



E6318 - Microwave Circuit Design

Columbia University

Spring 2006

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Outline of Lecture 4 & 5

- ◆ Short recap of lecture 3 (practical TL's),
- ◆ Discuss material not seen last time: μ -strip, CPW, wave velocity, power capacity
- ◆ Network analysis
 - Impedance and admittance matrix
 - Scattering matrix
 - Calculating S-parameters, signal flow graphs
- ◆ Impedance matching and tuning
 - Matching with lumped elements

Calendar

◆ Course: Th 4:10-6:40 PM, 1127 Mudd

- | | |
|------------------------|-------------------------|
| ■ 01/19 | ■ 03/16 Spring Holidays |
| ■ 01/26 | ■ 03/23 |
| ■ 02/02 rescheduled | ■ 03/30 |
| ■ 02/09 | ■ 04/06 |
| ■ 02/16 | ■ 04/13 |
| ■ 02/23 | ■ 04/20 |
| ■ 03/02 | ■ 04/27 |
| ■ 03/09 Midterm | ■ Final (05/11) |

Recap practical transmission lines

- ♦ **TEM:** $\beta = \omega \sqrt{\mu\epsilon} = k$ $k_c = 0$ $v_p = c / \sqrt{\epsilon_r}$
 - the transverse fields of TEM wave are same as static fields, 2 or more conductors needed, no TEM in closed conductor
 - voltage, current and impedance well-defined

- ♦ **TE or TM:**
 - closed conductor or higher order modes TEM
 - propagation constant β dependent frequency & geometry

$$\beta = \sqrt{k^2 - k_c^2} \quad k = \omega \sqrt{\mu\epsilon} = 2\pi / \lambda$$

- ♦ **quasi-TEM** (different ϵ_r under and above line)
 - leads to concept of effective dielectric constant

$$\beta = \frac{\omega}{v_p} = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_e} = \sqrt{\epsilon_e} k_0 \quad v_p = c / \sqrt{\epsilon_e}$$

Recap practical transmission lines (2)

- ◆ Attenuation in transmission lines

$$\boxed{\alpha = \alpha_d + \alpha_c} \xrightarrow{\alpha_c \sim R_s} \boxed{R_s = \frac{1}{\sigma \delta_s} = \sqrt{\frac{\omega \mu}{2 \sigma}}}$$

$$\begin{aligned} \alpha_d &\cong \frac{k^2 \tan \delta}{2 \beta} \text{ Np/m} && \text{(TE or TM waves)} \\ &\cong \frac{k \tan \delta}{2} \text{ Np/m} && \text{(TEM waves)} \end{aligned}$$

Coaxial line:

- ◆ TEM mode, use from DC-to mm-waves
- ◆ EM-fields from static fields using cylindrical coordinates
- ◆ First higher order mode is TE_{11} , cutoff frequency approx.

$$\boxed{k_c \cong \frac{2}{a+b} \quad f_c = \frac{ck_c}{2\pi \sqrt{\epsilon_r}}}$$

Recap (3): Waveguides, (micro)stripline

- ♦ **Rectangular waveguide** has limited bandwidth
- ♦ dominant mode is **TE₁₀**: $f_{c10} = \frac{1}{2a\sqrt{\mu\epsilon}}$ $\beta = \sqrt{k^2 - (\pi/a)^2}$
- ♦ For $f < f_c$, β is imaginary, all field components will decay exponentially: cut-off or evanescent modes
- ♦ Higher order modes: **TE₁₀**, for normal case $a > 2b$ waveguide BW typically factor of two
- ♦ **Strip-line**: integrated, TEM, low dispersion and loss
- ♦ **Microstrip**: integrated, quasi-TEM, most used, requires vias and insulating substrates

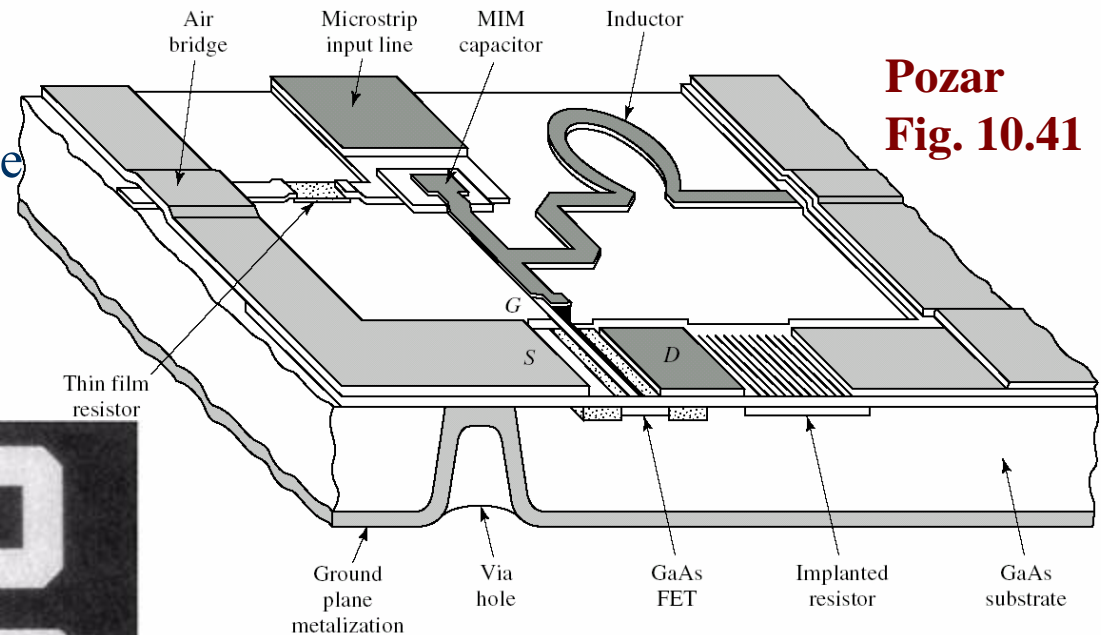
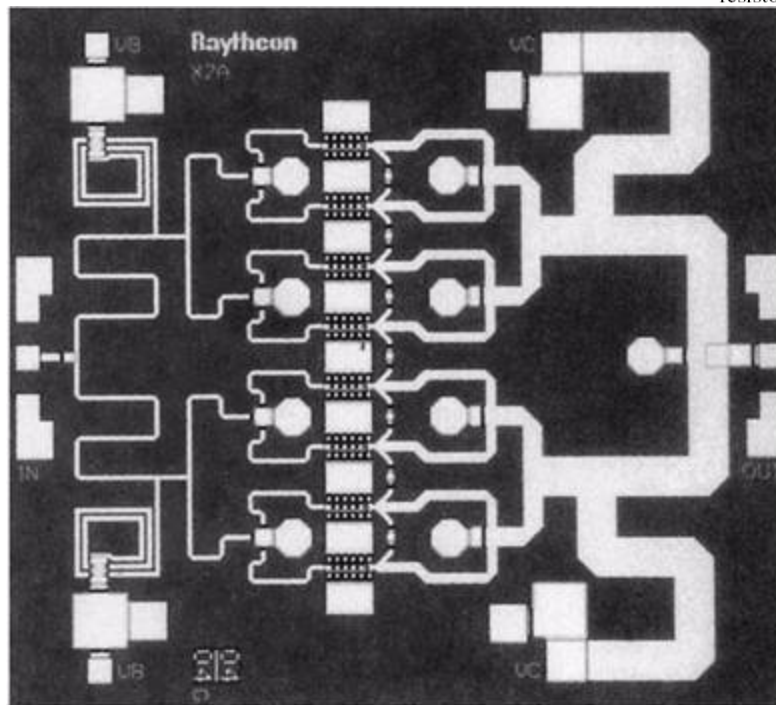
Microstrip substrate materials

<i>Material</i>	<i>Type of Material</i>	<i>Dielectric Constant</i>	<i>Loss Tangent</i>	<i>Other Characteristics</i>
<i>Fused Silica</i>	Amorphous form of quartz	3.78	<0.0001 to at least 20 GHz	Expensive, brittle, difficult metal adhesion
<i>Alumina</i>	Ceramic form of alumina	9.0-10.0	<0.0015 to 25 GHz	Characteristics depend on manufacture, k=9.8 is most common
<i>Sapphire</i>	Crystalline alumina	8.6 hor. 10.55 vert.	<0.0015 in all directions	Electrically anisotropic
<i>RT Duroid 5880</i>	Composite; PTFE-fiber-glass	2.20	0.0009 at 10 GHz	Low-cost “soft” substrate; widely used
<i>RT Duroid 5870</i>	Composite; PTFE-fiber-glass	2.33	0.0012 at 10 GHz	Low-cost “soft” substrate; widely used
<i>RT Duroid 6006</i>	Composite; ceramic-PTFE	6.15	0.0019 at 10 GHz	Not mechanically as good as other materials
<i>Silicon</i>	Crystal (Si)	11.9	Very Lossy	Dielectric loss problem for RF/MW circuits
<i>Gallium Arsenide</i>	Crystal (GaAs)	12.9	Typically 0.001	Used for monolithic circuits only
<i>Indium Phosphide</i>	Crystal (InP)	12.4	Typically 0.001	Used for monolithic circuits only

K. Chang, I. Bahl and V. Nair, “RF and Microwave Circuit and Component Design for Wireless Systems”, Wiley Series in Microwave and Optical Engineering

Microstrip Monolithic Integrated Circuits

Layout monolithic microwave integrated circuit (MMIC) again microstrip topology



**Pozar
Fig. 10.41**

Example of MMIC:
Integrated X-band power amplifier

Multiple HBT's combined to deliver 5W

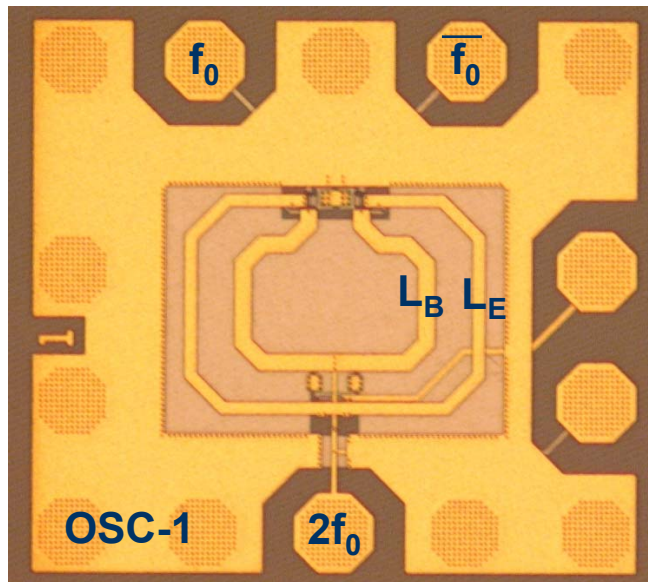
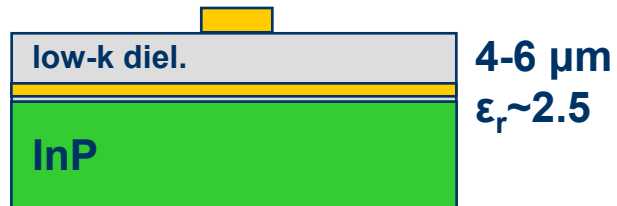
**Pozar
Fig. 10.42**

More recent: **thin-film** μ -strip

Thin-Film Microstrip (TF-MS)

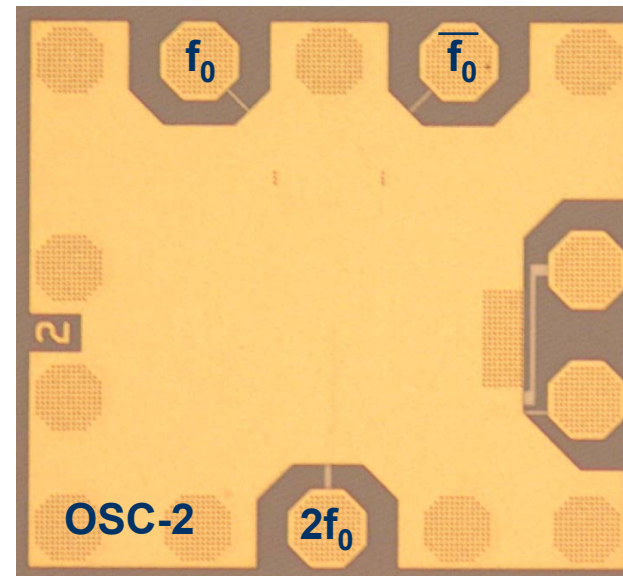
Conventional TFMS:

- + lower loss (fixed Z_0)
- + low $\epsilon_{r,\text{eff}}$ (digital)



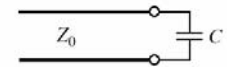
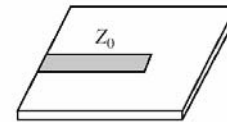
Inverted TFMS:

- + good shielding, easy flip-chip
- + high $\epsilon_{r,\text{eff}}$ (delay)

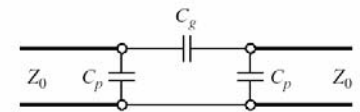
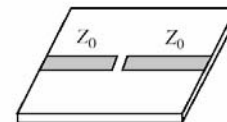


Microstrip discontinuities

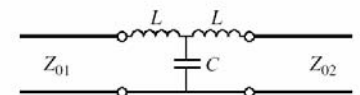
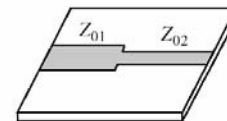
- ◆ Compared with ideal TL, additional parasitics associated with discontinuities such as:
 - open end and gap
 - Via-hole to ground
 - change in width (step)
 - T-junction & cross-junction
 - corner or bend
- ◆ Local change in E and H-field
 - fringing fields: capacitor
 - change current: inductance



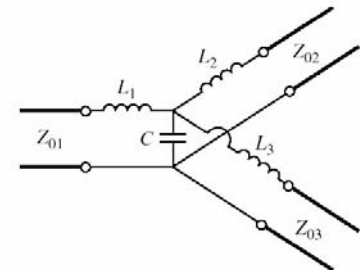
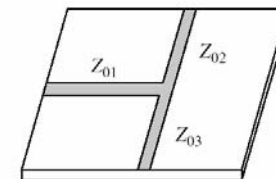
(a)



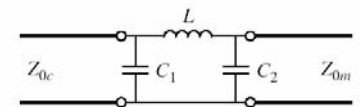
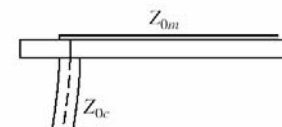
(b)



(c)



(d)

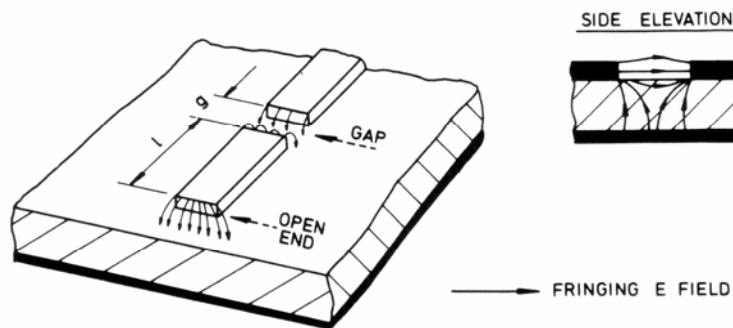


(e)

