E6318 - Microwave Circuit Design

Columbia University

Spring 2006

Yves Baeyens

Outline of Lecture 2

- Recap of lecture 1 (transmission line theory)
  - demonstrate TL program of Agilent
- The Smith Chart
- Quarter wavelength transformer
- Load and generator mismatches
- Terminated lossy lines
- Transients in TL's,
- Homework!

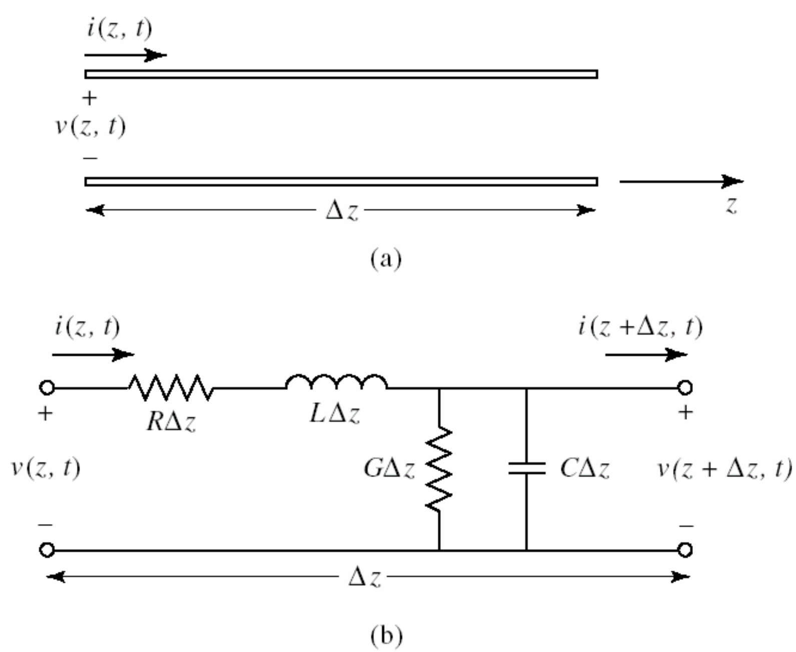
Announcements!

- TA/CA: Austin Chen
- Webpage: http://www.cisl.columbia.edu/~ee6318/
- Please check website regularly for updates on assignments, possible rescheduling of classes, etc…
- username: ee6318
  password: 4MWcour53only

Calendar

- Course: Th 4:10-6:40 PM, 1127 Mudd
  - 01/19
  - 01/26
  - 02/02
  - 02/09
  - 02/16
  - 02/23
  - 03/02
  - 03/09 **Midterm**
  - 03/16 Spring Holidays
  - 03/23
  - 03/30
  - 04/06
  - 04/13
  - 04/20
  - 04/27
  - **Final** (05/11)

Recap transmission line theory



General solution!

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$$

$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$$

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \frac{V_o^+}{I_0^+} = -\frac{V_o^-}{I_0^-}}$$

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-\gamma z} - \frac{V_o^-}{Z_0} e^{+\gamma z}$$

Complex propagation constant: $\boxed{\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}}$

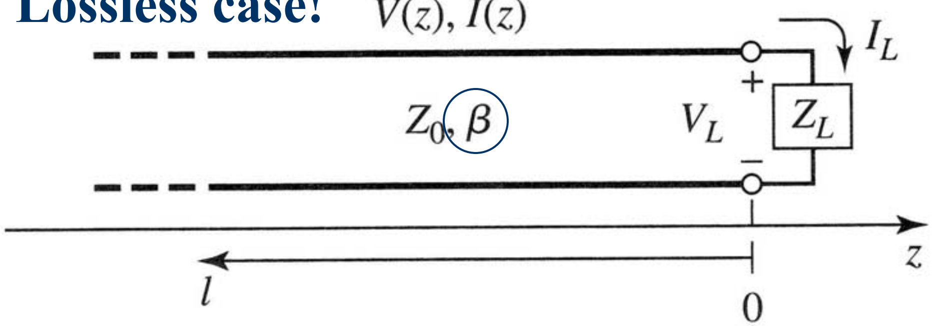
Wavelength: $\boxed{\lambda = \frac{2\pi}{\beta}}$

