^\beta} & \text{if } j \in \mathcal{N}_i^h \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $\mathcal{N}_i^h$ is the feasible neighborhood of ant h. η_{ij} is the heuristic information between nodes *i* and *j*; and in TSP, $\eta_{ij} = \frac{1}{d_{ij}}$ where d_{ij} is the Euclidean distance between *i* and *j*. α and β are controlled coefficients determining the balance between pheromone and heuristic information. Elitism was originally introduced in AS by storing a best-so-far solution that is used to update pheromone information.

Note that there are several variants of ACO introduced in the literature which have addressed a number of related issues such as pheromone boundaries, and global and local updates. In these works, ACO has been shown to be very effective in solving combinatorial problems. Recently, a number of attempts have also been given to extending ACO to the domain of continuous problems. For more detail, readers are referred to [1].

Because of the effectiveness of ACO in solving combinatorial problems, there have been an increasing number of works using ACO algorithms to deal with MOPs either with or without the use of elitism, such as Multiple Objective Ant-Q algorithm (MOAQ) [15] (which extends Ant-Q to multi-objective optimization by having a family of agents in charge of each objective), or Bicriterion Ant [16], P-ACO [17], MONACO [18], COMPE-Tants [19], and SACO [20] which uses local best solutions, or MOPACO [21]/CPACO [22] which uses global best solutions to update the pheromone information.

An effort was recently put forth to comprehensively investigate the performance of these approaches on the mTSP [9]. From that investigation, MACOs were shown to have better performance in comparison to other GA-based approaches such as NSGA-II and SPEA2 when multiple matrices of either pheromone or heuristic information were used.

The above studies focused mainly on organizing pheromone and heuristic information into one or more matrices (e.g. using different matrices for different objectives) and using single or multiple colonies. However, as stated in [23], elitism is a critical factor in the field of evolutionary multi-objective optimization which has been well-studied in the context of using Genetic Algorithms for solving MOPs. In this context, the best-so-far solutions are utilized in several aspects of evolution such as selection and reproduction. However, in the area of multi-objective ACO, it is not clear how the best solutions should be involved in the pheromone updating process since most MACO research has involved multiple "*ad hoc*" design changes with the goal of improving algorithm performance and not of understanding algorithm behavior.

III. Methodology

As stated in the introduction, this paper intends to:

- Investigate the behaviour of MACO and the effect of elitism. This will be done through establishing an unified method and using a variety of elitism techniques for updating pheromone information.
- Propose an adaptive strategy for the use of elitism. This provides a mechanism for deciding which solutions in the set of elitist solutions should be used or preferred during the updating process.

When using elitism, an important decision must be made in defining which solutions are to be labeled as elitist solutions. In multi-objective optimization, no single best solution exists for a given problem. Instead, there will be a set of non-dominated solutions where each non-dominated solution can be said to be optimal for a given context. From this point forward, we will use the terms the *best* and *non-dominated* solutions interchangeably. In the next section, we propose several methods that can be used to define the best solutions for MACOs.

A. Elitism implementation

Similar to single-objective ACO, the best solutions can be classified as one of two types: iteration-based and best-so-far. For the sake of simplicity, we refer to them as the local- and global-best solutions. In particular, after each iteration, we obtain two special sets of solutions: the non-dominated solutions within the iteration (the local best) and the non-dominated solutions found so far (the global best).

Based on this simple observation, we propose to investigate the effect of elitism using the approaches listed below. These approaches employ the same number of ants, pheromone and heuristic information. Each objective of the test problem is associated with two matrices of pheromone and heuristic values respectively. The only difference between these algorithms is in the way the pheromone information is updated. For each move from node *i* to node *j*, a moving probability is defined as follows:

$$p_{ij}^{h} = \sum_{k=1}^{M} w_k p_{ij}^{k} \tag{6}$$

where M is the number of objectives, p_{ij}^{k} is calculated as in Eq. 5, and w_k a weighting coefficient (set equal to 0.5 for all experiments).