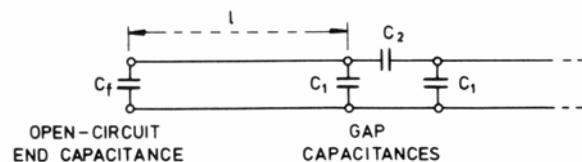
Pozar 4.23

Microstrip discontinuities (2)

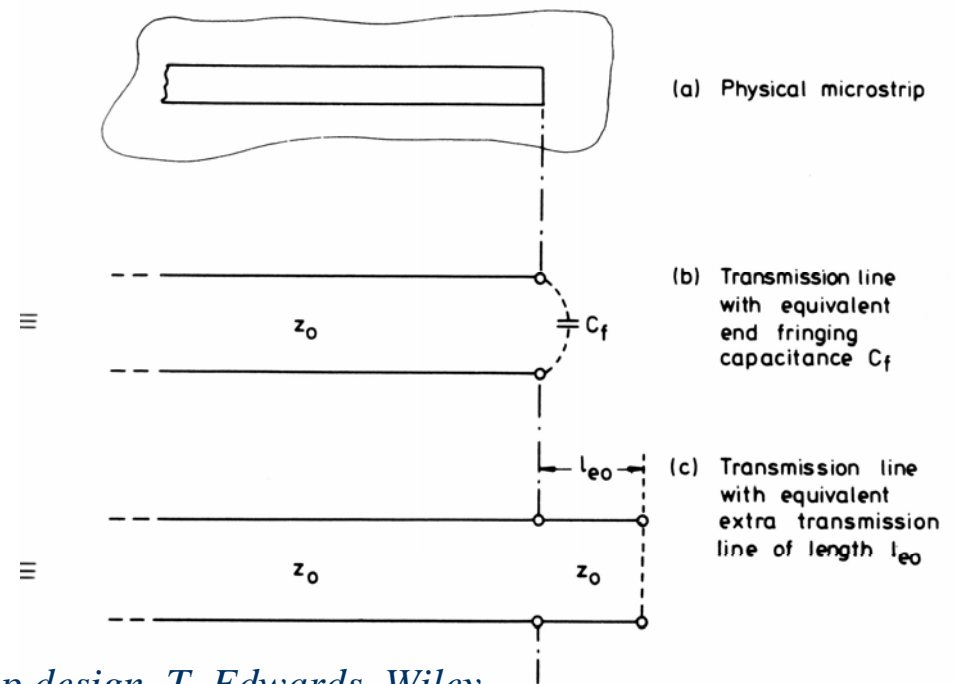
- ◆ In general, dimensions $\ll \lambda$: effect approximated by equivalent circuit model, parameters from rigorous EM-simulations
- ◆ Alternative : equivalent end effect (only distributed parameters)
- ◆ Models available in μ wave circuit simulators (ADS), use them!



(a) Physical open circuits



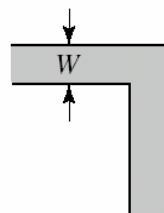
(b) Equivalent networks



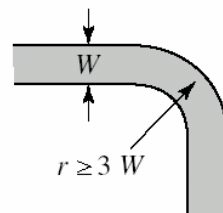
Foundations microstrip design, T. Edwards, Wiley

Microstrip discontinuities compensation

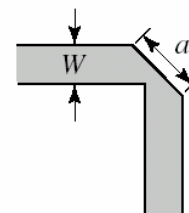
- ◆ Effect of discontinuities:
 - additional reactances can cause errors circuit design
 - conversion to other modes (surface-wave mode in μ strip)
 - radiation: loss mechanism, source EMI, coupling
- ◆ Whenever possible, effect discontinuity mitigated by making smoother transition or compensation for discontinuity



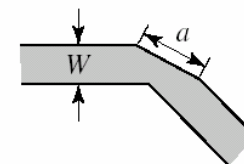
Right-angle bend



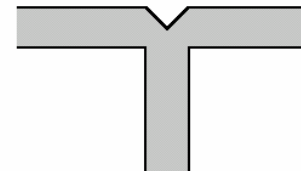
Swept bend
 $r \geq 3W$



Mitered bends



Mitered step

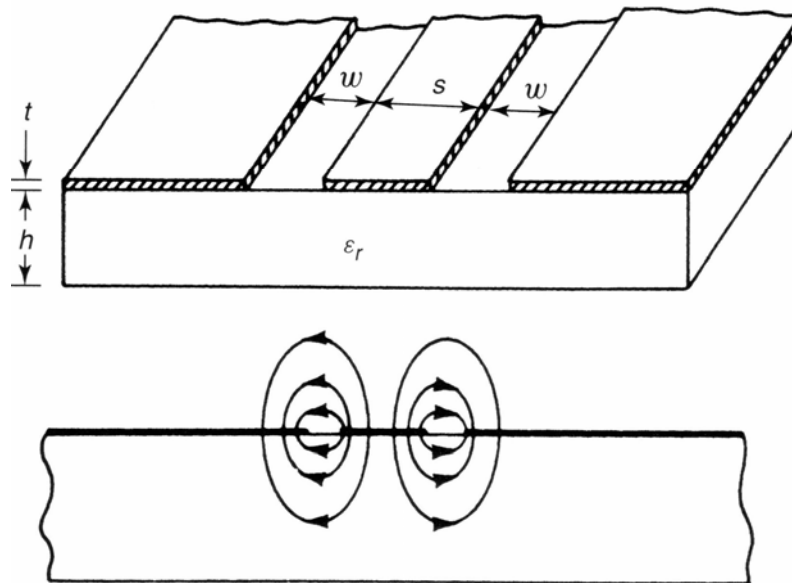


Mitered T-junction

Pozar p.204

Coplanar waveguide lines (CPW)

- ♦ originally introduced by C.P. Wen in 1969
- ♦ only more recently (from mid-90's) used in circuits, mainly due to lack of accurate modeling
- ♦ also does not support pure TEM (different dielectric constant under and above line) quasi-TEM



Conductor in gap between 2 ground planes (dual microstrip)
Variations:

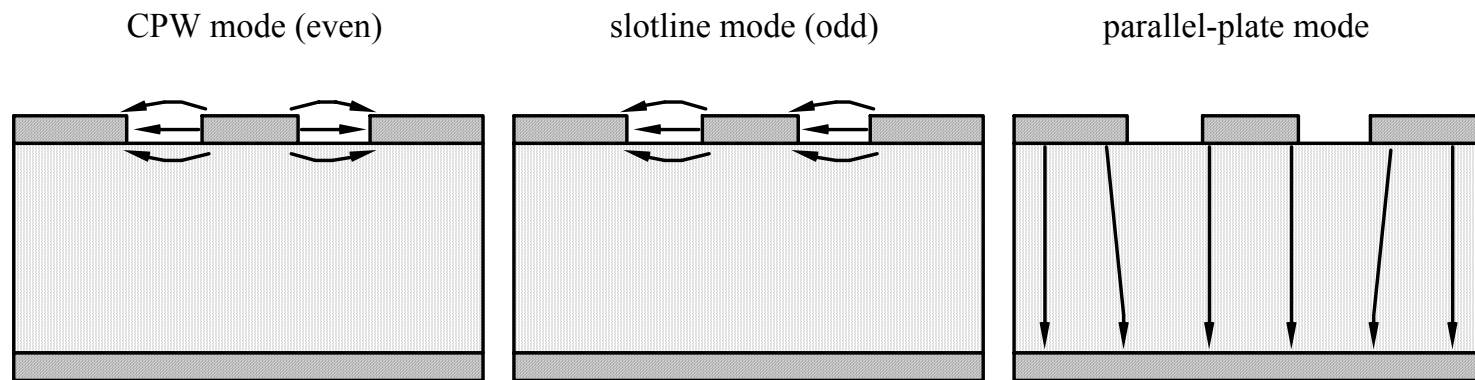
- CPS
- asymmetric CPW
- conductor backed CPW

Advantages CPW versus μ -stripline

- ◆ **Uniplanar technology:** signal & ground conductors at same side of substrate; no wafer thinning, via hole etching and backside metallisation needed, this reduces cost
- ◆ ground plane is accessible at front side of the wafer, easy implementation of active elements; especially advantageous at very high frequencies due to the **absence of via-hole inductance**;
- ◆ correctly designed, CPW lines have **low dispersion** (variation of the effective dielectric constant), important for broad band applications;
- ◆ the presence of the ground plane results in a **reduced coupling** between adjacent line, enables a further miniaturisation of the MMIC circuits
- ◆ **on-wafer measurement** technique based on coplanar probe tips is commercially available facilitating accurate measurements

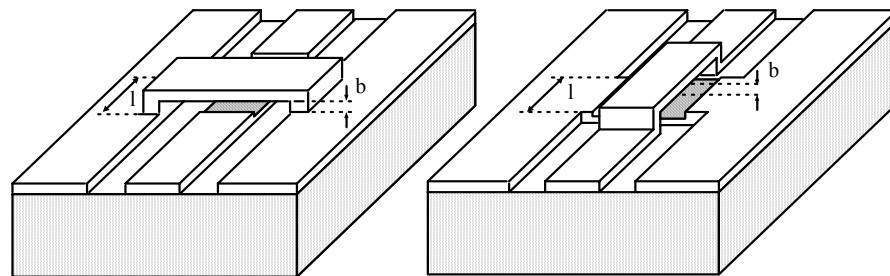
Disadvantages CPW versus μ -stripline

- ♦ in CPW the **electrical field is surface-oriented**, to determine accurately the coplanar line characteristics two-dimensional field needs to be solved.
- ♦ a coplanar circuit has normally a thick substrate, therefore **heat transfer** can become a problem in high-power applications,
- ♦ the CPW line consists of three unconnected conductors, such that both an **even and odd** transmission line mode can propagate, the symmetric CPW mode is the mode commonly used in coplanar circuits. At discontinuities, this mode can be converted into an asymmetric slot-line mode. Such multi-mode propagation should be prevented by the use of air bridges connecting the two ground metallisations to keep them at an equal potential. When a back metallisation is present, additionally a parallel-plate mode can be excited



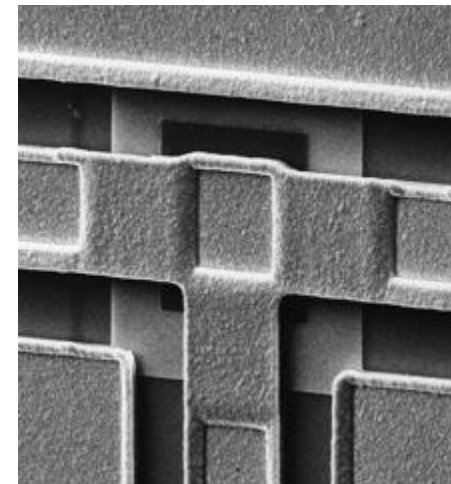
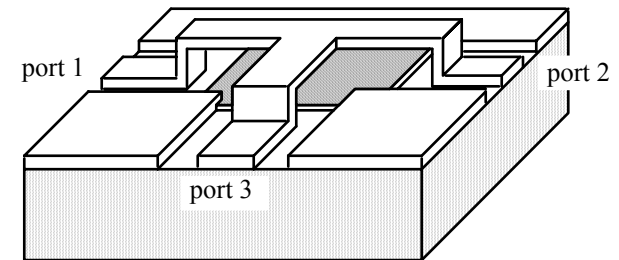
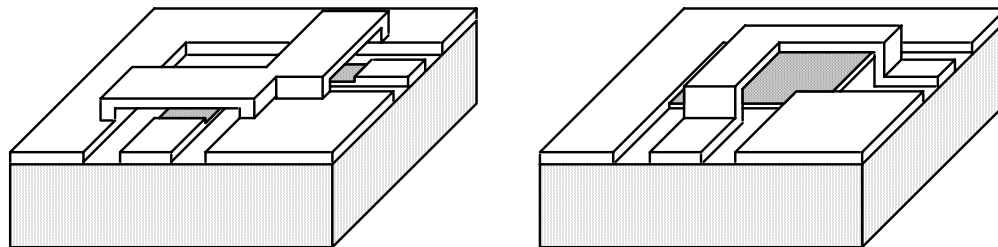
Discontinuities in CPW

Important to maintain ground continuity at CPW discontinuities: frequent use of airbridges at T- or cross-junctions, bends, at input and output ports lumped elements, etc...

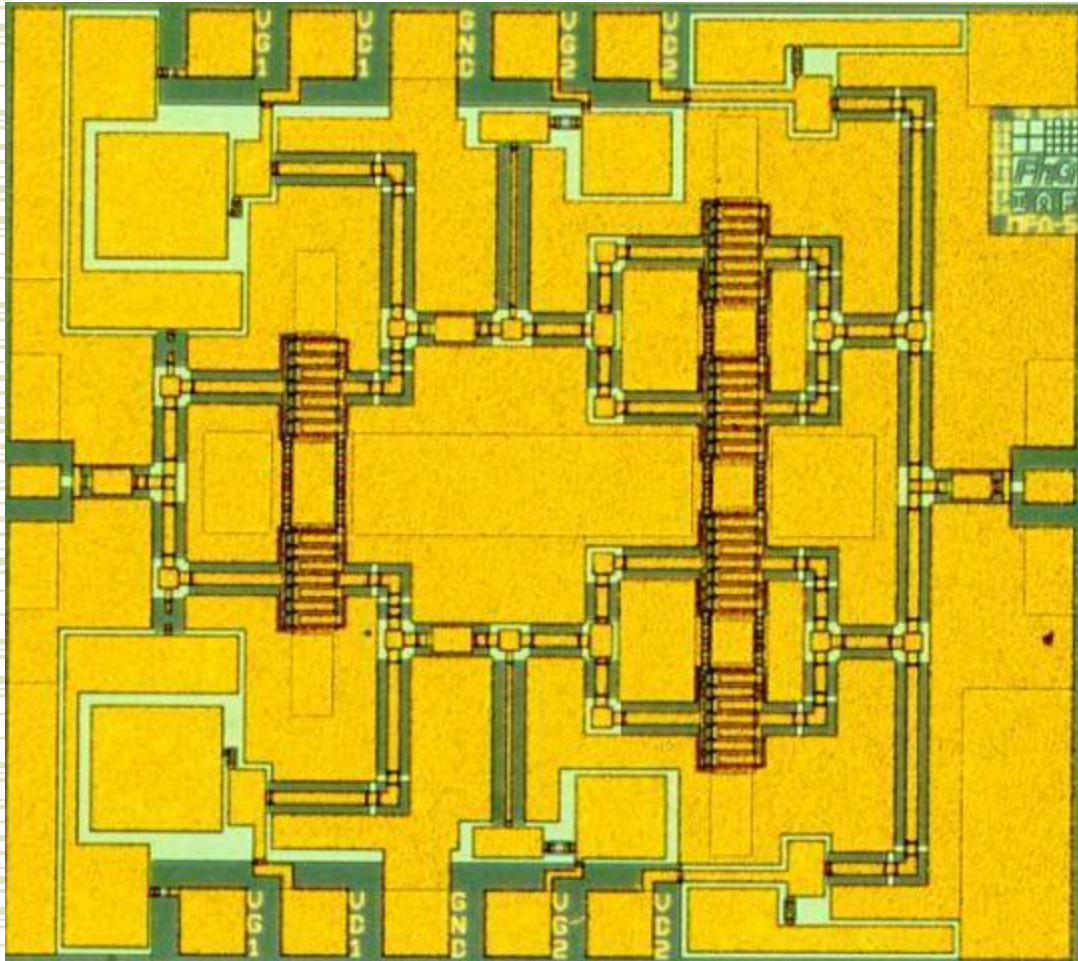


TYPE A

TYPE B



Example of coplanar MMIC (1)



