...{ MATHS }...

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Differential Equation

1. Ordinary differential eqn.
2. Partial differential eqn.

$$\Rightarrow \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^x$$

$$\Rightarrow y(m^2 + 2m + 1) = e^x$$

A.E. :- $m^2 + 2m + 1 = 0.$

$(m+1)^2 = 0.$

$m = -1, -1.$

C.F. :- $(c_1 + c_2 x)e^{-x}$

P.I. :- $\frac{e^x}{4}$.

P.D.E.

Homogeneous of Higher order | Non-Homogeneous

$$\Rightarrow a_0 \frac{\delta^n z}{\delta x^n} + a_1 \frac{\delta^{n-1} z}{\delta x^{n-1}\delta y} + a_2 \frac{\delta^{n-2} z}{\delta x^{n-2}\delta y^2} + \cdots + a_{n-1}\frac{\delta z}{\delta x \delta y^{n-1}} + a_n \frac{\delta z}{\delta y^n} = f(x,y).$$

$$\Rightarrow [a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} D^2 + \cdots + a_{n-1} D \cdot D^{n-1} + a_n D^n] z = f(x,y).$$

P.T.O.

complete solution = CF + PI.

$\rightarrow (D - m_1 D')(D - m_2 D') \cdots\cdots (D - m_n D') = f(x,y).$

$\Rightarrow D - m_1 D' = 0.$

$\Rightarrow p - m_1 q = 0.$

$$\frac{dx}{1} = \frac{dy}{-m_1} = \frac{dz}{0}, \Rightarrow dz = 0.$$
$$\Rightarrow z = c_2.$$

$\rightarrow \frac{dx}{1} = \frac{dy}{-m_1} \Rightarrow dy + m_1 dx = 0.$

Integrating,

$y + m_1 x = c_1.$

Sol.^{n} $z = \phi_1(y + m_1 x).$

$\longrightarrow$ (any arbitary function).

$\Rightarrow z = \phi_1(y + m_1 x) + \phi_2(y + m_2 x) + \cdots\cdots + \phi_n(y + m_n x).$

e.g. :- $D^2 + D \cdot D' - 6D'^2 = 0.$

$\Rightarrow$ ~~[illegible]~~

$\hookrightarrow$ Put $D = m$, $D' = 1$.

$\rightarrow m^2 + m - 6 = 0.$

$\Rightarrow m = -3, +2.$

$\rightarrow$ for Different roots.

$z = \phi_1(y + 3x) + \phi_2(y + 2x).$

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(Linear Homogeneous PDE)

P.I. :— $\phi(D, D')z = f(x, y)$.

$f(x, y) = \psi(\alpha x + \beta y)$.

→ e.g :— $\psi(\alpha x + \beta y) = \sin(\alpha x + \beta y)$.

$= \tan(\alpha x + \beta y)$.

$= e^{\alpha x + \beta y}$.

$$\Rightarrow \frac{1}{\phi(D, D')} f(x, y) = \frac{1}{\phi(D, D')} \psi(\alpha x + \beta y).$$

$$\Rightarrow \frac{1}{\phi(\alpha, \beta)} \psi(\alpha x + \beta y), \text{ provided } \phi(\alpha, \beta)$$

where $u = \alpha x + \beta y$

$$\Rightarrow \frac{1}{\phi(\alpha, \beta)} \underbrace{\int\int \cdots}_{n \text{ times}} \psi(u)\, \underbrace{du\, du \ldots du}_{(n \text{ times})}$$

E.g. :— $(D_x^2 + D_x D_y - 2D_y^2) Z = \sqrt{x + 2y}$.

CF :— $D^2 + DD' - 2D'^2 = 0$.

$m_1 = 1; \quad m_2 = -2$.

PI :—

$$Z_P = \frac{1}{\phi(D, D')} \psi(\alpha x + \beta y).$$

$$= \frac{1}{(D + 2D')(D - D')} \int\int \sqrt{u}\, du\, du.$$

$= D^2 + 2D \cdot D' - D \cdot D' - 2D'^2$

$= D(D + 2D') - D'(D + 2D')$

$= (D + 2D')(D - D')$

$$\Rightarrow \frac{1}{-5} \int \frac{u^{3/2}}{3/2} du.$$

$$\Rightarrow \frac{1}{5} \frac{u^{5/2}}{\frac{3}{2} \times \frac{5}{2}} = \frac{-4}{75} u^{5/2} \rightarrow$$

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Complete solution;

$z_c + z_p$

$= \phi_1(y+x) + \phi(y-2x) - \frac{4}{75}(x+2y)^{5/2}$.