Phase velocity: $\boxed{v_p = \frac{\omega}{\beta} = \lambda \cdot f}$ TEM $\boxed{v_p = c/\sqrt{\varepsilon_r}}$

Recap transmission line theory (2)

To obtain relation voltage amplitude incoming & reflected wave : boundary condition set: termination impedance

Lossless case!



$$V(z) = V_o^+ \left[e^{-j\beta z} + \Gamma e^{+j\beta z} \right]$$

$$I(z) = \frac{V_o^+}{Z_0} \left[e^{-j\beta z} - \Gamma e^{+j\beta z} \right]$$

$$\boxed{\Gamma(l) = \Gamma(0) e^{-2j\beta l}}$$

$$\boxed{\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

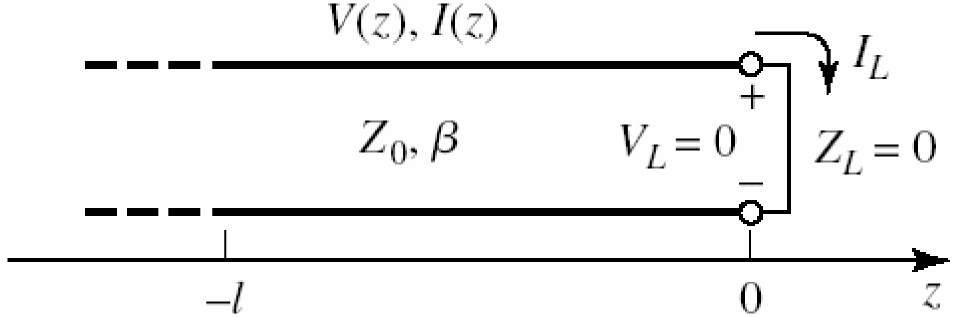
$$\boxed{P_{av} = \frac{1}{2} \frac{\left|V_o^+\right|^2}{Z_0} \left(1 - |\Gamma|^2 \right)}$$

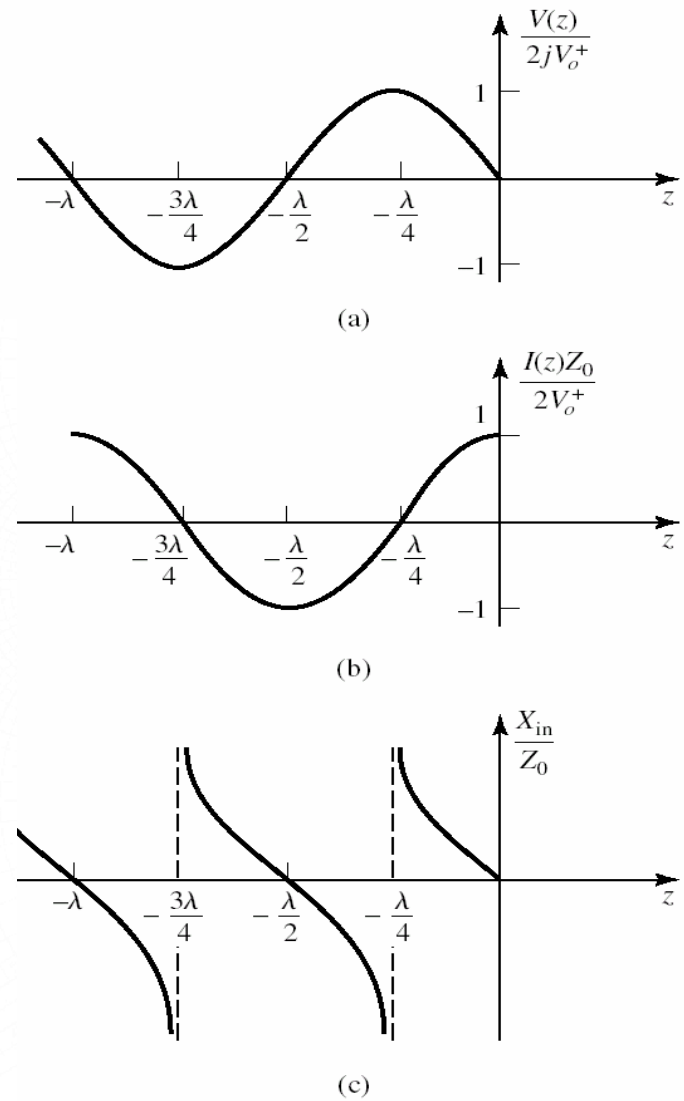
$$\boxed{\left|V(z)\right| = \left|V_o^+\right| \left| 1 + |\Gamma| e^{j(\theta - 2\beta l)} \right|}$$