- **Multi-objective AS**: There is no elitism in this approach and it is included as a baseline for investigating and comparing the behaviour of the elitist approaches. In this case, we use all solutions from the current iteration (i.e. the *local set*) to update the pheromone matrix. This represents the most straightforward extension of the AS algorithm to MOPs and so we call this **MAS**. This also acts
- **Combination of local-set and global-best**: This is the basic elitism version of MAS using both local and global information. It is referred to as **EMAS**.
- **Local-best MACO**: For this approach, we use only the non-dominated solutions found in the current iteration to update the pheromone matrix. This approach employs local elitism for updating the pheromone matrix and hence is called **LEMACO**. This allows us to investigate the impact of local elitism and its utility in guiding the search process of a MACO.
- **Global-best MACO**: This approach uses only solutions from the global non-dominated set to update the pheromone matrix. It can be seen as an instance of MPACO [21] which uses all non-dominated solutions instead of selecting a random subset of solutions as occurs in MPACO. Due to this similarity, we call this

approach **MPACOA**. Again, This allows us to investigate the pure effect of global elitism and its usefulness in guiding MACOs during the search process. Note that in this approach the archive size (i.e. size of Global best set) can not exceed the number of ants (i.e. size of Local set). If the number of the non-dominated solutions is over the archive size, truncation is applied based on the crowding distance metric which is taken from NSGA-II [13].

- **Mixup of local and global best**: This approach uses both the local and global non-dominated solutions to update the pheromone matrix. We called it the Local and Global ACO **LGACO**. This approach is very similar to EMAS with the only difference being that local information is taken from the local-best only instead of the entire *local set*.

B. Adaptive elitism technique

The global non-dominated solutions are stored in an external archive and can stay in that archive for an indefinitely long period of time. However, continually updating the pheromone information with the same solutions can potentially introduce a strong attractor to the state of the pheromone information which in turn could hinder exploration of the solution space. On the other hand, removing the solution from the archive would reduce the chance of exploiting this information in the future. An alternative is to develop an adaptive strategy that automatically makes decisions about which (and to what extent) solutions are used for updating pheromone information. In this study, we investigate a deterministic adaptive procedure where each solution in the archive is assigned an age, representing the number of iterations that the solution has resided in the archive, which is used to control how an archived individual contributes to pheromone information over time. The method itself can be understood as an *aging* strategy and is similar in spirit to the annealing schedule in Simulated Annealing. Formally, for a solution h in the archive, an adjusting coefficient δ at the time t is determined as follows:

$$\delta = \frac{m}{t - a_h + 1} \tag{7}$$

where a_h is the age of solution h, and m is the number of ants, and τ_{ij}^h is then updated as :

$$\tau_{ij}^h = \tau_{ij}^h + \frac{\delta}{C^p} \tag{8}$$

IV. EXPERIMENTAL STUDIES

The first series of experiments examines the proposed non-adaptive approaches: **MAS**, **EMAS**, **LEMACO**, **MPACOA**, **LGACO**. The investigation considers the behaviour of the algorithms both *during* as well as *at the end of* the optimization process by analyzing the obtained non-dominated solutions over time. A second set of experiments is also carried out to study the adaptive technique proposed in the previous section.

A. Test problems

Experiments were conducted on five instances of the multi-objective TSP which are referred to in the literature as *kroAB100*, *kroAC100*, *kroCB100*, *kroAB150*, and *kroAB200* and involve 100, 150, and 200 cities respectively. These problems have two objectives that must be minimized and have a discontinuous, convex Pareto optimal front. They are taken from the TSPLIB library [24] and have been used widely in the literature. For the sake of simplicity, we named them as **P1**, **P2**, **P3**, **P4**, and **P5**.

B. Performance measurement methods

Performance metrics are used to compare different algorithms and play an important role in assessing experimental results. However, how we decide to assess performance depends on a number of contextual factors such as the amount of computational resources available and the importance of solver reliability. In this work, we take the recommendations from [25], [26] and measure performance using the hyper-volume. The hyper-volume, H, is measured by summing the hyper-volumes of hyper-areas covered by the obtained POF. For this metric, the greater the value of H, the better convergence the algorithm has. Note that this metric is used to indicate both the *closeness and diversity* of the obtained POFs.

C. System settings

For all experiments, the number of ants was set as 100 and the number of iterations was set to 500 (for a total 50,000 function evaluations per run). All cases are tested in 30 separate runs with 30 different random seeds. The combined non-dominated set from these 30 runs is then used for comparisons between the different elitism approaches. Other ACO parameters are α = 1.0, β = 3.0 and ρ = 0.5 (see [1], *page 71* for more detail).

D. Performance Analysis

To analyze the performance of the approaches, we consider the convergence of hyper-volume values and the distribution of the obtained POFs for problems P1 and P5. Other problems provided similar results however we only present the final hyper-volume values for the other problems (see Tables I and II) due to space limitations.