II case of failures of $\phi(\alpha, \beta) = 0$.

$\psi(\alpha x + \beta y) = (\beta . D_x - \alpha D_y)^r g(D_x, D_y)$, $r \geq 1$.

$$z_p = \frac{1}{(\beta . D_x - \alpha D_y)} \times \frac{1}{g(D_x, D_y)} \psi(\alpha x + \beta y).$$

$$= \frac{x^r}{\beta^r \lfloor r} \times \left[\frac{1}{g(\alpha, \beta)} \underbrace{\iint}_{(n \text{ times})} \frac{u\,du\,du}{n \text{ times}} \cdots \right\} \rightarrow \text{form of } x.$$

$$= \frac{y^r}{(-\alpha)^r \lfloor r} \times \left[\text{---------}\ \text{''}\ \text{---------} \right] \Big\} \rightarrow \text{form of } y.$$

e.g. :– ① $(D_x^2 - 4D_x D_y + 4D_y^2) z = \tan(2x + y)$.

$\rightarrow \alpha = 2, \beta = 1.$

$\rightarrow D^2 - 4DD' + 4D'^2$.

$$PI :- \frac{1}{(D^2 - 4DD' + 4D'^2)} \psi(\alpha x + \beta y);\ \text{here } \alpha = 2;\ \beta = 1.$$

$$= \frac{1}{(D - 2D')^2},\ r = 2.$$

put in formula;
$$\frac{x^2}{\beta^2 \lfloor 2} \left[\frac{1}{g(\alpha, \beta)} \iint u\,du\,du \cdots n \text{ times} \right]$$

$\Rightarrow \frac{x^2}{1^2 \times L^2} \tan(\alpha x + \beta y)$. here, $g(Dx, Dy) = 1$.

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$\Rightarrow \frac{x^2}{1^2 \times L^2} \tan(\alpha x + \beta y)$.

$\Rightarrow \frac{x^2}{2} \tan(2x + y)$.

(2) $(D - 2D')(D + D')z = \tan(2x + y)$.

$\Rightarrow$ From CF :- $(2, -1)$.

$\alpha = 2 ; \beta = 1$.

$$z_p = \frac{1}{(D-2D')(D+D')} \tan(2x + y).$$

$(D-2D')$: (when $\alpha = 2; \beta = 1$ $= 0$)

$(D+D')$: (when $\alpha = 2; \beta = 1$ $\neq 0$.)

Hence, $\frac{x^1}{1'L1} \times \frac{1}{3} \iint \tan \; u \cdot du\,du$.

P.T.O.

Special Cases :-

1 : $\psi(\alpha x + \beta y) = e^{\alpha x + \beta y}$.

$$z_p = \frac{1}{\phi(Dx, Dy)} \psi(\alpha x + \beta y).$$

$$= \frac{1}{\phi(Dx, Dy)} \rightarrow \frac{1}{\phi(\alpha, \beta)} \cdot e^{\alpha x + \beta y}, \; \phi(\alpha, \beta) \neq 0.$$

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case II : $\psi(\alpha x+\beta y) = \sin(\alpha x+\beta y)$ or $\cos(\alpha x+\beta y)$

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put; $D_x^2 = -\alpha^2$.

$D_y^2 = -\beta^2$.

$D_x \cdot D_y = -\alpha\beta$.

ex :- ① $(3D_x^2 - 10D_x \cdot D_y + 3D_y^2)z = e^{2x-y}$.

$\rightarrow 3D^2 - 10DD' + 3D'^2 = 0$.

$\rightarrow 3D^2 - 9DD' - 1DD' + 3D'^2 = 0$.