- 1W power amplifier at 42 GHz
- 0.15 μm PHEMT
- size: 1.4x1.4 mm²
- 3.3V power supply
- Power-density 2-3x higher best μ -strip
- C-loaded lines

Summary of common TL's

<i>Characteristic</i>	<i>Coax</i>	<i>Waveguide</i>	<i>Stripline</i>	<i>Microstrip & CPW</i>
<i>Modes: Preferred</i>	TEM	TE ₁₀	TEM	Quasi-TEM
<i>Other</i>	TM, TE	TM, TE	TM, TE	Hybrid TM, TE
<i>Dispersion</i>	None	Medium	None	Low
<i>Bandwidth</i>	High	Low	High	High
<i>Loss</i>	Medium	Low	High	High
<i>Power Capacity</i>	Medium	High	Low	Low
<i>Physical Size</i>	Large	Large	Medium	Small
<i>Ease of Fabrication</i>	Medium	Medium	Fair	Easy
<i>Integration with other components</i>	Hard	Hard	Fair	Easy

- ◆ Other TL: ridge & dielectric WG, fin-line, balanced lines such as twisted pair, coplanar strip-line, slotline,
- ◆ Other important feature: radiation performance

Wave velocities and dispersion in TL

- ◆ Different velocities defined in TL:
 - Speed of light in medium: $1/\sqrt{\mu\epsilon}$
 - Phase velocity: $v_p = \omega/\beta$
 - Group velocity: $v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} \Big|_{\omega=\omega_0}$
- ◆ Phase velocity: speed at which a constant phase point travels. Dispersion of broadband signals in TL occurs when either phase velocity v_p or attenuation not constant afo frequency, from wave analogy: can be larger then speed of light
- ◆ Group velocity: velocity at which a narrow band signal propagates, related with information and power transport, needs to be smaller then c (for derivation, see p.151-153)

Example: waveguide wave velocities

? Calculate group velocity of waveguide mode propagating in air-filled guide. Compare to phase velocity and speed of light.

- ◆ First calculate propagation constant β :

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{(\omega/c)^2 - k_c^2}$$

- ◆ Taking derivative to frequency gives:

$$\frac{d\beta}{d\omega} = \frac{\omega/c^2}{\sqrt{(\omega/c)^2 - k_c^2}} = \frac{k_o}{c\beta}$$

- ◆ So group velocity becomes:

$$v_g = \left(\frac{d\beta}{d\omega} \right)^{-1} = \frac{c\beta}{k_o}$$

- ◆ Phase velocity is: $v_p = \omega/\beta = (k_o c)/\beta$
- ◆ Since $\beta < k_o$, we have $v_g < c < v_p$, which indicates phase velocity may be greater than speed of light, but group velocity will always be less than speed of light

Power capacity of TL's

- ◆ Power in TL's is limited by voltage breakdown, for air occurring at breakdown electric field $E_d = 3 \times 10^6$ V/m
- ◆ Calculation of capacity requires knowledge of E-field
- ◆ For air-filled coax: $E_\rho = V_0 / (\rho \ln b/a)$ this is max. for $\rho = a$

$$V_{\max} = E_d a \ln b/a$$

and maximum power capacity becomes:

$$P_{\max} = \frac{V_{\max}^2}{2Z_0} = \frac{\pi a^2 E_d^2}{\eta_0} \ln \frac{b}{a}$$

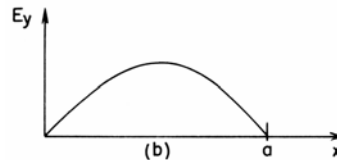
- ◆ As expected power capacity increases for larger diameter cable, limit is cut-off frequency of higher order mode TE_{11}

$$P_{\max} = \frac{0.025}{\eta_0} \left(\frac{c E_d}{f_{\max}} \right)^2 = 5.8 \times 10^{12} \left(\frac{E_d}{f_{\max}} \right)^2 \quad @10\text{GHz} = 520\text{kW}$$

Power capacity of waveguides

- ♦ For air-filled rectangular waveguide: $E_y = E_o \sin(\pi x/a)$ this is max. E_o at $x=a/2$ and maximum power capacity becomes:

$$P_{\max} = \frac{abE_o^2}{4Z_w} = \frac{abE_d^2}{4Z_w}$$



- ♦ As expected, power capacity increases for guide size, for most waveguides $b \cong a/2$, to avoid TE_{20} mode, $a < c/f_{\max}$, with f_{\max} the maximum operating frequency. Maximum power capacity of guide can be shown:

$$P_{\max} = \frac{0.11}{\eta_0} \left(\frac{cE_d}{f_{\max}} \right)^2 = 2.6 \times 10^{13} \left(\frac{E_d}{f_{\max}} \right)^2 \quad @10\text{GHz} = 2300\text{kW}$$

- ♦ In practice, safety factor of two + some care for reflections (for $|\Gamma|=1$, max. voltage can double)
- ♦ Higher breakdown using inert gas or dielectric

Microwave Circuit Analysis

- ◆ Circuit dimensions \ll wavelength
 - Lumped passive and active components.
 - Negligible phase change throughout the circuit.
 - Circuit theory — Kirchhoff's laws and Ohm's law.
- ◆ Circuit dimensions \approx wavelength
 - Distributed passive and active components.
 - Phase depends on position. Components are characterized by their dimensions, propagation constants and characteristic impedances.
 - Microwave network theory.

Impedance, voltage and current

- ◆ The voltage, current and characteristic impedance of transmission lines are defined as:

$$V = \Phi_+ - \Phi_- = \int_+^- \overline{E} \cdot d\overline{l}$$

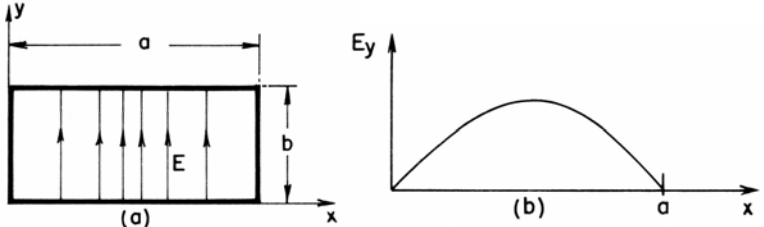
$$I = \oint_{C^+} \overline{H} \cdot d\overline{l}$$

$$Z_0 = \frac{V}{I}$$

- ◆ **TEM-type** TL have **unique V, I and Z_0** because:
 - The lines have well defined terminal pairs.
 - The above integrations are independent of path.

Characteristics & calculations non-TEM lines

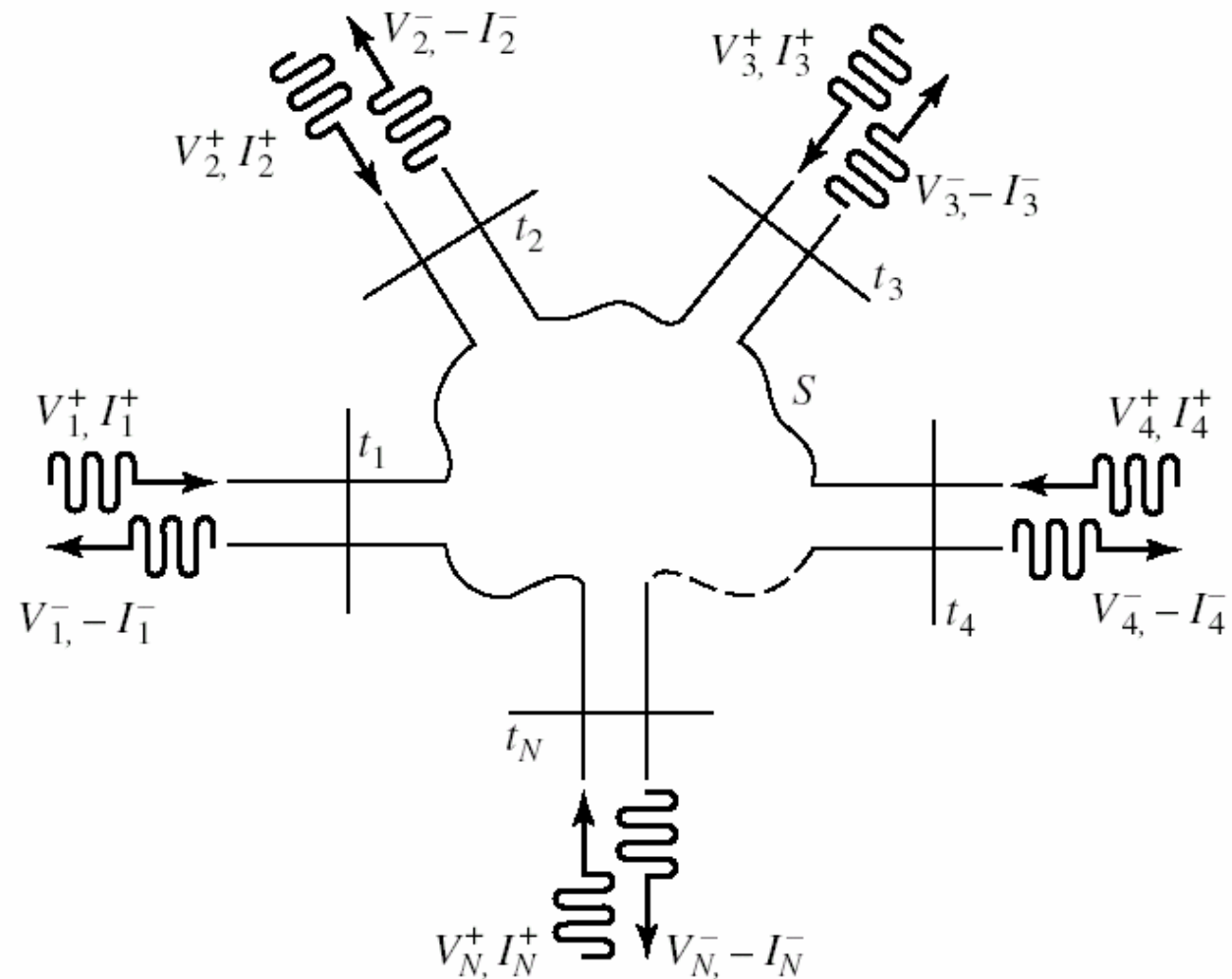
- ♦ **Non TEM-type** transmission lines such as rectangular waveguide do not have unique V , I and Z_0 values because:
 - The lines DO NOT have well defined terminal pairs.
 - The above integrations are path dependent.
- ♦ For the dominant TE_{10} mode in rectangular waveguide, voltage from the transverse fields can be written as:

$$V = \frac{-j\omega\mu a}{\pi} A \cdot \sin \frac{\pi x}{a} e^{-j\beta z} \int_y dy$$


The diagram consists of two parts. Part (a) shows a rectangular waveguide with width 'a' and height 'b'. Inside the waveguide, there are five vertical arrows pointing upwards, labeled 'E', representing the electric field. The horizontal axis is 'x' and the vertical axis is 'y'. Part (b) shows a graph of the electric field component E_y versus the position x . The curve is a sine wave starting at zero at $x=0$, reaching a maximum, and returning to zero at $x=a$.

- ♦ The above voltage depends on the position, x , as well as the length of the integration contour along the y -direction.
- ♦ For non-TEM: equivalent I , V & Z used (not discussed here)

An arbitrary N-port Microwave Network



Impedance Matrix

- ◆ Two-terminal pair \Rightarrow Port.
- ◆ V and $I \Rightarrow$ Equivalent V and I .
 - Reference planes are defined to provide a phase reference for the (equivalent) V and I phasors.
 - At the n^{th} reference plane, the total voltage and current are:

$$V_n = V_n^+ + V_n^- \quad I_n = I_n^+ - I_n^-$$

- ◆ The impedance matrix relates these voltages and currents:

$$[V] = [Z][I], \quad Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k = 0 \text{ for } k \neq j}$$

Z_{ii} : input impedance

Z_{ij} : transfer impedance between ports i and j , $i \neq j$

Admittance Matrix

- ◆ The admittance matrix is defined as:

$$[I] = [Y][V], \quad Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_k = 0 \text{ for } k \neq j}$$

Y_{ii} : input admittance

Y_{ij} : transfer admittance between ports i and j , $i \neq j$

- ◆ For **reciprocal** networks (no active devices, ferrites,..), the impedance and admittance matrices are symmetric: $Z_{ij} = Z_{ji}$ and $Y_{ij} = Y_{ji}$
- ◆ If the network is **lossless**, Z_{ij} and Y_{ij} are purely imaginary

Example: evaluation of impedance parameters

♦ Example: Find Z-parameters two-port T-network

♦ Solution:

- Z_{11} : input impedance port 1 when port 2 is open circuited

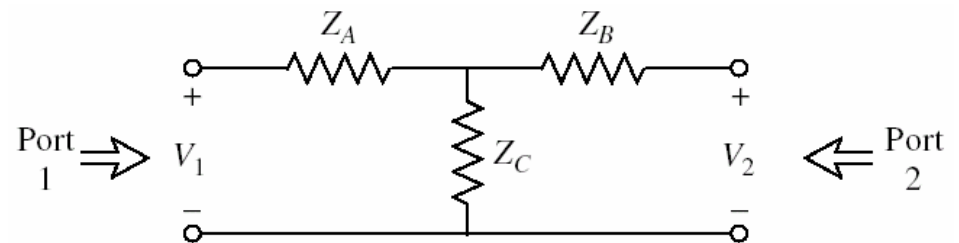
$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = Z_A + Z_C$$

- Transfer impedance Z_{12} : measure open-circuit voltage at port 1 when current I_2 applied at port 2:

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{V_2}{I_2} \frac{Z_C}{Z_B + Z_C} = Z_C$$

- $Z_{12}=Z_{21}$ and Z_{22} can be found:

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = Z_B + Z_C$$



The Scattering Matrix (S-parameters)

- ◆ Impedances and admittances are easy to work with; however these parameters cannot be measured easily:
 - VSWR, non-TEM complicate measurement
 - Short and open circuits are difficult to achieve over a broad-band of microwave frequencies.
 - Active devices, such as power transistors, very often are not open- or short-circuit stable
- ◆ Scattering parameters deal directly with incident, reflected and transmitted voltage waves.
- ◆ Scattering parameters can be measured directly with a vector network analyzer (VNA).
- ◆ Conversion from scattering parameters to other matrix parameters can be easily done.