$$\boxed{Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}}$$

Interference incoming & reflected wave causes variation total voltage (& current) magnitude and impedance along line

Special terminations (1): short circuit





- if Z_L=0, then Γ= -1, so

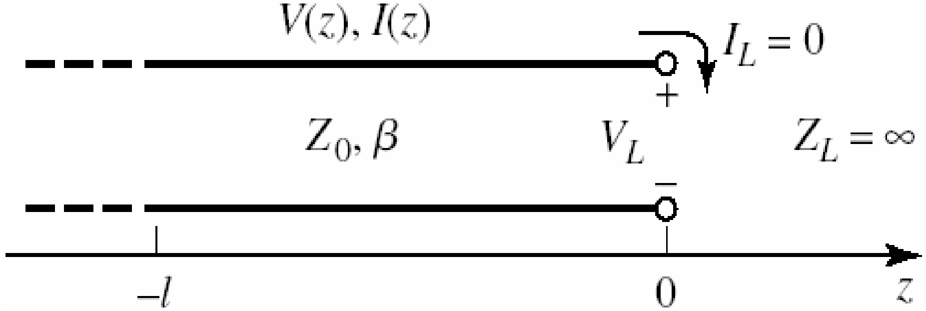
$$V(z) = V_o^+\left[e^{-j\beta z} - e^{+j\beta z}\right] = -2jV_o^+ \sin \beta z$$

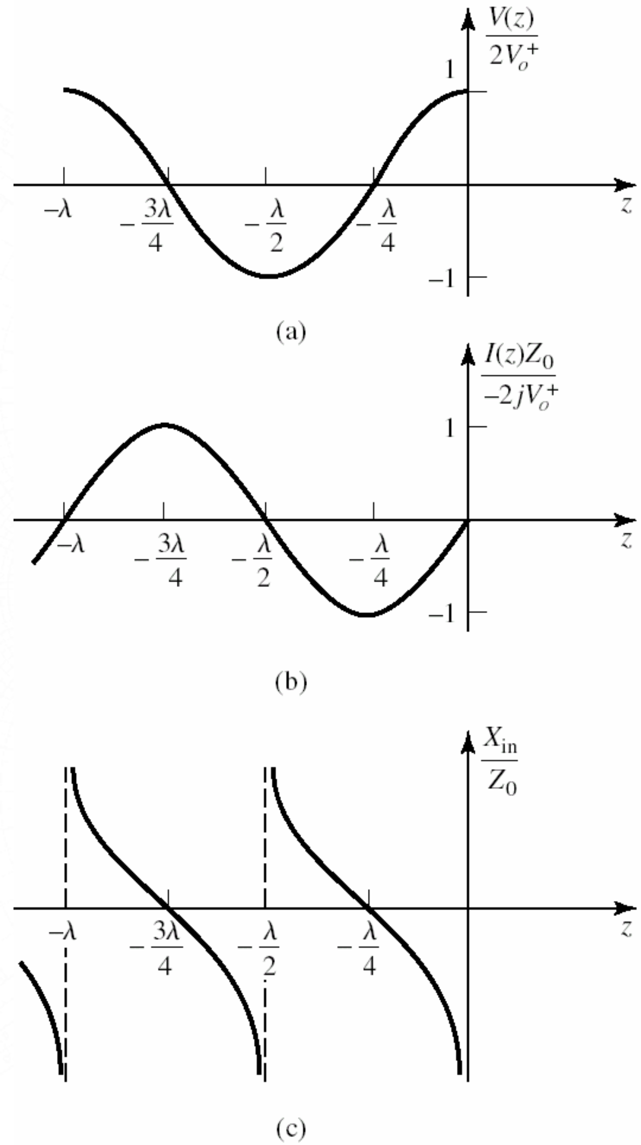
$$I(z) = \frac{V_o^+}{Z_0}\left[e^{-j\beta z} + e^{+j\beta z}\right] = \frac{2V_o^+}{Z_0} \cos \beta z$$