$\rightarrow 3(D^2 - 3DD') - 3(1DD' - D'^2) = 0$.

$$z_p = \frac{1}{3D^2 - 10DD' + 3D'^2} \cdot e^{2x-y}.$$

$$= \frac{1}{3(2)^2 - 10(2)(-1) + 3(-1)^2} e^{2x-y}. \quad (D=2;\ D'=-1).$$

$$= \frac{1}{12+20+3} e^{2x-y}.$$

$$= \frac{1}{35} \cdot e^{2x-y}.$$

② $\frac{1}{3D^2 - 10DD' + 3D'^2} = 10\sin(2x+y) + e^{(2x-y)}$

$\rightarrow$

③ $r + 2s + t = e^{2x+3y}$.

$\rightarrow r = \frac{\delta^2 z}{\delta x^2}, \quad s = \frac{\delta^2 z}{\delta x\, y}; \quad t = \frac{\delta^2 z}{\delta y^2}$.

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Ex 1 :- $(D^2 - 3DD' + 2D'^2)z = e^{2x+3y} + e^{x+y} + \sin(x-2y)$.

Ex 2 :- $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{2x+y}$.

CF :- put $D=m;\ D'=1$.

PI :- $D^2 - 3DD' + 2D'^2 = D^2 - 2D \cdot D' - DD' + 2D'^2$.

$= D(D-2D') - D'(D-2D')$.

$= (D-D')(D-2D')$.

Then,

$$z_p = \frac{1}{(D-D')(D-2D')} e^{2x+3y} + \frac{1}{(D-D')(D-2D')} e^{x+y} + \frac{1}{D^2 - 3DD' + 2D'^2} \sin(x-2y).$$

$$\Rightarrow \frac{e^{2x+3y}}{(2-3)(2-6)} + \frac{x}{1 \times 1} \times \frac{1}{D-2D'} e^{x+y} + \frac{\sin(x-2y)}{-1-3(-2)+2(-4)}.$$

Ex 3 :- $D^3 - 7D \cdot D'^2 - 6D'^3$.

Sol :- $\frac{1}{D^3 - 7D \cdot D'^2 - 6D'^3} \sin(x+2y) + \frac{1}{D^3 - 7D \cdot D'^2 - 6D'^3} e^{2x+y}$.

$$\Rightarrow \frac{1}{D \cdot D^2 - 7D \cdot D'^2 - 6D' D'^2} \sin(x+2y) + \frac{e^{2x+y}}{2^3 - 7 \cdot 2 \cdot 1 - 6 \cdot 1^3}.$$

$$\Rightarrow \frac{1}{D(-1) - 7(-2) - 6D'(-4)} \sin(x+2y) + \frac{e^{2x+y}}{2^3 - 7 \cdot 2 \cdot 1 - 6 \cdot 1^3}.$$

$$\Rightarrow \frac{(D + 24D' + 14)}{-(D - 24D' - 14)(D + 24D' + 14)} \sin(x+2y) + \frac{e^{2x+y}}{2^3 - 7 \cdot 2 \cdot 1 - 6 \cdot 1^3}.$$

$\rightarrow$ P.T.O.

SPECIAL CASE III :– $f(x,y) = x^m y^n$, $m, n > 0$.

RULE :– if $m > n$, then expand the binomial in the ascending form of D_y.

if $m < n$, then expand the binomial in the ascending form of D_x.

if $m = n$, then expand the binomial in the ascending form of D_x & D_y.

e.g. :– $(D_x^3 - D_y^3) z = x^2 y$

Sol :– put $D_x = m$; $D_y = 1$.

$\rightarrow D_x^3 - D_y^3 \Rightarrow m^3 - 1 = 0$.

$\therefore m = 1, \omega, \omega^2$.

$\Rightarrow (1-x)^{-1} = 1 + x + x^2 + \cdots$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$

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Ex1 :– $(D_x^3 - D_y^3) z = x^2 y \Rightarrow (D_x^3 - D_y^3) z$.

Sol :– $z_p = \dfrac{1}{(D_x^3 - D_y^3)} x^2 y$

$= \dfrac{1}{D_x^3 \left(1 - \frac{D_y^3}{D_x^3}\right)} x^2 y$

$= \dfrac{1}{D_x^3} \left\{1 - \dfrac{D_y^3}{D_x^3}\right\}^{-1} x^2 y$.