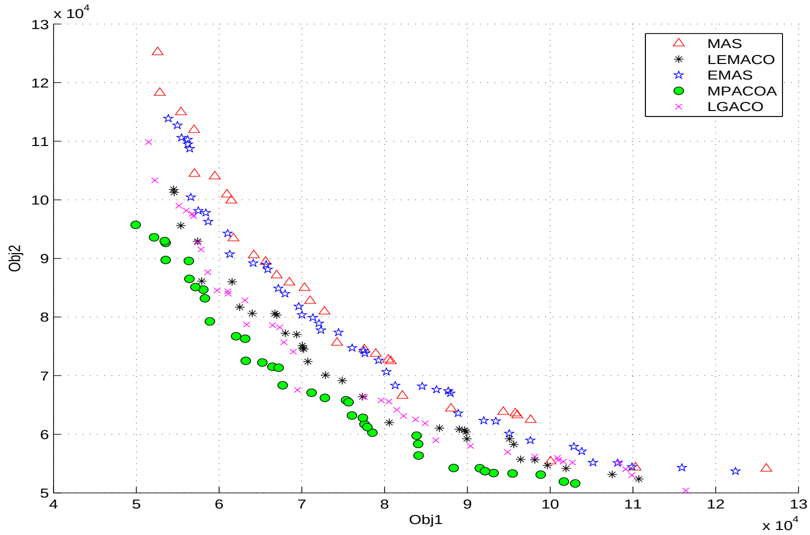

Fig. 1. Non-dominated solutions obtained by different approaches after 500 iterations and with P1.

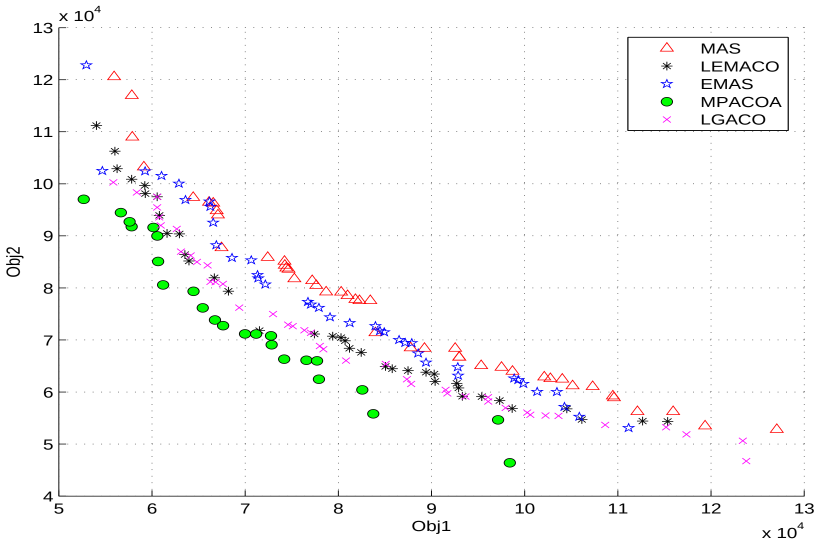

Fig. 2. Non-dominated solutions obtained by different approaches after 500 iterations and with P2.

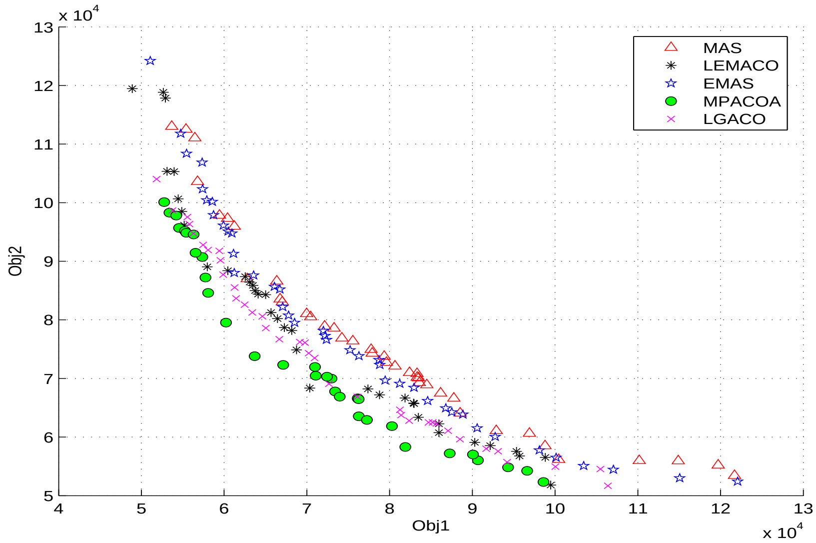

Fig. 3. Non-dominated solutions obtained by different approaches after 500 iterations and with P3.