$$Z_{in} = \frac{V(-l)}{I(-l)} = jZ_0 \tan \beta l$$

- So impedance purely imaginary
- For l=0: Z_{in}=0 but for l=λ/4: Z_{in}= ∞
- Impedance is periodic with λ/2

Special terminations (2): open circuit





- if $Z_L=\infty$, then $\Gamma= +1$, so

$$V(z) = V_o^+\left[e^{-j\beta z} + e^{+j\beta z}\right] = 2V_o^+ \cos \beta z$$

$$I(z) = \frac{V_o^+}{Z_0}\left[e^{-j\beta z} - e^{+j\beta z}\right] = -\frac{2jV_o^+}{Z_0}\sin \beta z$$

$$Z_{in} = \frac{V(-l)}{I(-l)} = -jZ_0 \cot \beta l$$

- So impedance purely imaginary
- For $l=0$: $Z_{in}=\infty$ but for $l=\lambda/4$: $Z_{in}=0$
- Impedance is periodic with $\lambda/2$

Special lengths of transmission line

- A half-wavelength line ($l=n\lambda/2$) does not alter or transform the load impedance, regardless of Z_0

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \quad \Rightarrow \quad Z_{in}(\lambda/2) = Z_L$$

- A quarter-wavelength line ($l= \lambda/4+n\lambda/2$) transforms the impedance, according to characteristic impedance of line

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = \frac{Z_0^2}{Z_L}$$

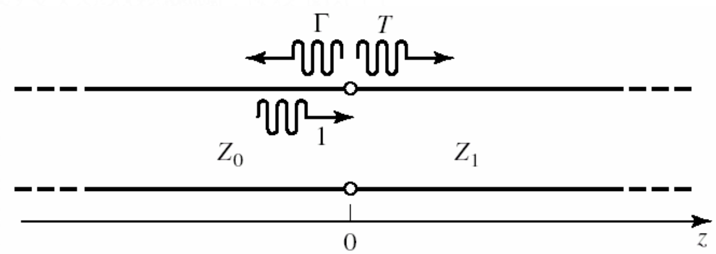
- This is called quarter-wave transformer, to be studied more in detail next lecture (2.5)

Termination with matched transmission line

- Line with characteristic impedance Z_0 feeds line with characteristic impedance Z_1, if this TL is infinitely long or terminated with Z_1, then $Z_L=Z_1$, so

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

- Not all incident wave reflected, part transmitted onto line 2 with voltage amplitude T



$$V(z) = V_o^+ \left[e^{-j\beta z} + \Gamma e^{+j\beta z} \right], \quad z < 0$$

$$V(z) = V_o^+ T e^{-j\beta z}, \quad z > 0$$

Equating both at z=0 gives $T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2 Z_1}{Z_1 + Z_0}$