Scattering Matrix

- ◆ The scattering matrix of a N-port network with the same characteristic impedance at all ports is defined as:

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & & S_{2N} \\ \vdots & & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

$$V^- = S \cdot V^+ \quad S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \text{ for } k \neq j}$$

- ◆ V_n^+ and V_n^- are the amplitudes of the incident and reflected voltage waves at the n^{th} port
- ◆ S_{ij} is found by driving port j with incident wave of voltage V_j^+ and measuring the reflected wave amplitude V_i^- , coming out of port i , incident waves on all other ports set to zero, terminated with matched load

Example: evaluation of S-parameters

♦ Example: Find S-parameters 3-dB attenuator network

♦ Solution:

- S_{ii} : reflection coefficient into port i with other ports terminated
- S_{ij} : transmission coefficient from port j to i, other terminated
- So, S_{11} : reflection coefficient port 1 when port 2 is terminated in matched load ($Z_0=50\Omega$)

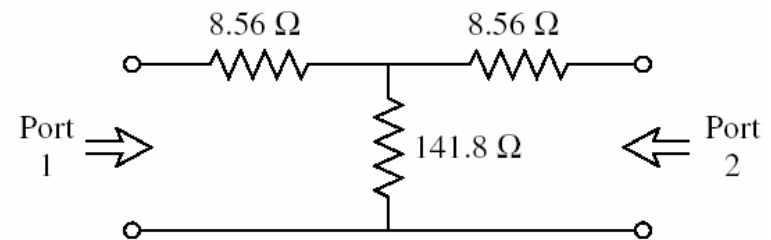
$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} = \Gamma^{(1)} \Big|_{V_2^+=0} = \left. \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0} \right|_{Z_0 \text{ on port 2}}$$

$$Z_{in}^{(1)} = 8.56 + [141.8(8.56 + 50)] / (141.8 + 8.56 + 50) = 50\Omega$$

- For S_{21} :

- apply incident wave at port 1, V_1^+
- measure outgoing wave at port 2, V_2^-

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0}$$



3-db attenuator

Example: evaluation of S-parameters

- ♦ $S_{11}=S_{22}=0$, so $V_1^- = 0$ if port 2 terminated in $50\ \Omega$ ($V_2^+ = 0$) $\Rightarrow V_1^+ = V_1$ & $V_2^- = V_2$
- ♦ So applying V_1 and calculating V_2 (2x voltage division) :

$$V_2^- = V_2 = V_1 \left(\frac{141.8 // 58.56}{141.8 // 58.56 + 8.56} \right) \left(\frac{50}{50 + 8.56} \right) = 0.707 V_1$$

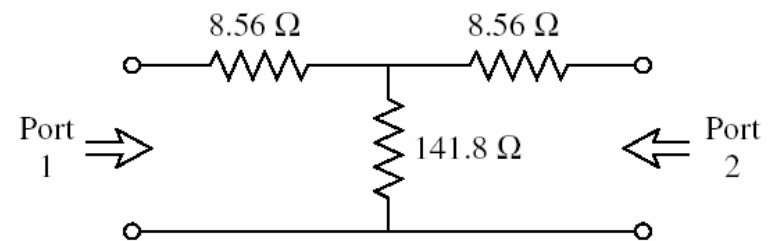
- ♦ So, $S_{21}=S_{12}=0.707$
- ♦ If input power is $|V_1^+|^2 / 2Z_0$ then output power is:

$$S = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

$$|V_2^-|^2 / 2Z_0 = |S_{21}V_1^+|^2 / 2Z_0 = |S_{21}|^2 |V_1^+|^2 / 2Z_0 = |V_1^+|^2 / 4Z_0$$

so attenuator effectively attenuates with 3-dB

(half power put into 2-port is transmitted,
other half is dissipated in resistors)



3-dB attenuator

Determination [S] from [Z] or [Y] (Z_{0n} equal)

- ◆ Total voltage and current at n^{th} port (and set $Z_{0n}=1$):

$$V_n = V_n^+ + V_n^- \quad I_n = I_n^+ - I_n^- = V_n^+ - V_n^-$$

- ◆ then: $[Z][I] = [Z][V^+] - [Z][V^-] = [V] = [V^+] + [V^-]$

- ◆ rewritten as: $([Z] + [U])[V^-] = ([Z] - [U])[V^+]$

with [U] the unit or identity matrix,

- ◆ [S] can be determined as: $[S] = ([Z] + [U])^{-1}([Z] - [U])$

- ◆ for one-port this becomes: $s_{11} = \frac{z_{11} - 1}{z_{11} + 1}$ in agreement with reflection coefficient

- ◆ For [Z] as function of [S] $[Z] = ([U] + [S])([U] - [S])^{-1}$

Generalized scattering Matrix

- The scattering matrix of a N-port network with characteristic impedance Z_{0n} at the n^{th} port is defined as:

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & & S_{2n} \\ \vdots & & \ddots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \begin{aligned} a_n &= V_n^+ / \sqrt{Z_{0n}} \\ b_n &= V_n^- / \sqrt{Z_{0n}} \\ Z_{0n} &= Z_0 \text{ at port } n \end{aligned}$$

$$[b] = [S] \cdot [a] \quad S_{ij} = \left. \frac{b_i}{a_j} \right|_{V_k^+ = 0 \text{ for } k \neq j} = \left. \frac{V_i^- \sqrt{Z_{0j}}}{V_j^+ \sqrt{Z_{0i}}} \right|_{V_k^+ = 0 \text{ for } k \neq j}$$

- V_n^+ and V_n^- are the amplitudes of the incident and reflected voltage waves at the n^{th} port

Power Power Delivered

- ◆ a_n and b_n can be expressed in terms of V_n and I_n :

$$a_n = V_n^+ / \sqrt{Z_{0n}} = \sqrt{Z_{0n}} I_n^+ \quad b_n = V_n^- / \sqrt{Z_{0n}} = \sqrt{Z_{0n}} I_n^-$$

$$a_n + b_n = V_n / \sqrt{Z_{0n}} \quad a_n - b_n = \sqrt{Z_{0n}} I_n$$

$$a_n = \frac{1}{2\sqrt{Z_{0n}}} [V_n + Z_{0n} I_n] \quad b_n = \frac{1}{2\sqrt{Z_{0n}}} [V_n - Z_{0n} I_n]$$

- ◆ Average power delivered to the port n is:

$$\begin{aligned} P_n &= \frac{1}{2} \operatorname{Re}\{V_n I_n^*\} = \frac{1}{2} \operatorname{Re}\left\{ (V_n^+ + V_n^-) \left(\frac{V_n^+ - V_n^-}{Z_{0n}} \right)^* \right\} \\ &= \frac{1}{2} \operatorname{Re}\left\{ \sqrt{Z_{0n}} (a_n + b_n) \left(\frac{a_n - b_n}{\sqrt{Z_{0n}}} \right)^* \right\} \\ &= \frac{1}{2} \operatorname{Re}\{ |a_n|^2 - |b_n|^2 + (b_n a_n^* - b_n^* a_n) \} = \underbrace{\frac{|a_n|^2}{2}}_{\text{incident}} - \underbrace{\frac{|b_n|^2}{2}}_{\text{reflected}} \end{aligned}$$

Reciprocal and lossless networks

- ◆ For reciprocal networks (no active elements, ferrites), $[Z]$ and $[Y]$ are symmetric.
- ◆ Similarly, $[S]$ -matrix of **reciprocal** network is **symmetric**:
 $[S]=[S]^t$ ($[S]^t$ is transpose matrix)
- ◆ For lossless networks, $[Z]$ and $[Y]$ are purely imaginary
- ◆ The S-parameters of a **lossless** network form a **unitary** matrix:
 $[S]^t[S]^*=[U]$, product any column $[S]$ with own conjugate gives unity, product with conjugate different column gives zero

$$\sum_{k=1}^N S_{ki} S_{ki}^* = 1$$

$$\sum_{k=1}^N S_{ki} S_{kj}^* = 0 \quad \text{for } i \neq j$$

Application of S-parameters

- ♦ Measured S-parameters 2-port: $[S] = \begin{bmatrix} 0.1\angle 0 & 0.8\angle 90^\circ \\ 0.8\angle 90^\circ & 0.2\angle 0 \end{bmatrix}$
- ♦ Determine if 2-port is reciprocal or lossless, calculate return loss at port1 for short at port 2
- ♦ **Solution:**
 - [S] is symmetric, so 2-port is reciprocal
 - Not lossless: evaluation 1st row: $|S_{11}|^2 + |S_{12}|^2 = (0.1)^2 + (0.8)^2 = 0.65 \neq 1$
 - Calculation reflection coefficient for shorted port 2 ($V_2^+ = -V_2^-$) from definition S-parameters:

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ = S_{11}V_1^+ - S_{12}V_2^-$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ - S_{22}V_2^-$$

Application of S-parameters (2)

- ♦ From last equation: $V_2^- = \frac{S_{21}}{1 + S_{22}} V_1^+$

- ♦ Dividing 1st equation by V_1^+ , inserting V_2^- :

$$\begin{aligned}\Gamma = \frac{V_1^-}{V_1^+} &= S_{11} - \frac{S_{12}S_{21}}{1 + S_{22}} \\ &= 0.1 - \frac{(j0.8)(j0.8)}{1 + 0.2} = 0.633\end{aligned}$$

- ♦ So return loss becomes:

$$RL = -20 \log |\Gamma| = 3.97 \text{ dB}$$

Shift in Reference Planes

- ◆ Phase reference planes needed for each port of network
- ◆ S-parameters transformed when reference planes moved from original locations
- ◆ For original & new reference:

$$[V^-] = [S] \cdot [V^+]$$

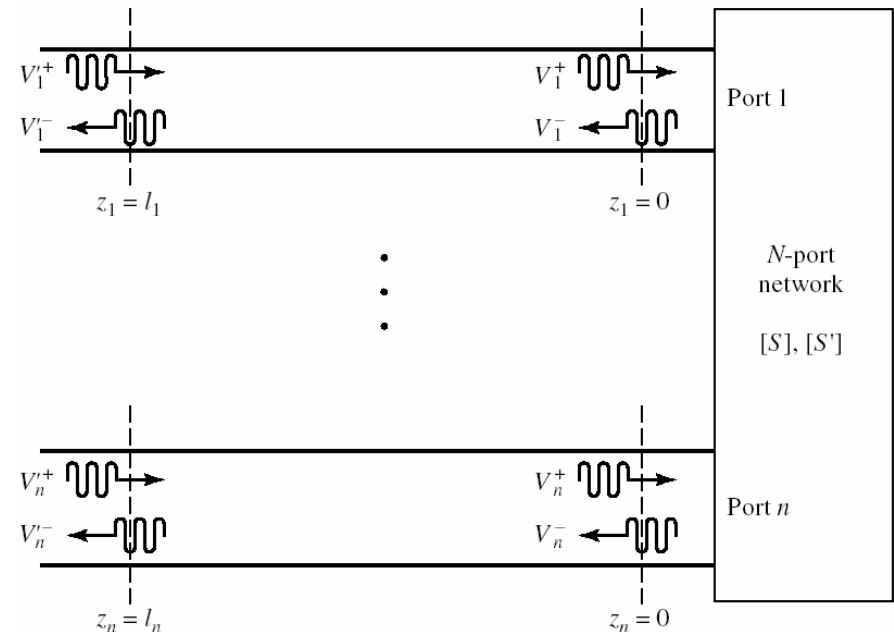
$$[V'^-] = [S'] \cdot [V'^+]$$

- ◆ From transmission line theory:

$$V_n'^+ = V_n^+ e^{j\theta_n}$$

$$V_n'^- = V_n^- e^{-j\theta_n}$$

- ◆ With $\theta_n = \beta_n l_n$ electrical length of outward shift reference plane



Shift in Reference Planes

- ♦ Writing S-parameter equation in matrix form:

$$\begin{bmatrix} e^{j\theta_1} & & 0 \\ & e^{j\theta_2} & \\ & & \ddots \\ 0 & & & e^{j\theta_n} \end{bmatrix} [V'^-] = [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & & \ddots \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [V'^+]$$

- ♦ Multiplying with inverse matrix on left:

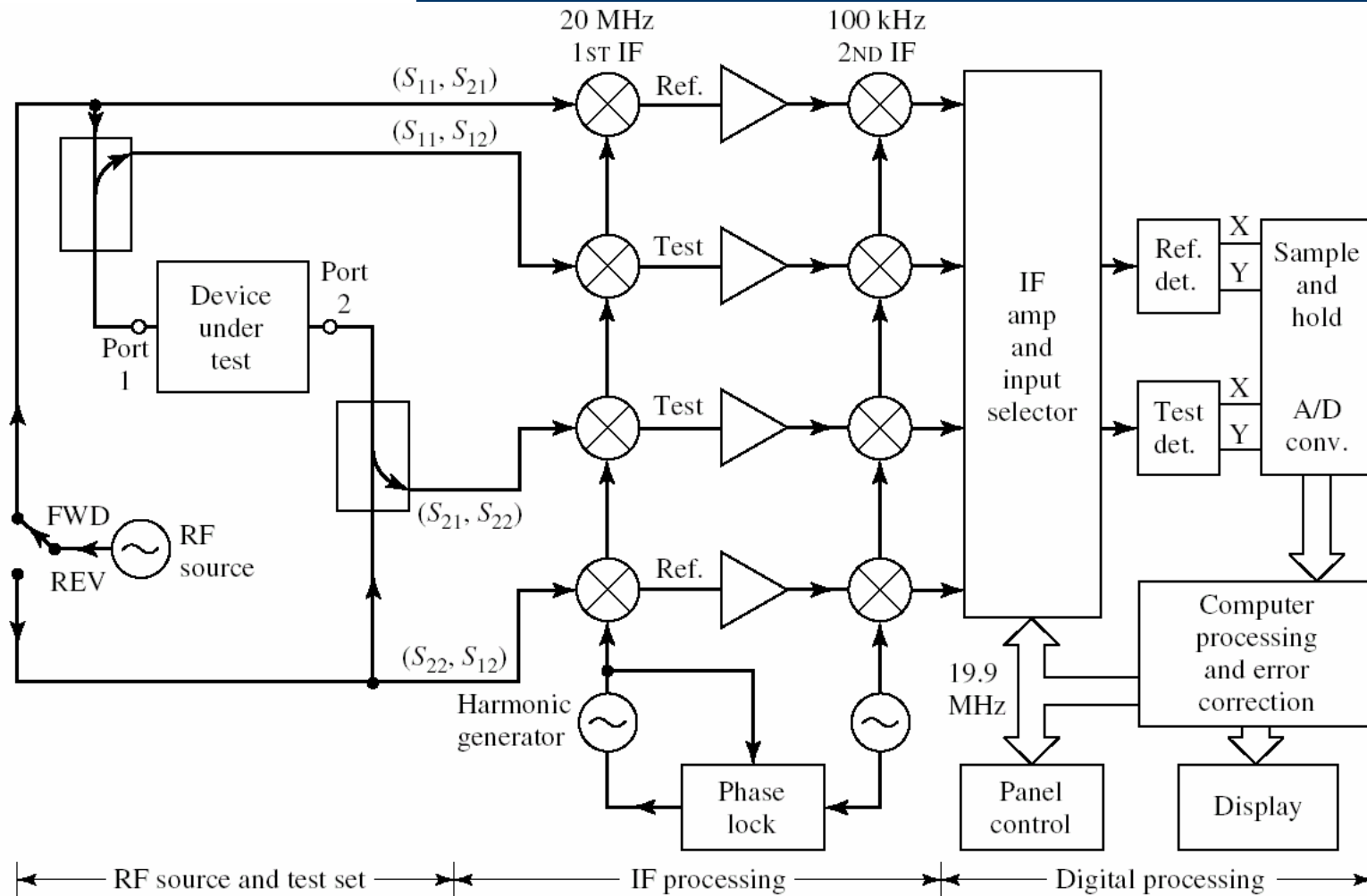
$$[V'^-] = \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & & \ddots \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & & \ddots \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [V'^+]$$