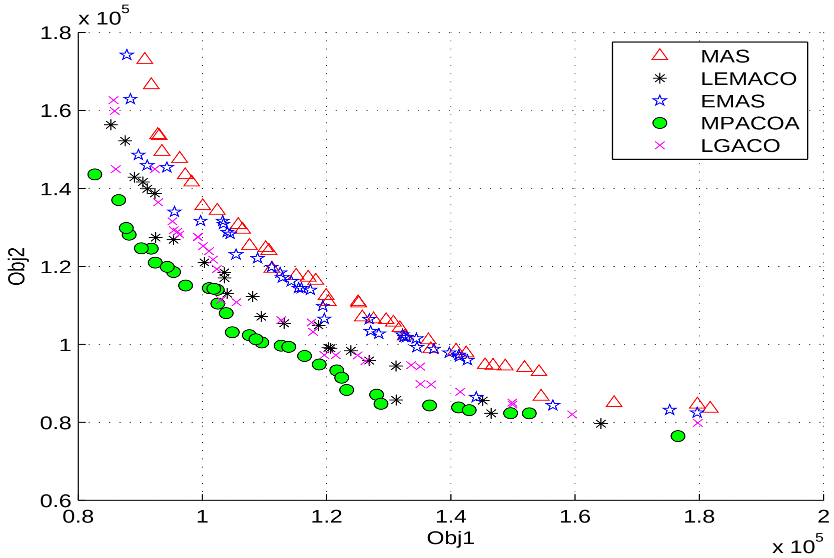

Fig. 4. Non-dominated solutions obtained by different approaches after 500 iterations and with P4.

The results in Figures 1, 2, 3, 4, 5 indicate that MAS and EMAS have poor performance in comparison with the other algorithms. For these two approaches, all current solutions (the local set) located by the ants are allowed to update the pheromone information which is similar to what takes place in AS. Previous studies [1] have shown that AS is outperformed by many approaches in the single-objective domain. It is possible that the poor performance of AS in single objective problems and the poor performance of MAS for MOPs are both due to the use of the local set which is expected to create a more explorative and less focused search process. By treating MAS as the baseline approach to compare with the elitist methods, it is clear from the figures that elitism provides a strong benefit to MACO performance. The remainder of the approaches use only elitist solutions to update the pheromone information and from these it appears that using only the global best set (MPACOA) provides significantly better results compared to when the local best set is used (LEMACO and LGACO).

These findings are more clearly indicated in Figure 6 where the hype-volume occupied by the non-dominated

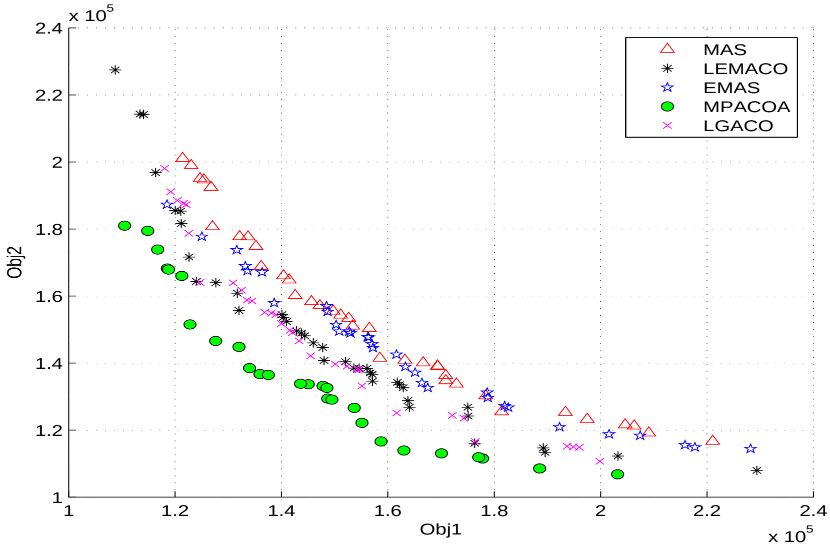

Fig. 5. Non-dominated solutions obtained by different approaches after 500 iterations and with P5.

sets is plotted over time. Here we find that all approaches start with very similar performance over the first 10 iterations but over longer time scales, the elitist approaches steadily become better than the non-elitist approaches and converge more quickly to the POF.