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$

$= \cancel{\left[1 + \left(\frac{D_y}{D_x}\right)^3 + \left(\frac{D_y}{D_x}\right)^6 \cdots\right] x}$

$\Rightarrow \dfrac{1}{D_x^3} \left[1 + \left(\dfrac{D_y^3}{D_x^3}\right) + \left(\dfrac{D_y^3}{D_x^3}\right)^2 + \cdots\right] x^2 y$

$\Rightarrow \dfrac{1}{D_x^3} \left[x^2 y + \left(\dfrac{D_y^3}{D_y^3}\right) x^2 y\right]$

$\Rightarrow \dfrac{1}{D^3 x} (x^2 y)$.

integrate three times, with respect to x;

$\iiint \dfrac{1}{D^3 x} (x^2 y) = \dfrac{(x^5 y)}{(60)}$.

Ex 2 :– $(D_x^2 - D_y^3) z = x^2 y^3$.

Sol :– $\dfrac{x^2 y^3}{D_x^2 \left\{1 - \left(\frac{D_y}{D_x}\right)^3\right\}}$.

$\Rightarrow \left[1 - \left(\dfrac{D_y}{D_x}\right)\right]^{-2} \dfrac{1}{D_x^2} x^2 y^3$.

$\Rightarrow \left[1 + \left(\dfrac{D_y}{D_x}\right)^3 + \left(\dfrac{D_y}{D_x}\right)^6 \cdots\right] \dfrac{x^2 y^3}{D_x^2}$.

$\Rightarrow \dfrac{1}{D_x^2} \left[x^2 y^3\right]$.

Here, $m = 2$; $n = 3$; $m < n$.

P.I. :– $\dfrac{-1}{D_y^2} \left[1 - \dfrac{D_x^2}{D_y^2}\right]^{-1} x^2 y^3 = \dfrac{1}{D^2 y} \left[1 + \dfrac{D^2 x}{D^2 y} + \left(\dfrac{D^2 x}{D^2 y}\right)^4 \cdots\right] x^2 y^3$.

general form of P.I. :–

$\phi(D_x, D_y) = f(x, y)$.

$\phi(D_x, D_y) = (D_x - \alpha_1 D_y)(D_x - \alpha_2 D_y) \cdots$

P.I. :– $\dfrac{1}{D_x - \alpha_1 D_y} f(x, y) = \int f(x, c - \alpha_1 x)$

where $y = c - \alpha_1 x$.

Ex 1 :– $\{D_x^2 + D_x \cdot D_y - 6 D^2 y\} z = y \sin x$.

Sol :– $D_x^2 + D_x \cdot D_y - 6 D^2 y = (D_x + 3D_y)(D_x - 2D_y)$.

P.I. :– $z_p = \dfrac{1}{(D_x + 3D_y)(D_x - 2D_y)} \cdot y \sin x$

$= \dfrac{1}{(D_x + 3D_y)} \int f(x, c - \alpha_1 x) \cdot dx$

$\boxed{\text{Here } \alpha_1 = 2}$

$f(x,y) = y\sin x$.

$f(x, c-\alpha_1 x) = (c-\alpha_1 x)\sin x$.

$\Rightarrow \frac{1}{D_x+3D_y}\int (c-2x)\sin x \, dx$.

$\Rightarrow z_p = \frac{1}{D_x+3D_y}\left[(c-2x)(-\cos x) - \frac{2\cos x}{\sin x}\right]$

$= \frac{y}{D_x+3D_y}\left[-y\cos x - \frac{2\cos x}{\sin x}\right]$.

$= \int f(x, c+3x)\,dx$, where $f(x,y) = -y\cos x - 2\sin x$.

Non-Homogeneous Linear Equation with constant coefficients :–

$\phi(D_x, D_y) = (a_1D + b_1D' + C_1)(a_2D + b_2D' + C_2) \text{-----} \text{-----}(a_nD + b_nD' + C_n)$.

For CF ;

$a_1D + b_1D' + C_1 = 0 \therefore \Rightarrow a_1D + b_1D' = C_1 z$.

$\frac{dx}{a_1} = \frac{dy}{b_1} = \frac{dz}{c_1}$

Take first two terms ;

$\frac{dx}{a_1} = \frac{dy}{b_1}$.

