Part 1 :

1. $$y = \sigma(x) = \frac{1}{1+e^{-x}}$$

1.(a) : $$\frac{\partial \sigma(x)}{\partial x} = \frac{d}{dx}\frac{1}{1+e^{-x}} = \frac{d}{dx}(1+e^{-x})^{-1}$$

$$= -(1+e^{-x})^{-2} \cdot -(e^{-x}) = (1+e^{-x})^{-2}(e^{-x})$$

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$$

$$= \sigma(x) \cdot (1-\sigma(x))$$ ※

1.(b) : $$\sigma(-x) = \frac{1}{1+e^{x}}$$

$$= \frac{\frac{1}{e^x}}{\frac{1}{e^x}+\frac{e^x}{e^x}} = \frac{e^{-x}}{1+e^{-x}} = 1 - \frac{1}{1+e^{-x}}$$

$$= 1 - \sigma(x)$$ #

1.(c) : $$y = \frac{1}{1+e^{-x}}$$

$$\Rightarrow \frac{1}{y} = 1+e^{-x}$$

$$\Rightarrow e^{-x} = \frac{1}{y} - 1 = \frac{1-y}{y}$$

$$\Rightarrow e^{x} = \frac{y}{1-y}$$

$$\Rightarrow \ln(e^x) = \ln\left(\frac{y}{1-y}\right)$$

$$\Rightarrow x = \ln\left(\frac{y}{1-y}\right)$$ #

2. $\begin{cases} \hat{y}_n = \sigma(W^T \phi_n) \\ L(W) = -\sum_{n=1}^{N} \{ y_n \ln(\hat{y}_n) + (1-y_n)\ln(1-\hat{y}_n) \} \\ \frac{\partial \sigma(x)}{\partial x} = \sigma(x)\cdot(1-\sigma(x)) \end{cases}$

$$\nabla L(W) = \left[\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_2}, \dots, \frac{\partial L}{\partial W_N} \right]$$

先推導 $L(W)$ 對單一權重參數 W_i 的導數 $\frac{\partial L}{\partial W_i}$，

$$\frac{\partial L}{\partial W_i} = \left(\frac{\partial L}{\partial \hat{y}_n}\right)\cdot\left(\frac{\partial \hat{y}_n}{\partial W_i}\right), \quad \frac{\partial L}{\partial W_1} = \left(\frac{\partial L}{\partial \hat{y}_n}\right)\cdot\left(\frac{\partial \hat{y}_n}{\partial W_1}\right)$$

首先，$\frac{\partial L}{\partial \hat{y}_n} = -\frac{y_n}{\hat{y}_n} + \frac{1-y_n}{1-\hat{y}_n}$

$$= \frac{\hat{y}_n - y_n}{\hat{y}_n(1-\hat{y}_n)}$$

$$\frac{\partial \hat{y}_n}{\partial W_1} = \sigma(W^T\phi_n)\cdot(1-\sigma(W^T\phi_n))\cdot\frac{\partial(W^T\phi_n)}{\partial W_1}$$

$= \hat{y}_n\cdot(1-\hat{y}_n)\cdot\phi_{n,i}$，其中 $\phi_{n,i}$ 是輸入特徵向量 ϕ_n 的第 i 個元素。

再將兩個偏微分代回 $\frac{\partial L}{\partial W_i}$ 中，

$$\frac{\partial L}{\partial W_1} = \left(\frac{\partial L}{\partial \hat{y}_n}\right)\cdot\left(\frac{\partial \hat{y}_n}{\partial W_1}\right)$$

$$= \frac{\hat{y}_n - y_n}{\cancel{\hat{y}_n(1-\hat{y}_n)}}\cdot\cancel{\hat{y}_n(1-\hat{y}_n)}\cdot\phi_{n,i}$$

$$= (\hat{y}_n - y_n)\cdot\phi_{n,i}$$

$$\therefore \nabla L(W) = \left[\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_2}, \dots, \frac{\partial L}{\partial W_N} \right]$$

$$= \sum_{n=1}^{N} (\hat{y}_n - y_n)\cdot\phi_n$$

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3.(a)

第2題的 Loss function 是應用在 $0 \le \hat{y_n} \le 1$ 且 label $y_n \in \{0,1\}$ 時，
$\hat{y_n}$、y_n 都不包含負值，所以可以使用包含 log 的 cross-entropy 的 $L(w)$，
但此題 $-1 \le \hat{y_n} \le 1$ 且 label $y_n \in \{-1,1\}$，若 $L(w)$ 中有 log 可能導致出錯，
所以若使用 MSE 或 RMSE 等方法，

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y_i})^2$$

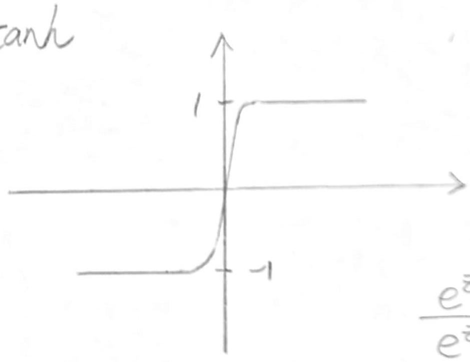
$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y_i})^2}$$

則不包含 log 即可。

3.(b)

需選擇 $-1 \le \hat{y_n} \le 1$ 的激勵函數，例如

tanh



$$\frac{e^z - e^{-z}}{e^z + e^{-z}}$$

或 SELU、ELU 等。