- ♦ Gives expression for new S-parameter matrix

$$[S'] = \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & & \ddots \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & & \ddots \\ 0 & & & e^{-j\theta_n} \end{bmatrix}$$

- ♦ Note: $S'_{nn} = e^{-2j\theta_n}$, phase twice shifted by electrical length of shift in terminal plane n (wave travels twice along length)

The Vector Network Analyzer



Critical component is directional coupler (will see later)

The transmission (ABCD) Matrix

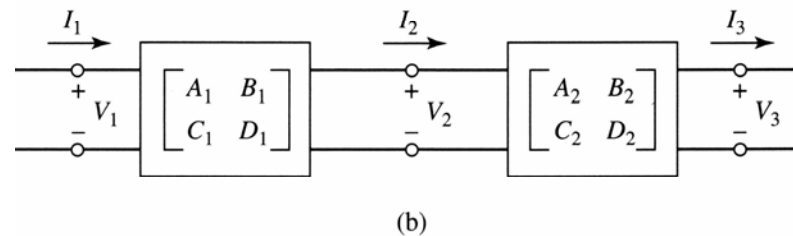
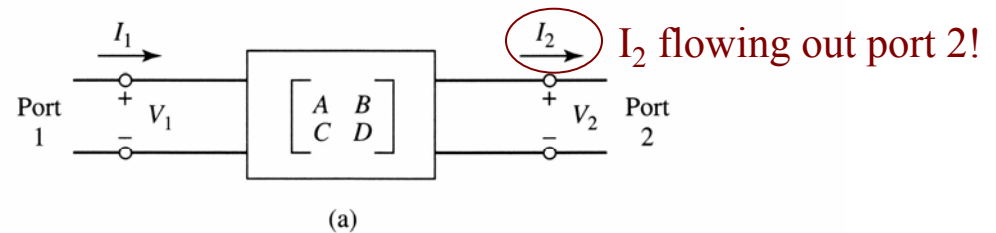
- ◆ Used to calculate cascade connection of networks by multiplying ABCD matrices of individual two-ports

- ◆ Defined as:

$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



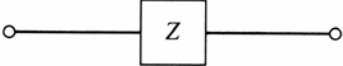
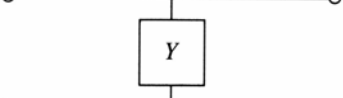
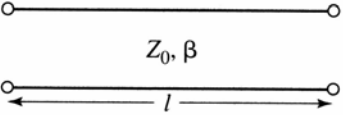
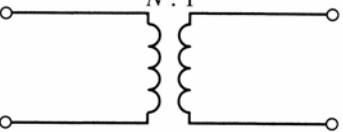
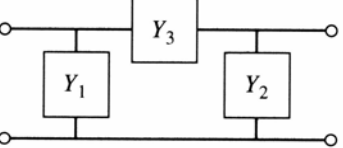
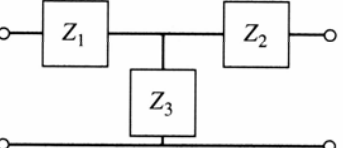
- ◆ Cascade connection:

(a) A two-port network; (b) a cascade connection of two-port networks.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

So:
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

ABCD-parameters of some useful 2-ports

Circuit	ABCD Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta l$ $C = jY_0 \sin \beta l$	$B = jZ_0 \sin \beta l$ $D = \cos \beta l$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

- ◆ Library building blocks
- ◆ Not commutative
- ◆ Example 1st network:

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} = 1$$

$$B = \left. \frac{V_1}{I_2} \right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} = 0$$

$$D = \left. \frac{I_1}{I_2} \right|_{V_2=0} = \frac{I_1}{I_1} = 1$$

Relation Transmission and Impedance Matrix

- ◆ Z-parameters (consistent with sign convention I_2 ABCD):

$$V_1 = I_1 Z_{11} - I_2 Z_{12}$$

$$V_2 = I_1 Z_{21} - I_2 Z_{22}$$

- ◆ Results in calculation ABCD:

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} = \frac{I_1 Z_{11}}{I_1 Z_{21}} = \frac{Z_{11}}{Z_{21}}$$

$$\begin{aligned} B &= \left. \frac{V_1}{I_2} \right|_{V_2=0} = \left. \frac{I_1 Z_{11} - I_2 Z_{12}}{I_2} \right|_{V_2=0} = \left. \frac{I_1 Z_{11}}{I_2} \right|_{V_2=0} - Z_{12} \\ &= Z_{11} \frac{I_1 Z_{22}}{I_1 Z_{21}} - Z_{12} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} \end{aligned}$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} = \frac{I_1}{I_1 Z_{21}} = \frac{1}{Z_{21}}$$

$$D = \left. \frac{I_1}{I_2} \right|_{V_2=0} = \frac{Z_{22}}{Z_{21}}$$

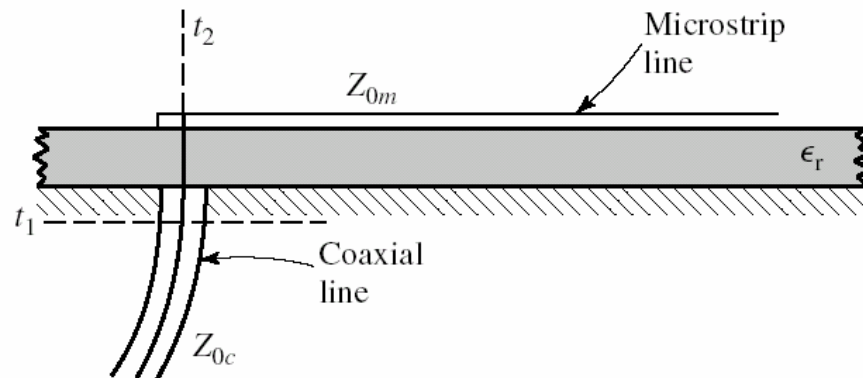
Reciprocal: $AD - BC = 1$

Conversions between 2-port parameters (p.211)

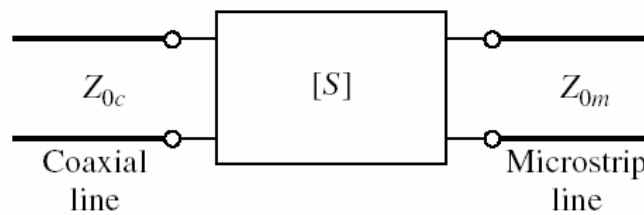
	<i>S</i>	<i>Z</i>	<i>Y</i>	<i>ABCD</i>
<i>S</i> ₁₁	<i>S</i> ₁₁	$\frac{(Z_{11} - Z_0)(Z_{22} + Z_0) - Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0 - Y_{11})(Y_0 + Y_{22}) + Y_{12}Y_{21}}{\Delta Y}$	$\frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}$
<i>S</i> ₁₂	<i>S</i> ₁₂	$\frac{2Z_{12}Z_0}{\Delta Z}$	$\frac{-2Y_{12}Y_0}{\Delta Y}$	$\frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$
<i>S</i> ₂₁	<i>S</i> ₂₁	$\frac{2Z_{21}Z_0}{\Delta Z}$	$\frac{-2Y_{21}Y_0}{\Delta Y}$	$\frac{2}{A + B/Z_0 + CZ_0 + D}$
<i>S</i> ₂₂	<i>S</i> ₂₂	$\frac{(Z_{11} + Z_0)(Z_{22} - Z_0) - Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0 + Y_{11})(Y_0 - Y_{22}) + Y_{12}Y_{21}}{\Delta Y}$	$\frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$
<i>Z</i> ₁₁	$Z_0 \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$	<i>Z</i> ₁₁	$\frac{Y_{22}}{ Y }$	$\frac{A}{C}$
<i>Z</i> ₁₂	$Z_0 \frac{2S_{12}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$	<i>Z</i> ₁₂	$\frac{-Y_{12}}{ Y }$	$\frac{AD - BC}{C}$
<i>Z</i> ₂₁	$Z_0 \frac{2S_{21}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$	<i>Z</i> ₂₁	$\frac{-Y_{21}}{ Y }$	$\frac{1}{C}$
<i>Z</i> ₂₂	$Z_0 \frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$	<i>Z</i> ₂₂	$\frac{Y_{11}}{ Y }$	$\frac{D}{C}$
<i>Y</i> ₁₁	$Y_0 \frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}$	$\frac{Z_{22}}{ Z }$	<i>Y</i> ₁₁	$\frac{D}{B}$
<i>Y</i> ₁₂	$Y_0 \frac{-2S_{12}}{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}$	$\frac{-Z_{12}}{ Z }$	<i>Y</i> ₁₂	$\frac{BC - AD}{B}$
<i>Y</i> ₂₁	$Y_0 \frac{-2S_{21}}{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}$	$\frac{-Z_{21}}{ Z }$	<i>Y</i> ₂₁	$\frac{-1}{B}$
<i>Y</i> ₂₂	$Y_0 \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}$	$\frac{Z_{11}}{ Z }$	<i>Y</i> ₂₂	$\frac{A}{B}$
<i>A</i>	$\frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	<i>A</i>
<i>B</i>	$Z_0 \frac{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}{2S_{21}}$	$\frac{ Z }{Z_{21}}$	$\frac{-1}{Y_{21}}$	<i>B</i>
<i>C</i>	$\frac{1}{Z_0} \frac{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{- Y }{Y_{21}}$	<i>C</i>
<i>D</i>	$\frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	<i>D</i>

211 $|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}; \quad |Y| = Y_{11}Y_{22} - Y_{12}Y_{21}; \quad \Delta Y = (Y_{11} + Y_0)(Y_{22} + Y_0) - Y_{12}Y_{21}; \quad \Delta Z = (Z_{11} + Z_0)(Z_{22} + Z_0) - Z_{12}Z_{21}; \quad Y_0 = 1/Z_0$

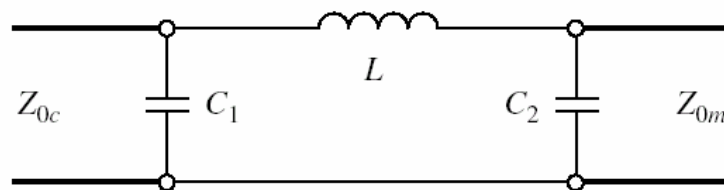
Equivalent circuits for two-ports



(a)



(b)



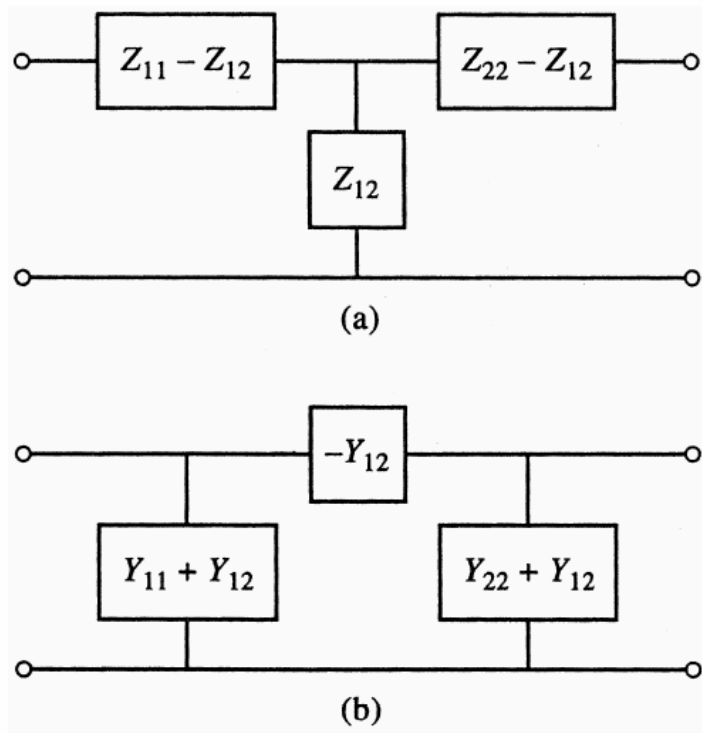
(c)

Discontinuity: storage
electrical-magnetic energy:
results in reactances

Example: coax-to- μ strip
transition representation:

- Black-box S-parameters
- Equivalent circuit with # idealized components

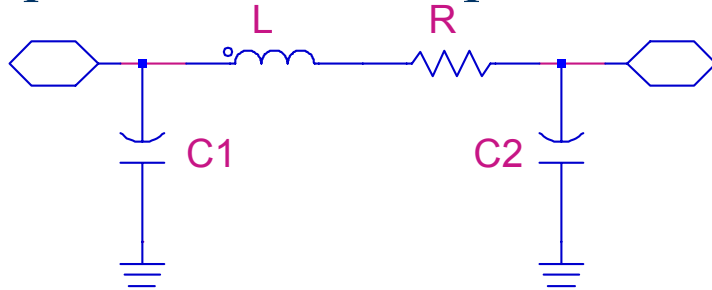
T and π -equivalent networks reciprocal 2-ports



- ◆ For reciprocal networks, six independent parameters needed (real, imag. 3 matrix elements)
- ◆ Leads to two possible equivalent networks:
 - using impedance: T
 - using admittance: π
- ◆ Lossless networks: elements purely reactive