$\Rightarrow b_1x - a_1y = d_1$.

Take first & last terms ;

$\frac{dx}{a_1} = \frac{dz}{-c_1 z}$.

$\Rightarrow -c_1 x = a_1 \log z + \log dz$.

$\Rightarrow -\frac{c_1}{a_1}x = \log(z\,dz)$.

$\Rightarrow e^{-c_1/a_1 x} = z \cdot dz$.

$\Rightarrow z e^{\frac{c_1}{a_1}x} = dz$.

$Z e^{\frac{c_1}{a_1}x} = \phi_1(b_1x - a_1y)$.

formula ;

$z = e^{-\frac{c_1}{a_1}x}\phi_1(b_1x - a_1y) + e$.

or; $z = e^{-\frac{c_1}{b_1}y}\phi_1(b_1x - a_1y) - e$.

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Non-Homogeneous Linear Equations :–

$\phi(D, D')z = f(x,y)$ — can be factorized linearly — non repeated / repeated. — can't be factorized.

For CF ;

$\rightarrow z = e^{-\frac{c_1}{a_1}x}\phi_1(b_1x - a_1y) + e^{-\frac{c_2}{a_2}x}\phi_2(b_2x - a_2y) + \text{------}$

repeated case $z = e^{-\frac{c_1}{a_1}x}\left[\phi_1(b_1x - a_1y) + x\phi_2(b_1x - a_1y)\right.$

$\left. + \text{----------}\right]$

Ex:1 :– $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$.

Ex:2 :– $s + p = q + z$.

Ex:1 :– solution :– $D^2 + 2DD' + D'^2 - 2D - 2D' = 0$.

$D = m$; $D' = 1$.

$m^2 + 2m(1) + (1)^2 - 2m - 2(1) = 0$.

$m^2 + 2\cancel{m} - 2\cancel{m} - 2 = 0$.

$m^2 = 2$.

$m = \pm\sqrt{2}$

factors :– $(D+D')(D+D'-2)z = \sin(x+2y)$.

For $(D+D')z = 0$, $a_1 = 1$, $b_1 = 1$, $c_1 = 0$.

$z = e^{-c_1/a_1 x}\phi_1(b_1x - a_1y)$.

$z = e^{-0}\phi_1(x-y)$.

$= \phi(x-y)$.

For ; $(D+D'-2)z = 0$.

here, $a_2 = 1$; $b_2 = 1$; $c_2 = -2$.

$z = e^{+2x}\phi_2(x-y)$.

$\therefore$ Sol^n is $z = \phi_1(x-y) + e^{2x}\phi_2(x-y)$.

Ex:2 :– $s + p = q + z$.

Sol :– $s = D_x \cdot D_y$.

$D_xD_y + D_x = D_y + z$.

$D_xD_y + D_x - D_y - z = 0$.

$\Rightarrow D_x D_y + D_x - D_y - z = 0.$

$\Rightarrow D_x(D_y+1) - 1(D_y+1)z = 0.$

$\Rightarrow (D_y+1)(D_x-1)z = 0.$

For $(D_y+1)z=0$, here $a_1=0, b_1=1, c_1=-1$.

For $(D_x-1)z=0$, here $a_2=1, b_2=0, c_2=-1$.

sol :- $y = e^{x}\phi_1(y) + e^{-y}\phi_2(x)$.

Ex:3 :- $(D_x - D_y)(D_x + D_y + 1)z = 0.$

Sol :- $D_x - D_y = 0$.

$a_1 = 1;\ b_1 = -1;\ c_1 = 0.$

$z = e^{0}\phi(-x-y).$

$z = \phi(-x-y).$

$$\begin{cases} p = D_x \\ q = D_y \\ r = D_{xx} \\ s = D_x D_y \\ t = D^2 y \end{cases}$$

SOLVE :-

(1) $4r + 12s + 9t = 0.$

(2) $\frac{\delta^2 z}{\delta x^2} - \frac{\delta^2 z}{\delta x \delta y} = \cos x \cos 2y.$

(3) $4\frac{\delta^2 z}{\delta x^2} - 4\frac{\delta^2 z}{\delta x \delta y} + \frac{\delta^2 z}{\delta y^2} = 16\log(x+2y).$

