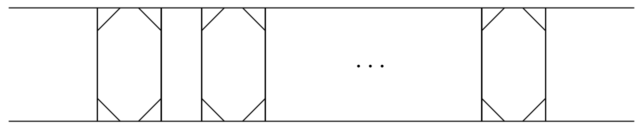

Figure 1: A guillotine subdivision with $n = 6k + 2$ segments that requires $4k$ 0-transmitters. Each of the $4k$ triangular faces must have a 0-transmitter on its boundary and no two triangular faces share a boundary.

Guillotine Subdivisions

A *guillotine subdivision* S is obtained by inserting a sequence $s_1, \ldots, s_n$ of line segments. Each inserted segment s_i splits a face of the current subdivision S_{i-1} into two new faces yielding a new subdivision S_i.

As the example in Figure 1 shows, a guillotine subdivision with n segments can require $2(n-2)/3$ 0-transmitters. In this section, we show that no guillotine subdivision requires more than $(n+1)/2$ 1-transmitters. We begin with a lemma:

Lemma 1. *Let F be a face in a guillotine subdivision S. If there are 1-transmitters on every face that shares an edge with F then these 1-transmitters see all of F.*

Proof. Consider the segment s_i whose insertion created the face F. Before the insertion of s_i, the subdivision S_{i-1} contained a convex face that was split by s_i into two faces F and F' (Figure 2.a). No further segments were inserted into F, but F' may have been further subdivided, so that there are now several faces $F'_1, \ldots, F'_k$, with $F'_j \subseteq F'$ and F'_j incident on s for all $j \in \{1, \ldots, k\}$ (Figure 2.b).

We claim that the 1-transmitters in $F'_1, \ldots, F'_k$ guard the interior of F. To see this, imagine removing s_i from the subdivision and instead, constructing a guillotine subdivision $\tilde{S}$ from the sequence $s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n$ (Figure 2.c). In this case, each face F'_j in S becomes a larger face $\tilde{F}'_j$ in $\tilde{S}$ and together $\bigcup_{j=1}^{k} \tilde{F}'_j \supseteq F$. Finally, we observe that each 1-transmitter in S in face F'_j guards at least $\tilde{F}'_j$, so together, the 1-transmitters in $F'_1, \ldots, F'_k$ guard all of F (Figure 2.d). □

Theorem 1. *Any guillotine subdivision can be guarded with at most $(n + 1)/2$ 1-transmitters.*

Proof. Consider the dual graph T of the subdivision. T is a triangulation with $n + 1$ faces. Let M be any maximal matching in T. Consider the unmatched vertices of T. Each such vertex is adjacent only to matched vertices (otherwise M would not be maximal). Let G be the set of 1-transmitters obtained by placing a single 1-transmitter on the primal edge associated with each edge $e \in M$. Then $|G| = |M| \leq (n+1)/2$. For every face F of S, F either contains a 1-transmitter in G, or all faces that share an edge with F contain a 1-transmitter in G. In the former case, F is obviously guarded. In the latter case, Lemma 1

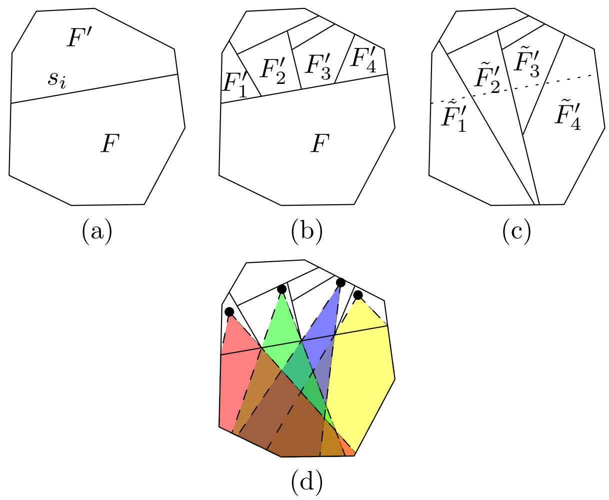

Figure 2: The proof of Lemma 1.

ensures that F is guarded. Therefore, G is a set of 1-transmitters that guards all faces of F and has size at most $(n+1)/2$. $\square$