Example: equivalent network spiral inductor

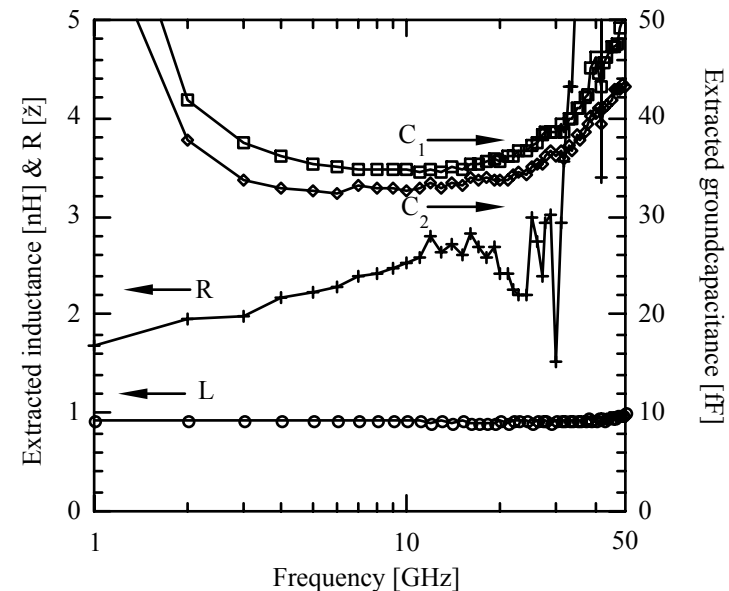
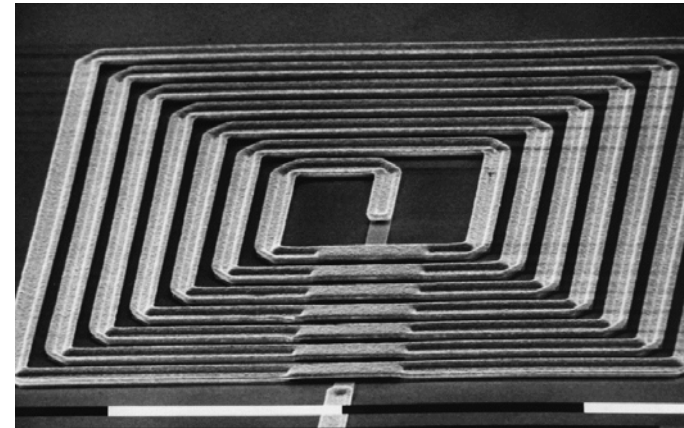
Π -equivalent circuit spiral inductor



$$L = -\text{Im} \left(\frac{1}{Y_{12}} \right) / \omega \quad R = -\text{Re} \left(\frac{1}{Y_{12}} \right)$$
$$C_1 = \frac{\text{Im} (Y_{11} + Y_{12})}{\omega} \quad C_2 = \frac{\text{Im} (Y_{22} + Y_{12})}{\omega}$$

Extraction procedure:

- ◆ Measure S-parameters (well-defined reference planes)
- ◆ Calculate L,R,C,.. afo frequency
- ◆ Take average where constant

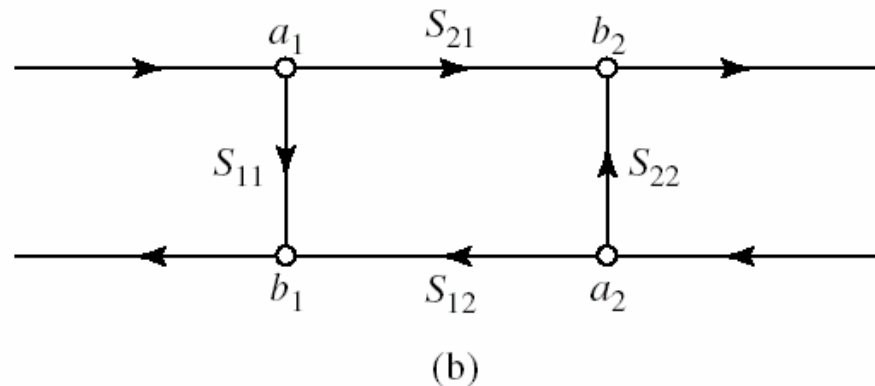
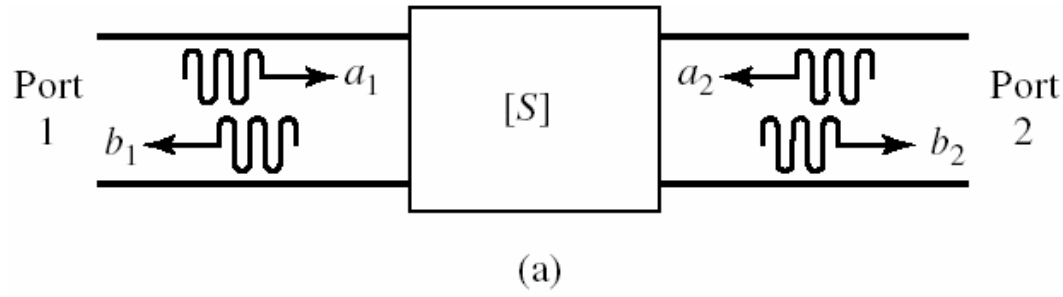


Signal flow graphs

- ◆ Technique for analysis of microwave networks in terms of transmitted and reflected waves
- ◆ Construction of signal flow graph: primary components are nodes and branches
 - **Nodes:** each port I of microwave network has two nodes a_i and b_i . Node a_i is identified with wave entering port i , b_i with wave reflected from port I
 - **Branches:** directed path between a-node and b-node representing signal flow, each branch has associated S-parameter or reflection coefficient

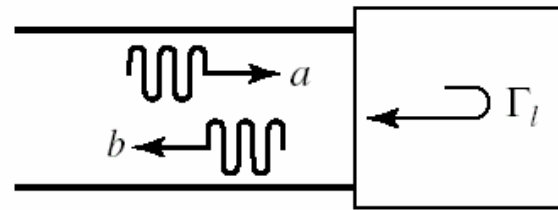
See also Agilent's application note on S-parameters posted on website

Signal flowgraph of two-port

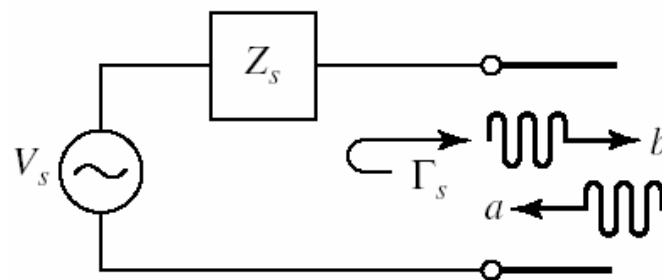
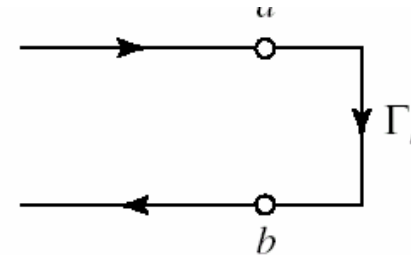


Wave a_1 incident at port 1 split, part through S_{11} and out port 1 as reflected wave, part transmitted through S_{21} to node b_2 . At node b_2 wave goes out port 2, can be partly reflected by load re-enter two-port at a_2 , reflected back out port 2 through S_{22} , part transmitted out port 1 through S_{12}

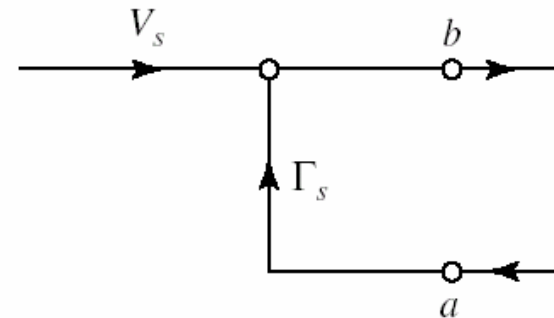
Network for one-port network and source



(a)



(b)

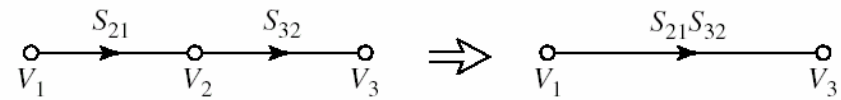


- ♦ Signal flow graph of microwave network can be solved for ratios of combination wave amplitudes using decomposition rules (or using Mason's rule control system theory)

Decomposition rules signal flow graphs

◆ Series rule

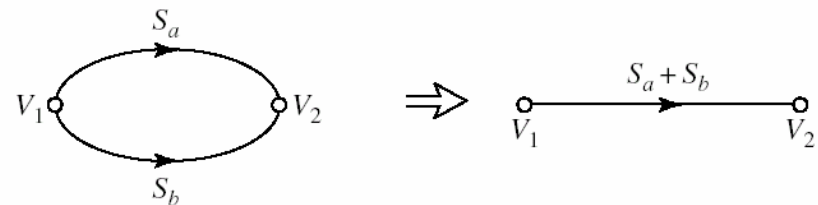
$$V_3 = S_{32}V_2 = S_{32}S_{21}V_1$$



(a)

◆ Parallel rule

$$V_2 = S_a V_1 + S_b V_1 = (S_a + S_b) V_1$$

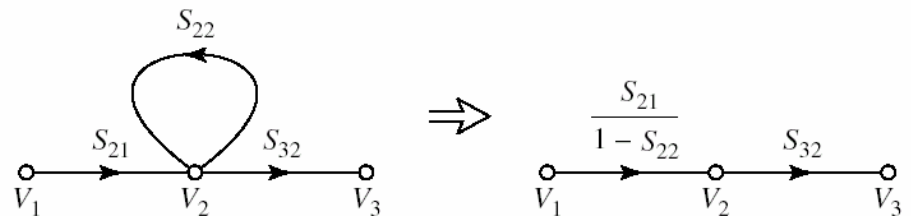


(b)

◆ Self-loop rule

$$V_2 = S_{21}V_1 + S_{22}V_2$$

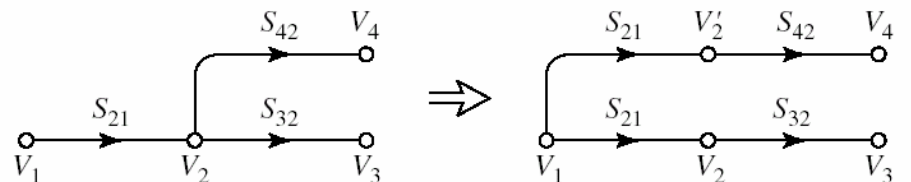
$$V_2 = \frac{S_{21}}{1 - S_{22}} V_1$$



(c)

◆ Splitting rule

$$V_4 = S_{42}V_2 = S_{21}S_{42}V_1$$



(d)

Example signal flow graph (1)

— EXAMPLE 4.7 Application of Signal Flow Graph

Derive the expression for Γ_{in} for the terminated two-port network shown in Figure 4.17 using signal flow graphs and the above decomposition rules.

Solution

The signal flow graph for the circuit of Figure 4.17 is shown in Figure 4.18. We wish to find $\Gamma_{\text{in}} = b_1/a_1$. Figure 4.19 shows the four steps in the decomposition of the flow graphs, with the final result that

$$\Gamma_{\text{in}} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_{\ell}}{1 - S_{22}\Gamma_{\ell}},$$

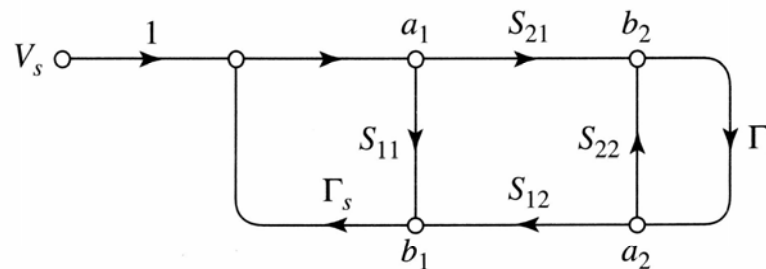


FIGURE 4.18 Signal flow path for the two-port network with general source and load impedances of Figure 4.17.

Example signal flow graph (2)

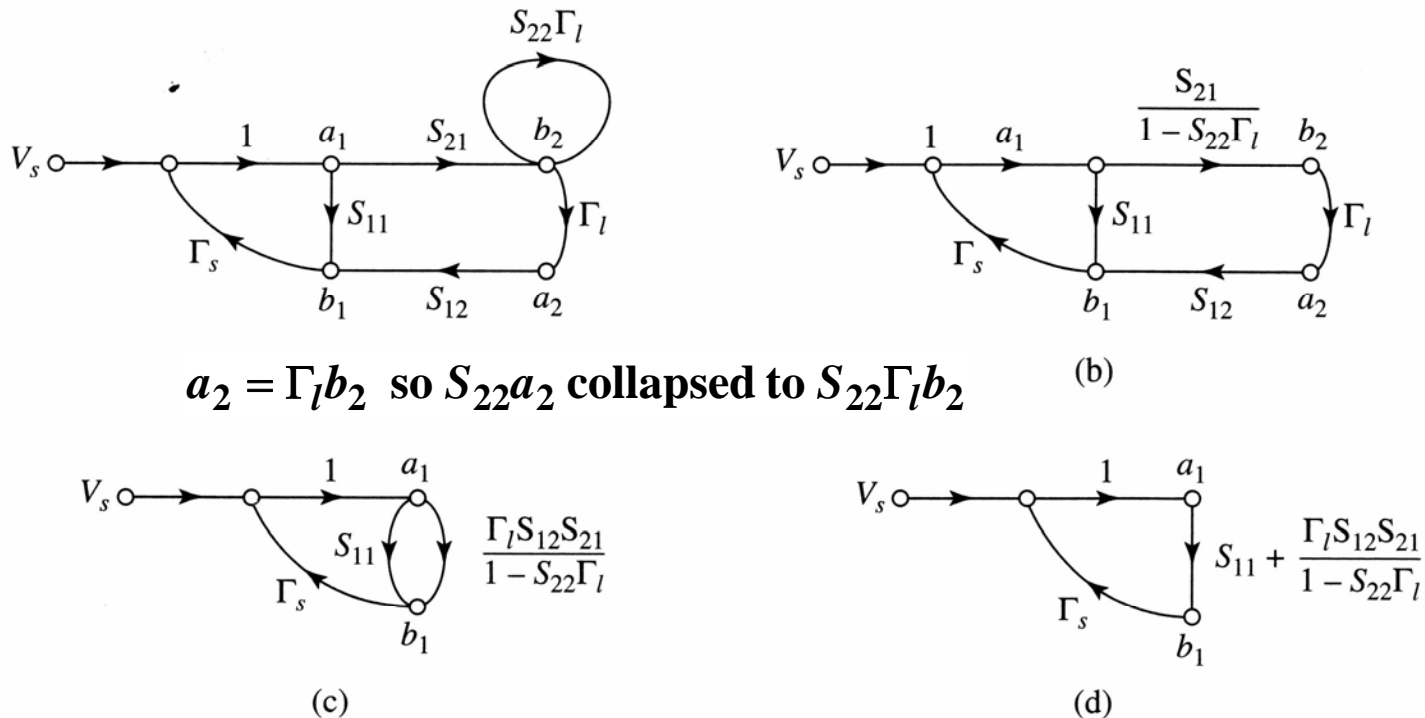


FIGURE 4.19 Decomposition of the flow graph of Figure 4.18 to find $\Gamma_{in} = b_1/a_1$. (a) Using Rule 4 on node a_2 . (b) Using Rule 3 for the self-loop. (c) Using Rule 1. (d) Using Rule 2.