(4) $\frac{\delta^3 z}{\delta x^3} - 4\frac{\delta^3 z}{\delta x^2 \delta y} + 5\frac{\delta^3 z}{\delta x \delta y^2} - 2\frac{\delta^3 z}{\delta z^3} = e^{2x+y}.$

(5) $\frac{\delta^3 z}{\delta x^3} - 4\frac{\delta^2 z}{\delta x \delta y} + 2\frac{\delta^2 z}{\delta y^2} = e^{2x-y} + e^{x+y} + \cos(x+2y).$

* ...FOR PI...*

case IV :- $\phi(D, D')z = f(x,y)$.

when $f(x,y) = e^{\alpha x + \beta y} V(x,y)$.

$$z_p = \frac{1}{\phi(D_x, D_y)} e^{\alpha x + \beta y} \cdot V(x,y).$$

$$z_p = e^{\alpha x + \beta y} \frac{1}{(D_x + \alpha, D_y + \beta)} V(x,y).$$

$\{a = 1;\ b = -3;\ c = -2.\}$

Ex1 :- $(D_x - 3D_y - 2)^2 z = 4e^{2x}\tan(3x+y)$.

Sol :- CF; $z = e^{2x}[\phi_1(x-y) + x\phi_2(x-y)]$.

PI; here, $\alpha = 2;\ \beta = 0;\ V(x,y) = \tan(3x+y)$.

$$z_p = \frac{1}{(D-3D'-2)} e^{2x}\tan(3x+y).$$

$$z_p = \frac{4e^{2x}}{(D+2-3D'-2)^2}\tan(3x+y).$$

$$z_p = \frac{e^{2x}}{(D-3D)'} x\, y \tan(3x+y).$$

$$= 4e^{2x}\frac{x^2}{1^2 \times L_2}\tan(3x+y).$$

$$= 4e^{2x}\cdot\frac{x^2}{2}\, x \tan(3x+y). \Rightarrow 2e^{2x} * x^2\tan(3x+y).$$

Ex:1 :- (1) $4r + 12s + 9t = 0$.

Sol :- PI = 0.

For CF :- $\Rightarrow 4D^2 + 12D.D' + 9D'^2 = 0.$

$\rightarrow (2D + 3D')^2 = 0.$

$\rightarrow$ here $a_1 = 2;\ b_1 = 3;\ c_1 = 0.$

$\rightarrow z = \phi_1(3x-2y) + x\,\phi_2(3x-2y).$

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* Method of separation of Variables :-

$\Rightarrow \frac{2y''}{y} = K$

Integrating;

$= Be^{-}$

$z(x,y) = X(x)Y(y) = xy.$

$\frac{\delta z}{\delta x} = x'(x)y(y) = x'y.$

$\frac{\delta z}{\delta y} = xy'.$

Eg :- Solve $3\frac{\delta z}{\delta x} + 2\frac{\delta z}{\delta y} = 0$, $z(x,0) = 4e^{-x}$.

Ans. Put $z(x,y) = X(x)\,Y(y)$

$\frac{\partial z}{\partial x} = x'y$, $\frac{\partial z}{\partial y} = xy'$

$\Rightarrow 3(x'y) + 2(xy') = 0$

$\Rightarrow 3x'y = -2xy'$

$\Rightarrow \frac{3x'}{x} = \frac{-2y'}{y} = k$ (const.) say

$\Rightarrow \frac{3x'}{x} = k \Rightarrow \frac{3\frac{dx}{dx}}{x} = k, \Rightarrow \frac{3dx}{x} = k\,dx$