The results also show that EMAS is consistently better than MAS as indicated in the above figures and from Tables I, and II. In conclusion, our findings from all tested problems strongly indicate a benefit from using elitism in MACO. Among the elitist schemes, the global-elitist scheme consistently resulted in the best performance.

TABLE I

HYPER-VOLUME VALUES OBTAINED AFTER 500 GENERATIONS WITH P1, P2, AND P3 (THE MEAN AND STD ERROR (*E+09)

Algs	P1	P2	P3
MAS	5.98 (0.02)	5.45 (0.02)	5.65 (0.01)
EMAS	6.16 (0.02)	5.64 (0.02)	5.81 (0.02)
LEMACO	6.57 (0.02)	6.04 (0.02)	6.10 (0.02)
MPACOA	6.98 (0.03)	6.40 (0.04)	6.35 (0.02)
LGACO	6.64 (0.02)	6.08 (0.02)	6.17 (0.01)
AMPACOA	7.18 (0.03)	6.61 (0.03)	6.53 (0.02)

TABLE II

HYPER-VOLUME VALUES OBTAINED AFTER 500 GENERATIONS WITH P4, AND P5 (THE MEAN AND STD ERROR (*E+09)

Algs	P4	P5
MAS	10.90 (0.03)	14.27 (0.04)
EMAS	11.27 (0.04)	14.76 (0.04)
LEMACO	12.14 (0.04)	16.03 (0.04)
MPACOA	12.73 (0.05)	17.24 (0.08)
LGACO	12.07 (0.03)	15.98 (0.04)
AMPACOA	13.26 (0.06)	17.72 (0.06)

E. Adaptation of elitism

In this section, we discuss the effect of our proposed adaptation strategy on the elitism schemes. We selected MPACOA for this study since it was found to be the best among all the elitist techniques tested thus far. The adaptive version of this approach is labeled as **AMPACOA**.

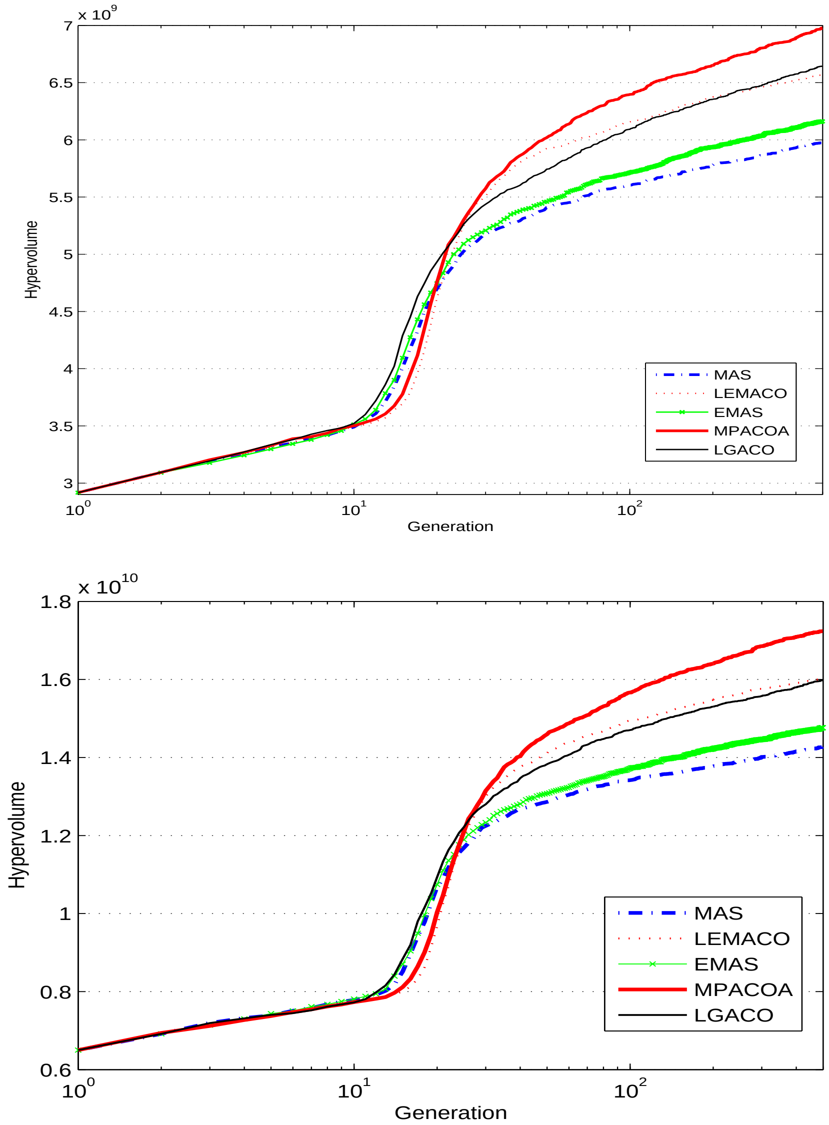

Fig. 6. Hypervolume values obtained by approaches over time with P1 and P5.