- Transmission coefficient expressed as insertion loss

$$\boxed{IL = -20 \log |T| \quad \text{dB}}$$

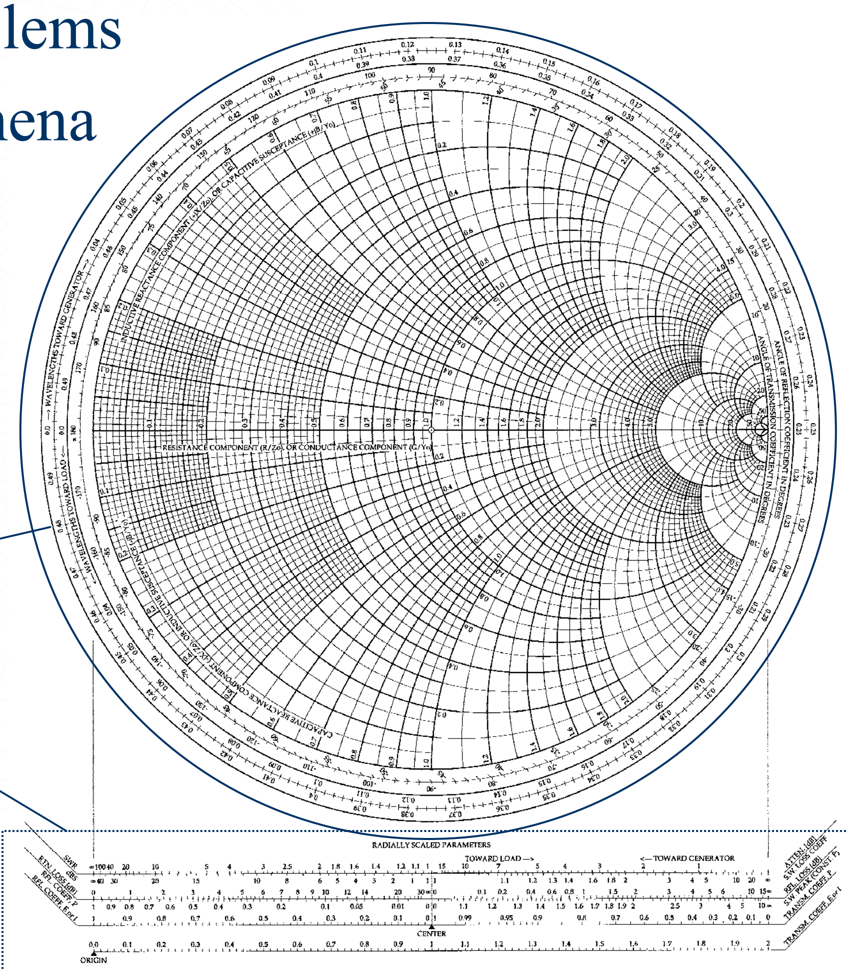
Transmission-Line Fundamentals Course

- The Transmission-Line Fundamentals Course can be found in Agilent Educator's Corner (http://www.educatorscorner.com) When you have completed this course, you should have a basic knowledge of the technology, terminology, and measurement techniques of transmission lines.
- This course can be found in the RF Corner under RF Computer-Based Training Modules section. Please follow-up instructions to download and install this program.
- Demonstrates intuitively the concept of TL-properties such as SWR by animation of incident and reflected waves
- Another helpful tool is the animated Wave Propagation along a Transmission Line Java applet in the RF Corner

The Smith Chart

Smith Chart: Polar plot of voltage reflection coefficient Γ

- Graphical aid for solving TL problems
- Useful for visualizing TL phenomena
- Easy conversion from Γ to normalized impedances using impedance (or admittance) circles printed on chart
- Two important scales:
  - Radial scale (in λ)
  - Scales for VSWR, RL, etc..
- See website for example (PDF)



Resistance and reactance circles on Smith Chart

- Reflection at load Z_L (z_L normalized) for lossless TL

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta} \quad \textbf{with} \quad z_L = Z_L / Z_0$$

- Solving for z_L in terms of Γ

$$z_L = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}$$

- Complex equation reduced to 2 real equations by writing Γ and z_L in real and imaginary parts ($z_L = r_L + jx_L$, $\Gamma = \Gamma_r + j\Gamma_i$)

$$r_L + jx_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}$$

Resistance and reactance circles on Smith Chart

- Yields equations for real and imaginary parts:

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} \qquad x_L = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

- Re-arranged:

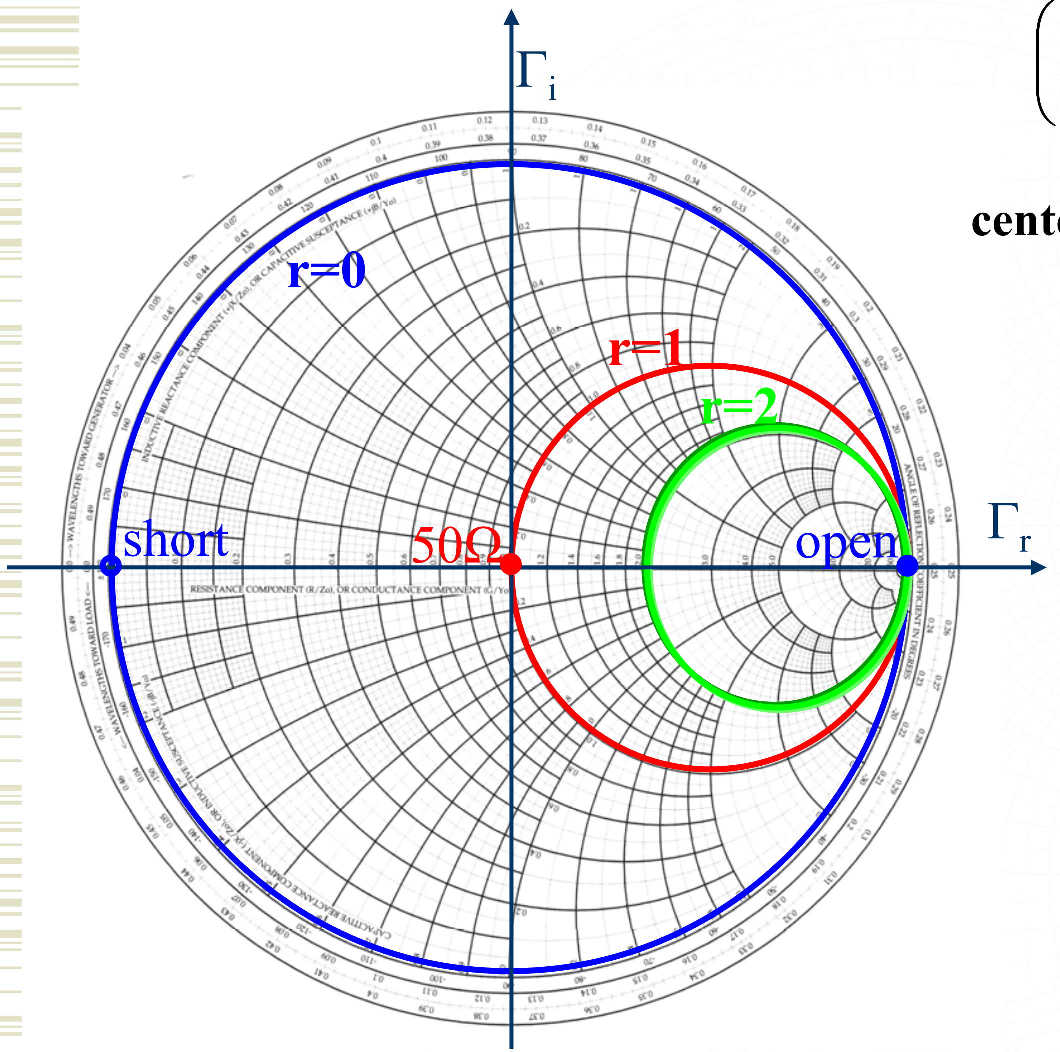
$$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2 \qquad (\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

- These formulas describe 2 families of circles in the Γ_r, Γ_i – plane: resistance circles and reactance circles:

$$\textbf{resistance circle has center} : \left(\frac{r_L}{1 + r_L}, 0\right), \ \textbf{radius} : \frac{1}{1 + r_L}$$

$$\textbf{reactance circle has center} : \left(1, \frac{1}{x_L}\right), \ \textbf{radius} : \frac{1}{x_L}$$