$\int^n$, we get

$\log x_1 = \frac{k}{3}x + c_1$

$= \frac{k_1}{3}x + c_1$

$\Rightarrow x = e$

$= e^{c_1} e^{k/3\,x} = A e^{k/3\,x}$, where $A = e^{c_1}$

Hlly,

$\frac{2y'}{y} = -k$

$\int^n$, we get,

$\log y = \frac{-k}{2}y + c_2$

$y = e^{-k/2\,y + c_2} = Be^{-k/2\,y}$ where $B = e^{c_2}$

Complete solution $z(x, y) = xy$

$= Ae^{k/3\,x} \, Be^{-k/2\,y} = Ce^{k(x/3 - y/2)}$

By the given cond^n

$z(x, 0) = 4e^{-x}$

$\Rightarrow 4e^{-x} = Ce^{k(x/3 - 0/2)}$

$\Rightarrow c = 4, \; k\left(\frac{x}{3}\right) = -1$

$\Rightarrow k = -3 \Rightarrow$ sol^n $= 4e^{-3(x/3 - y/2)}$

(2). Solve; $\frac{\delta u}{\delta x} = 2\frac{\delta u}{dt} + u, \; u(x, 0) = 6e^{-3x}$

Solve

(1). $\frac{\delta^2 z}{\delta x^2} - 2\frac{\delta z}{\delta x} + \frac{\delta z}{\delta y} = 0$

(2). $4\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} = 3u$

$u(0, y) = 3e^{-y} - e^{-5y}$

(3). $\frac{\delta u}{\delta x} = \frac{\delta u}{\delta t} + u,$

$u(x, 0) = 3e^{-4x}$

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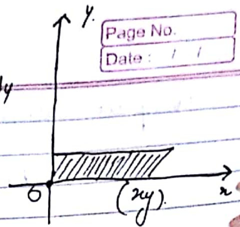
① Heat eqn. in one dimension;

$$\frac{\delta u}{\delta t} = \frac{c^2 \delta^2 u}{\delta x^2}, \; c^2 = \text{constant of diffusivity of material.}$$



② Laplace eqn. in two dimension;

$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0.$$

③ Wave equation;

$$\frac{\delta^2 u}{\delta t^2} = c^2 \frac{\delta^2 u}{\delta x^2}.$$

$$\frac{\delta u}{\delta t} = c^2 \frac{\delta u}{\delta x^2} \quad \text{—} ①$$

put $u(x,t) = x(x)\,t(t)$.

$$\frac{\delta u}{\delta t} = XT', \; \frac{\delta^2 u}{\delta x^2} = x''T.$$

From ①; $XT' = e^2 X''T$.

$$\Rightarrow \frac{X''}{X} = \frac{T'}{e^2 T} = K \text{ (say)}.$$

CASE 1 :- $K = 0$.

$\frac{X''}{X} = 0 \Rightarrow X'' = 0.$

$\Rightarrow X' = C_1.$

$\Rightarrow X = C_1 x + C_2.$

$\frac{T'}{e^2 T} = 0. \quad T' = 0.$

$T = C_3.$

Soln :- $u(x,t) = XT = C_3(C_1 x + C_2).$

CASE 2 :- when $K = p^2$.

$\frac{X''}{X} = p^2.$

$\Rightarrow X'' = p^2 X$

$(D^2 - p^2)X = 0.$

A.E.; $m^2 - p^2 = 0.$

$\Rightarrow m = \pm p.$

$\Rightarrow X = C_1 e^{px} + C_2 e^{-px}.$

P.T.O.

Soln :- $u(x,t) = XT = A(C_1 e^{px} + C_2 e^{-px}) c^2 e^{p^2 t}.$

$\Rightarrow \frac{T'}{e T} = p^2 p^2 t.$

$\Rightarrow T = A c^2 e.$

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CASE 3 :- $k = -p^2$.

$$\frac{X''}{X} = -p^2.$$

$\Rightarrow X'' + p^2 X = 0.$

A.E. $\Rightarrow m^2 + p^2 = 0.$

$\Rightarrow m = \pm ip.$

Sol :- $X = C_1 \cos px + C_2 \sin px.$

complete soln :-

$u(x,t) = XT.$

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...MATHS...

$u(0,t) = 0 = u(l,t).$

$u(x,0) = u_0.$

$u(x,t) = \sum b_n \sin \frac{n\pi x}{l} \, e^{-\frac{c^2 \pi^2 n^2}{l^2} t}.$

$u_0 = \sum b_n \sin \frac{n\pi x}{l}.$

$$b_n = \frac{2}{l}\int_0^l u_0 \cdot \sin \frac{n\pi x}{l} \cdot dx = \frac{2u_0}{l}\left[\frac{\cos n\pi x}{l}\right]_0^l \times \frac{l}{n\pi}$$

$$= \frac{2u_0}{n\pi}(1 - \cos n\pi) = \begin{cases} \frac{4u_0}{n\pi}, & n \text{ odd.} \\ 0, & n \text{ even.} \end{cases}$$

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* Wave Eqn :-

$$\frac{\delta^2 u}{\delta t^2} = c^2 \cdot \frac{\delta^2 u}{\delta t^2}$$

put $u(x,t) = X(x)T$.