The adaptation strategy has been designed to give extra weighting to those solutions which have most recently entered into the global archive and to give increasingly less weighting to those which have been in the archive for longer periods of time. It is anticipated that this will help to inject more explorative behavior into the search process by preventing solutions from introducing permanent attractive forces towards regions of the search space.

From the results in Figures 7, 8, 9, 10, 11, we can see that the adaptive strategy does indeed provide some improvements to algorithm performance (although the benefit is less clearly observed in the figures). This result is confirmed in Tables I and II where hypervolume values obtained by AMPACOA after 500 generations were clearly large than that of MPACOA. A further indication can be found in Figure 12 where it can be seen that AMPACOA converges to the POF more quickly than MPACOA. It is speculated here that this improvement from the adaptive mechanism is a consequence of better diversity maintenance during the search process.

V. Conclusion

In this paper, we investigated the effect of elitism, an important factor in population-based multi-objective optimization, on the performance of MACO. Our studies indicated that elitism is an essential design attribute

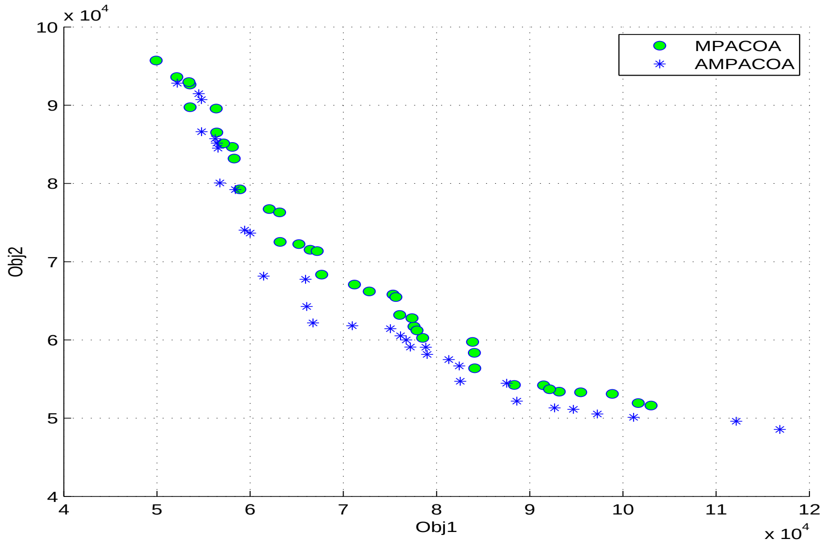

Fig. 7. Non-dominated solutions obtained by MPACOA and AMPACOA and with P1.

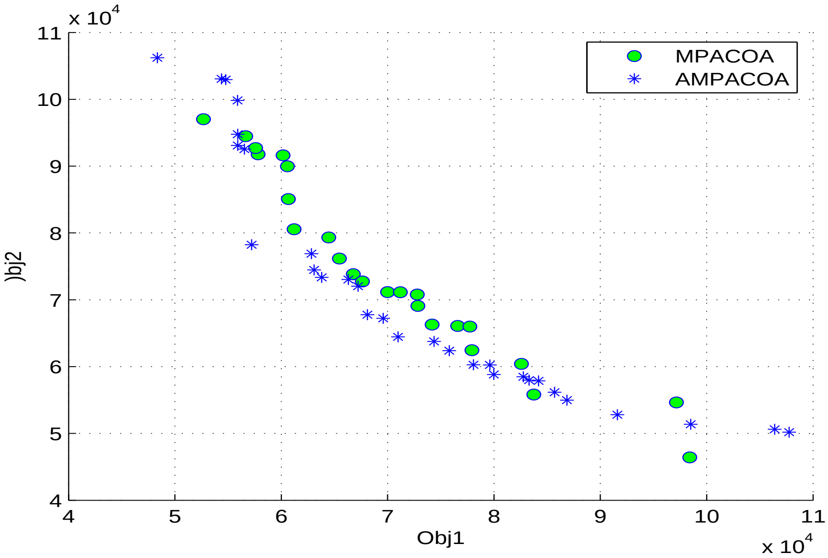

Fig. 8. Non-dominated solutions obtained by MPACOA and AMPACOA and with P2.

and significantly impacts the success of an MACO in solving MOPs. Further, global elitism provides the most effective information for pheromone updating in MACOs.