Another example in book involves application of signal flow graphs to determine error boxes of TRL-calibration VNA

Summary: Calculation of Microwave Networks

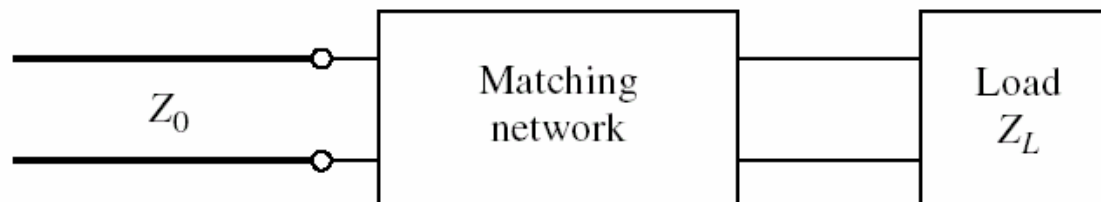
- ◆ Write out the equations for S-parameter matrix

$$V^- = S \cdot V^+ \quad S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \text{ for } k \neq j}$$

- S_{ii} : reflection coefficient into port i with other ports terminated
- S_{ij} : transmission coefficient from port j to i, other terminated
- ◆ Use signal Flow Graph Techniques
 - Four decomposition rules to simplify network
 - Mason's rule
- ◆ Calculate other parameters such as Z-Y or ABCD (cascade) and convert back to S-parameters

Impedance Matching and Tuning

- ◆ Matching network: lossless (ideally) network matching arbitrary load impedance (non-zero real part) to a TL

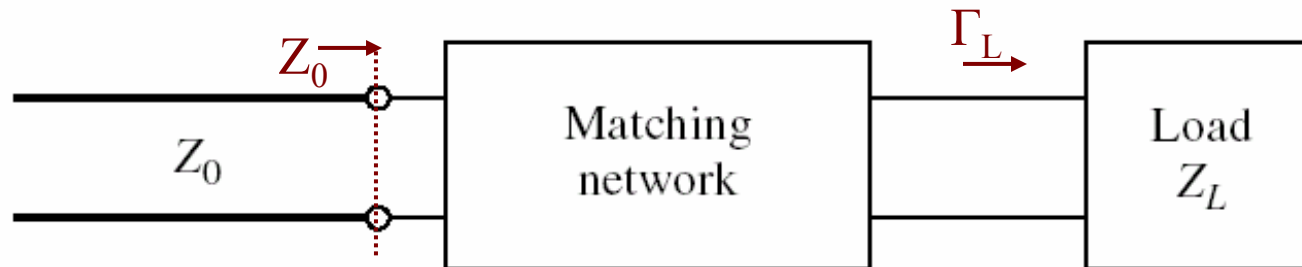


- ◆ Maximum **power** is delivered when the load and generator are matched to the line.
- ◆ Proper input impedance transformation of sensitive **receiver** components (antenna, LNA, etc.) improves the S/N ratio
- ◆ For power amplifier often transformation load to optimum load line needed to increase power output active device
- ◆ Impedance matching in a power distribution network (such as antenna array feed network) will reduce amplitude and phase **errors**.

Transforming Network Selection Criteria

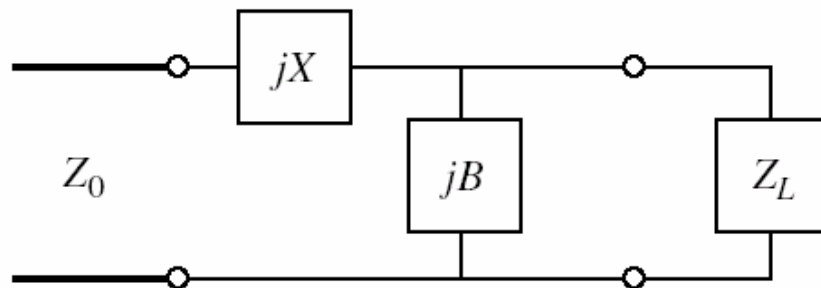
- ◆ **Complexity** — A simpler impedance transformation network is usually cheaper, more reliable, and less lossy than a more complex design.
- ◆ **Bandwidth** — typical matching network gives only match at single frequency, larger BW → increase in complexity (for instance multi-section transformers).
- ◆ **Implementation** — Short-circuited stubs in coax and waveguide (shorting stubs easy to implement in waveguide). Open-circuited stubs in stripline and microstrip.
- ◆ **Adjustability** — some applications may require adjustments (tuning stubs with micrometer in waveguides).

Lossless Matching Network

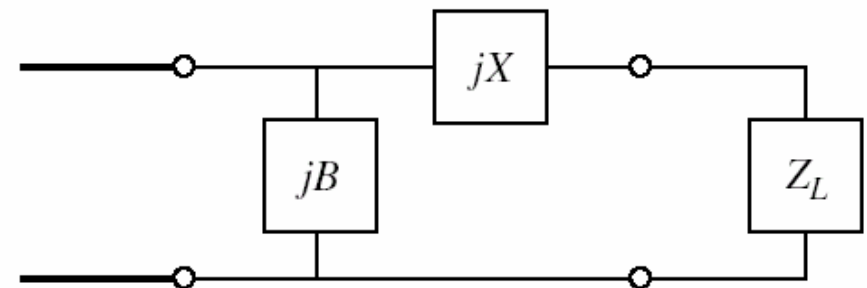


- ◆ In general, for network matching an arbitrary load impedance to a transmission line:
 - To avoid unnecessary power loss, matching network is ideally lossless.
 - The impedance looking in to the matching network is Z_0 .
 - Reflections are eliminated on the transmission line to the left of the matching network.
 - There will be multiple reflections between the matching network and the load.

Matching with lumped elements (L networks)

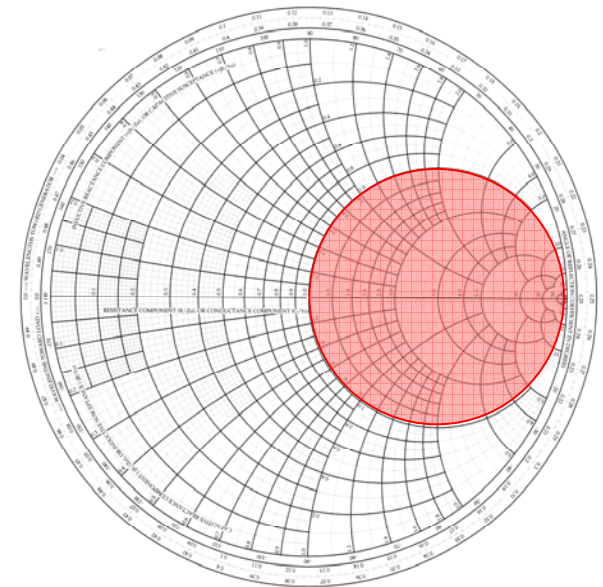


(a)

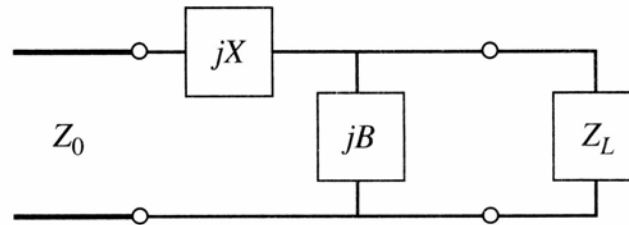


(b)

- ◆ Simplest type matching is L-section with 2 reactive elements
- ◆ Two possible configurations:
 - (a): network for z_L within $1+jx$ circle
 - (b): network for z_L outside $1+jx$ circle
- ◆ Reactive elements: capacitor or inductor
- ◆ Parasitics limit usable frequency range
- ◆ Solutions analytical or using Smith Chart



Analytical Solution Lumped Element Matching



$$Z_L = R_L + jX_L$$

$$R_L > Z_0 \quad (\text{within } r = 1 \text{ circle})$$

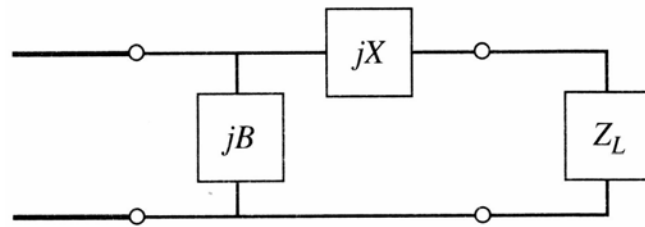
(a)

- For match to TL: $Z_0 = jX + \frac{1}{jB + 1/(R_L + jX_L)}$
- Separating into real and imaginary parts gives 2 equations for X and B, solving gives quadratic equation for B:

$$B = \frac{X_L \pm \sqrt{R_L/Z_0} \sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}$$

- X becomes: $X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}$
- Two solutions physically possible for B (pos. B: C, neg. B: L) and X (pos. X: L, neg. X: C), one solution often preferred in terms of size, bandwidth or SWR line feeding load

Analytical Solution Lumped Element Matching



$$Z_L = R_L + jX_L$$

$$R_L < Z_0 \quad (\text{outside } r = 1 \text{ circle})$$

(b)

- For match to TL: $\frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)}$
- Separating into real and imaginary parts gives 2 equations for X and B, solving gives :

$$X = \pm \sqrt{R_L(Z_0 - R_L)} - X_L$$

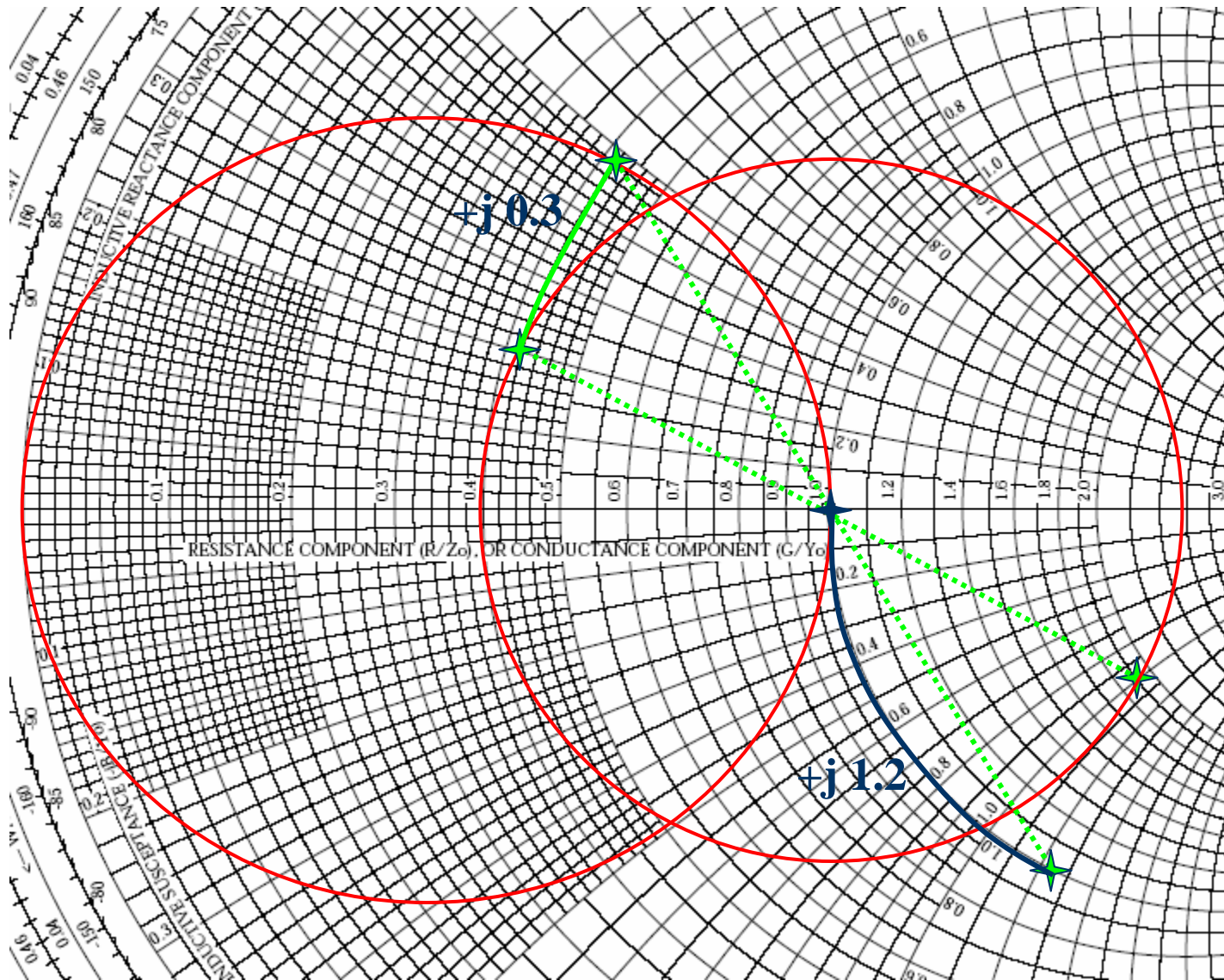
$$B = \pm \frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$

- Again two solutions possible
- In general, two degrees of freedom needed for matching (two equations to satisfy: $\text{Re}\{Z_{\text{in}}\} = Z_0$; $\text{Im}\{Z_{\text{in}}\} = 0$)