$XT'' = c^2 X'' T$.

$\Rightarrow \frac{X''}{X} = \frac{T''}{c^2 T} = \lambda$ {say}.

$\Rightarrow \frac{X''}{X} = \lambda$, $\frac{T''}{c^2 T} = \lambda$.

CASE 1 :- $\lambda = 0$.

$\frac{X''}{X} = 0$, $\frac{T''}{c^2 T} = 0$.

$\Rightarrow x = 0$; $T = d_1 t + d_2$.

$\Rightarrow x = c_1 x + c_2$.

$u(x,t) = (c_1 x + c_2)(d_1 t + d_2)$.

CASE 2 :- $\lambda = p^2$.

$\frac{X''}{X} = p^2$; $\frac{T''}{c^2 T} = p^2$.

$\Rightarrow X'' = p^2 X$.

$\Rightarrow X'' - p^2 X = 0$

A.E. :- $M^2 - p^2 = 0$.

$M = \pm p$.

$X(x) = c_1 e^{px} + c_2 e^{-px}$.

$T'' = c^2 p^2 T$.

$\rightarrow m = \pm cp$.

$\therefore T(t) = d_1 e^{cpt} + d_2 e^{-cpt}$.

Then, soln. is $u(x,t) = (c_1 e^{px} + c_2 e^{-px})(d_1 e^{cpt} + d_2 \cdot e^{-cpt})$.

CASE 3 :- $\lambda = -p^2$.

LAPLACE EQN :-

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$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0.$$

$u(x,y) = X(x) \cdot Y(y)$.

$X'' Y + Y'' X = 0$.

$X'' Y = -Y'' X$.

$\Rightarrow \frac{X''}{X} = \frac{-Y''}{Y} = \lambda$ (say).

CASE 1 :- $\lambda = 0$.

$\frac{X''}{X} = 0$, $\frac{-Y''}{Y} = 0$.

$\Rightarrow X = c_1 x + c_2$, $Y = d_1 y + d_2$.

Sol :- $u(x,y) = (c_1 x + c_2)(d_1 y + d_2)$.

CASE 2 :- $\lambda = p^2$.

$\frac{X''}{X} = p^2$; $\frac{-Y''}{Y} = p^2$.

$X'' - p^2 X = 0$; $Y'' + p^2 Y = 0$.

Sol :- $u(x,y) = (c_1 e^{px} + c_2 e^{-px})(d_1 \sin py + d_2 \cos py)$.

CASE 3 :- $\lambda = -p^2$.

$\frac{X''}{X} = -p^2$; $\frac{-Y''}{Y} = -p^2$.

$X = (c_1 \sin px + c_2 \cos px)(d_1 e^{py} + d_2 e^{-py})$.

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Q.1. Solve the laplace eqn :— $\frac{\delta^2 u}{\delta x^2} + \delta^2 u = 0.$

subject to conditions.

$u(0, y) = u(l, y) = 0.$

$u(x, 0) = 0$ &

$u(x, a) = \sin \pi x\, l.$

Sol :- when put $= u(0,y) = (c_1 + c_2)(d_1 \sin py + d_2 \cos py).$

$u(0,y) = 0 \Rightarrow c_2(d_1 e^{py} + d_2 e^{-py}) = 0.$

$\Rightarrow c_2 = 0.$

$u(l, y) = 0 \Rightarrow c_1 \sin pl\, (d_1 e^{py} + d_2 e^{-py}) = 0.$

$\Rightarrow \sin pl = 0 = \sin n\pi.$

$\Rightarrow p = \frac{n\pi}{l}.$

$u(x, y) \Rightarrow b_n \sin \frac{n\pi x}{l} \left(e^{\frac{n\pi y}{l}} \cdot e^{-\frac{n\pi y}{l}}\right).$