We also propose a simple adaptation strategy to control the effect of elitism. With this strategy, newly obtained non-dominated solutions are given a greater role in defining the pheromone information compared with older solutions. The obtained results on the tested mTSPs indicate strong performance of elitist MACO when using the adaptive strategy compared to the non-adaptive version. It is important for future work to analyze algorithm performance in greater detail in order to get a better understanding of how benefits are attained from using the adaptive strategy. There are many other ways in which an adaptive strategy could be implemented and future work will also investigate how an effective yet diverse search can be achieved by adaptively grouping and cycling through solutions from the global archive as they are used in pheromone information updating.

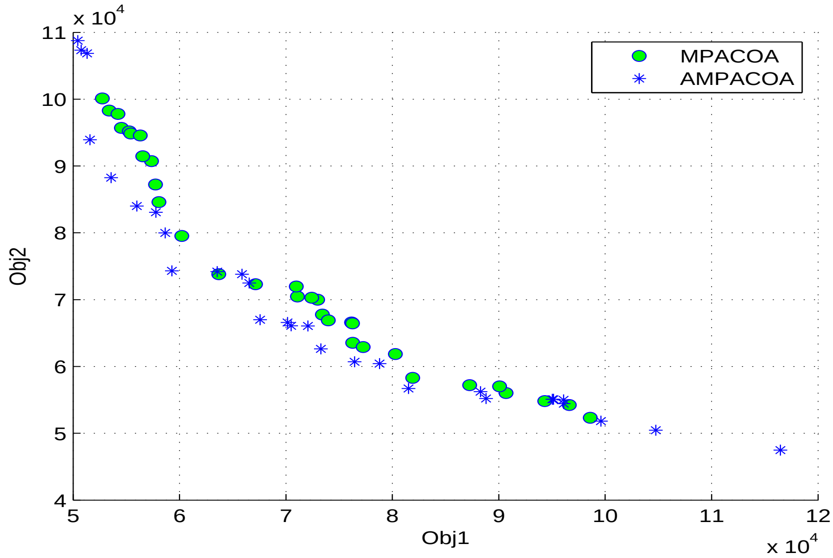

Fig. 9. Non-dominated solutions obtained by MPACOA and AMPACOA and with P3.

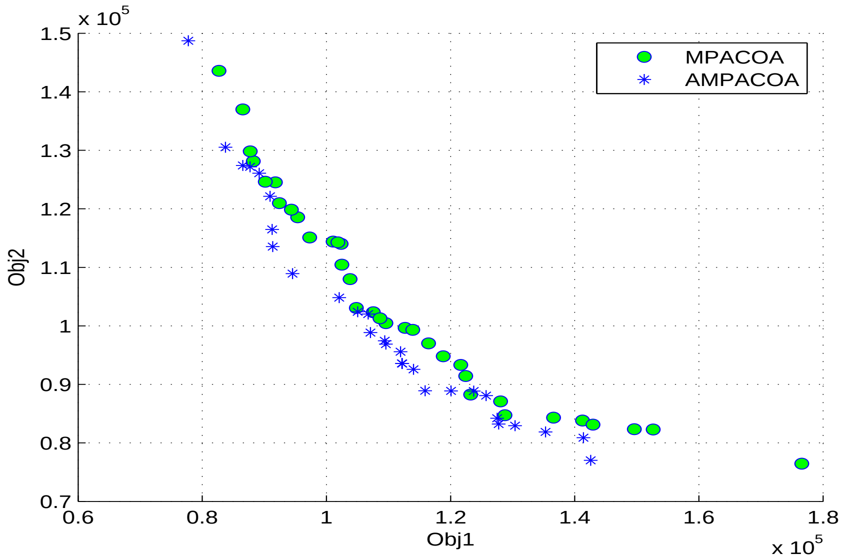

Fig. 10. Non-dominated solutions obtained by MPACOA and AMPACOA and with P4.