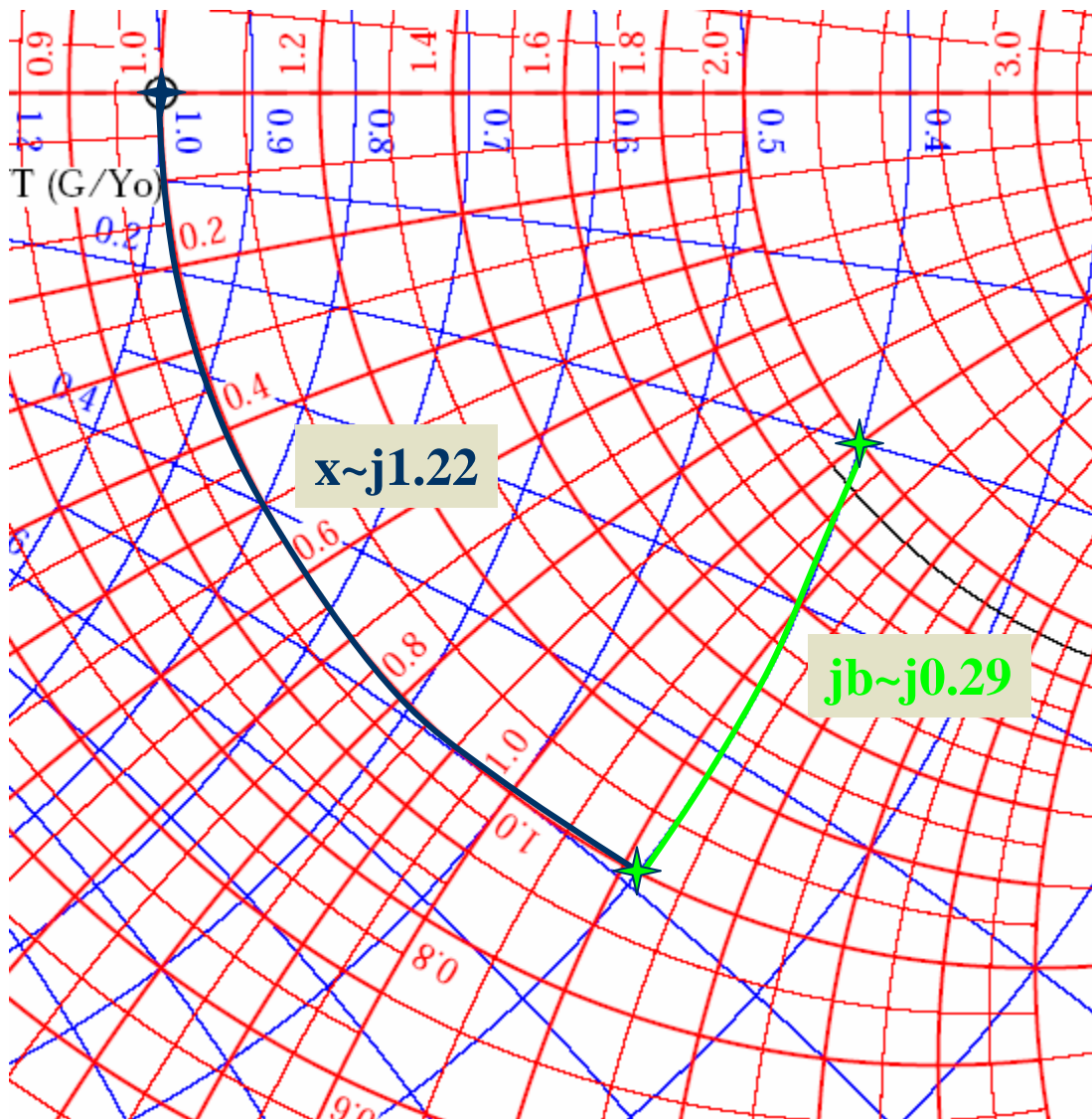
Matching solution on Smith Chart

- ◆ Ex. 5.1: load $Z_L = 200 - j100$, match to $100\ \Omega$ line at 500 MHz
- ◆ First: normalized load impedance z_L put on Smith chart
- ◆ z_L is within unit resistance circle: first shunt susceptance (a)
- ◆ to add shunt susceptance: z_L converted to y_L by imaging
- ◆ shunt susceptance will follow constant resistance circle on Z-chart, constant conductance circle for admittance Y-chart
- ◆ after imaging back impedance needs to be on $1 + jx$ circle: so shunt susceptance needs to move from y_L to $1 + jx$ circle on admittance chart (mirrored version circle Z-chart)
- ◆ after adding susceptance, imaging brings back to impedance
- ◆ from impedance on $1 + jx$ circle, value series reactance to reach $50\ \Omega$ can be determined

Solution LC-matching on Z Smith Chart



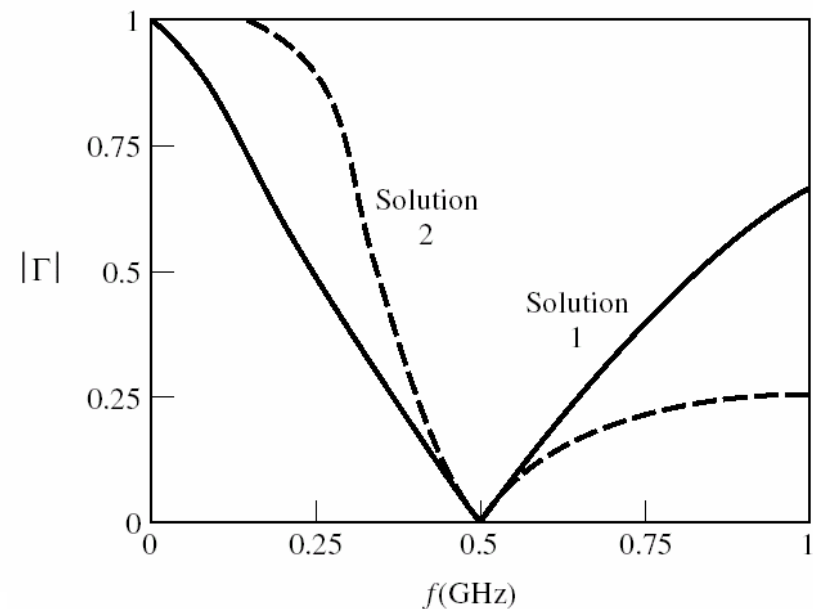
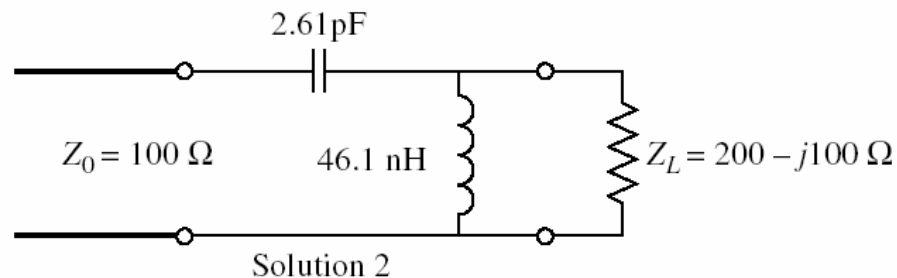
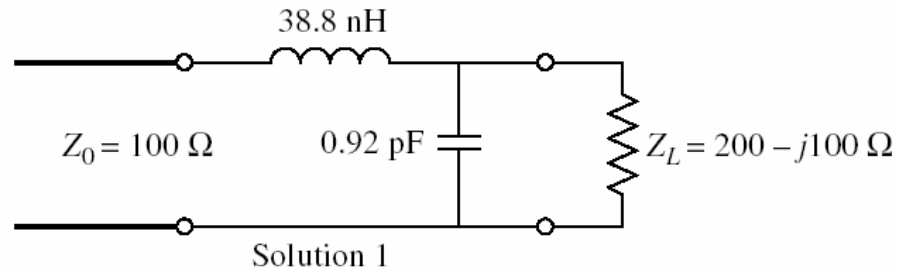
Same on combined ZY chart, only bit faster.....



$$C = \frac{b}{2\pi f Z_0} = 0.92 \text{ pF}$$

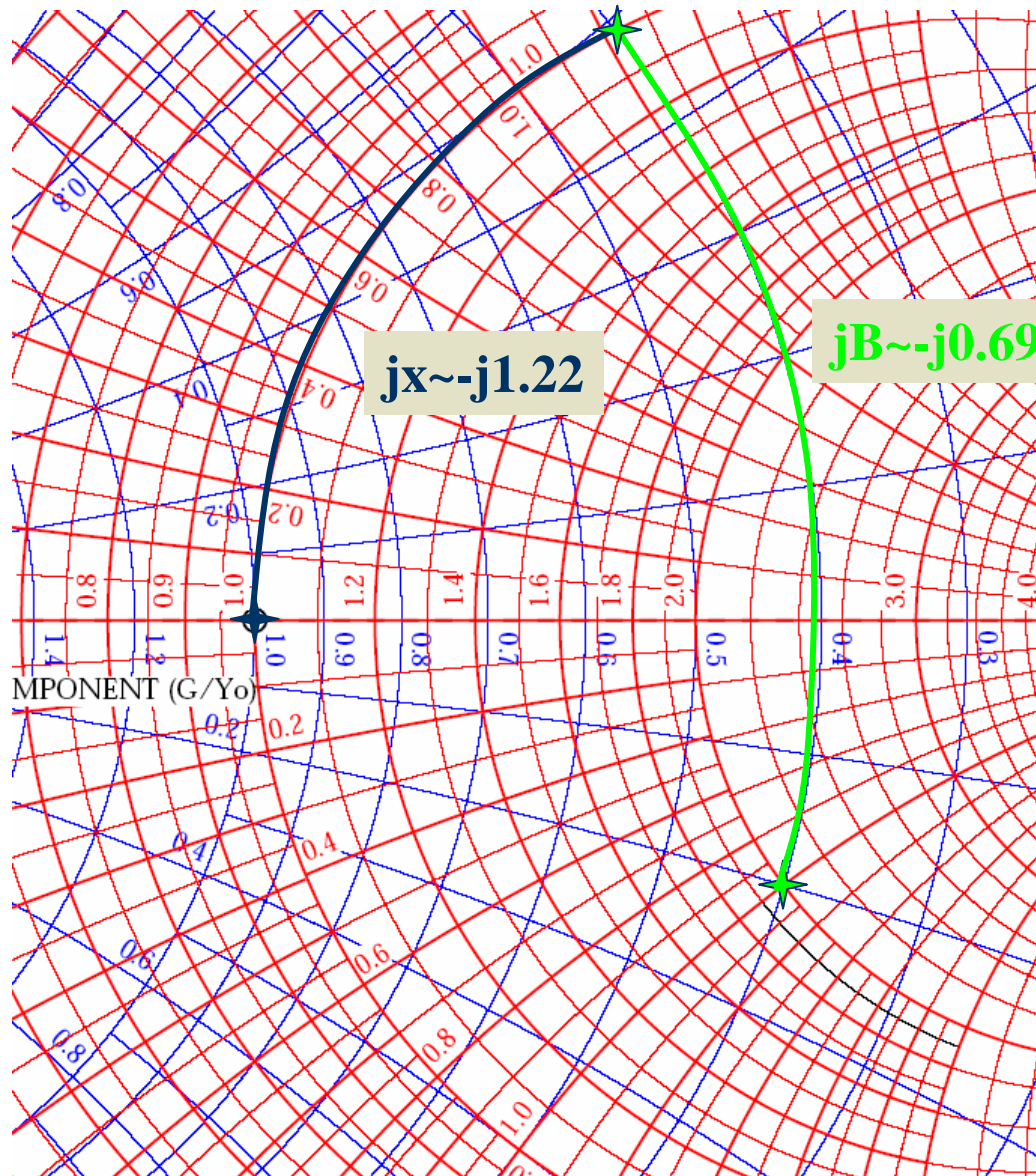
$$L = \frac{x Z_0}{2\pi f} = 38.8 \text{ nH}$$

Possible L matching circuits



- ◆ For alternative solution see next slide
- ◆ Choice solution based on component values (typically the smaller the better for losses & resonance frequency) or BW
- ◆ Also the ability to bias active device can be important

Alternative solution on ZY chart

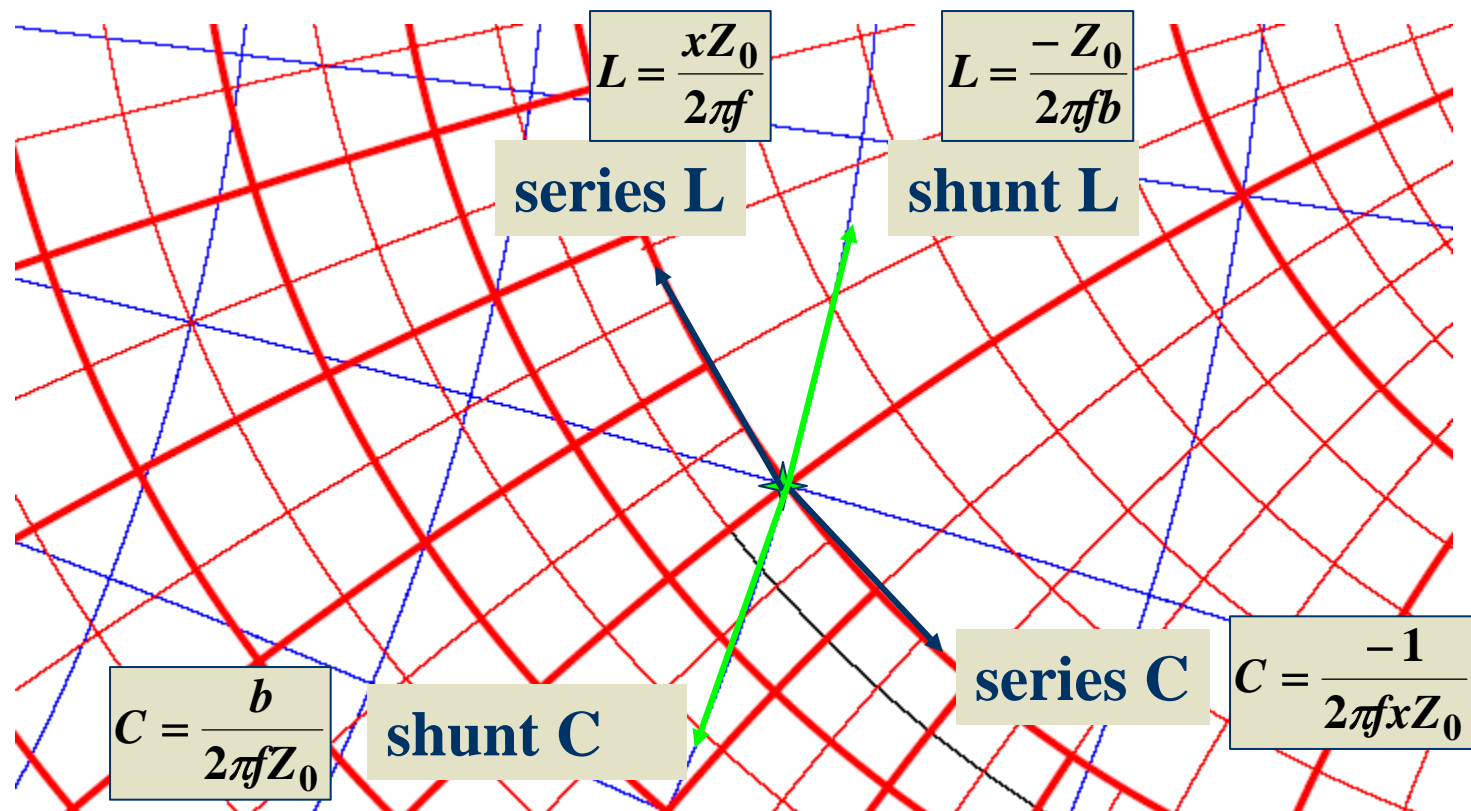


$$C = \frac{-1}{2\pi f x Z_0} = 2.61 \text{ pF}$$

$$L = \frac{-Z_0}{2\pi f b} = 46.1 \text{ nH}$$

LC matching on ZY chart

- ◆ Adding series reactance: rotate along circle of constant resistance (clockwise:L, counterclock:C)
- ◆ Adding shunt susceptance: rotate along circle constant conductance (clockwise:C, counterclock:L)



Homework 3 & next lecture!!

- ◆ Pozar, “Microwave Engineering” (3rd Ed.!) Will put on site!
 - 4.10
 - 4.16
- ◆ Due date: 2/23
- ◆ Next week we’ll review finish impedance matching and review HW1&2