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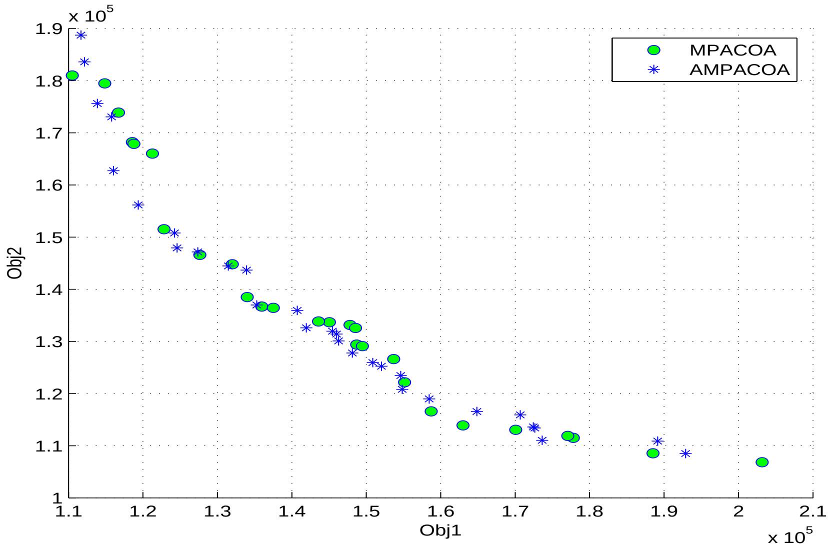

Fig. 11. Non-dominated solutions obtained by MPACOA and AMPACOA and with P5.

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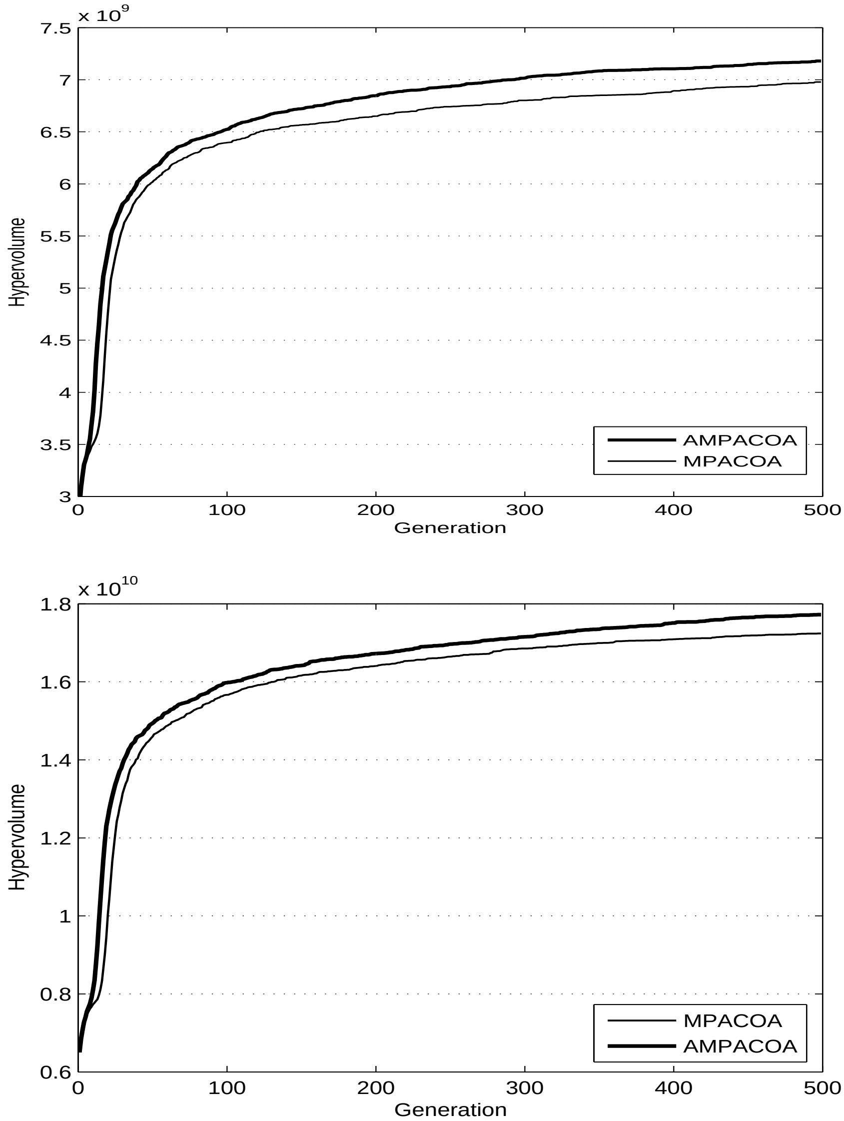

Fig. 12. Hypervolume values obtained by MPACOA and AMPACOA over time and with P1 and P5.