Resistance circles on Smith Chart

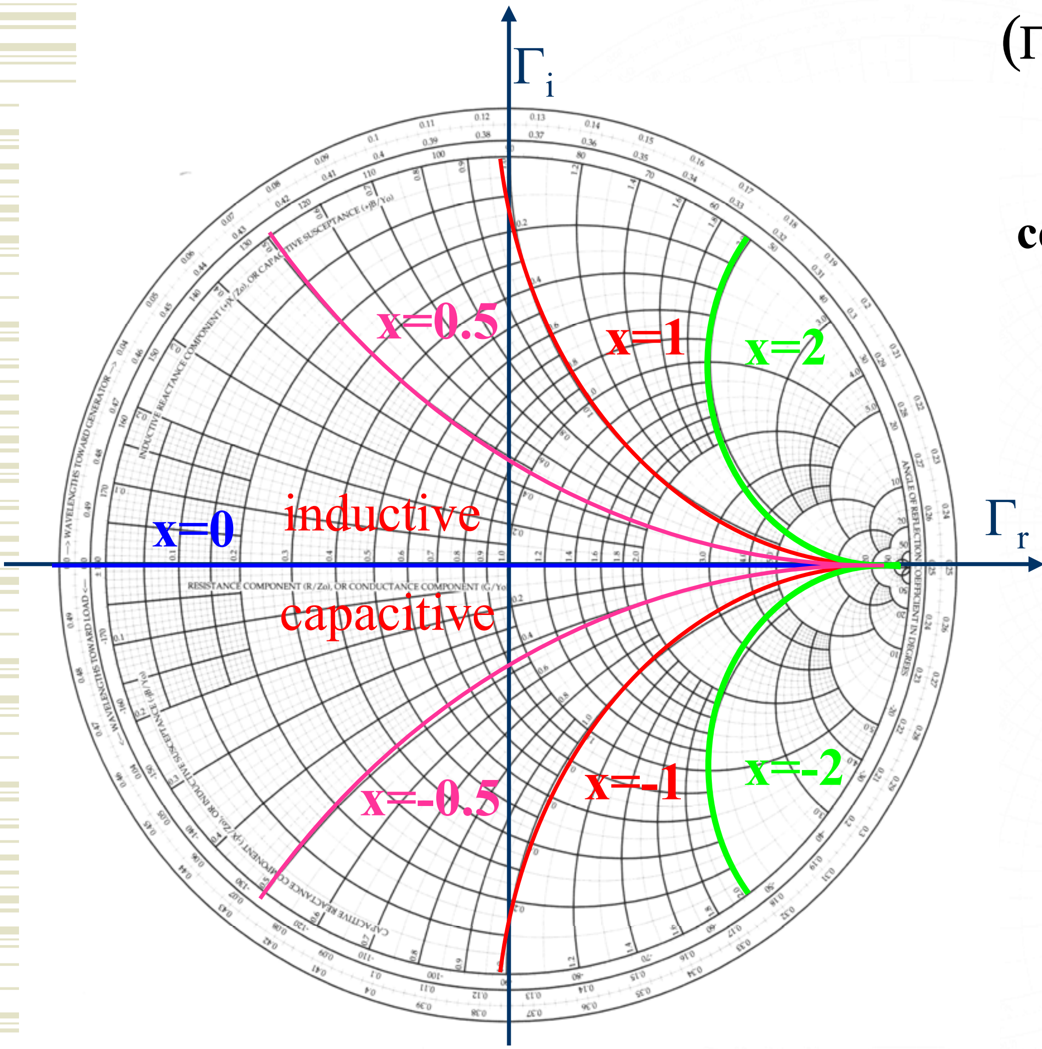


$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

$$\textbf{center} \quad : \left(\frac{r_L}{1+r_L}, 0\right), \quad \textbf{radius} \quad : \frac{1}{1+r_L}$$

- Centers lie on real axis
- Radius ↓ as r ↑
- All circles pass through (Γ_r=1, Γ_I=0)
- Only impedances with r>0 (passive) within chart

Reactance circles on Smith Chart

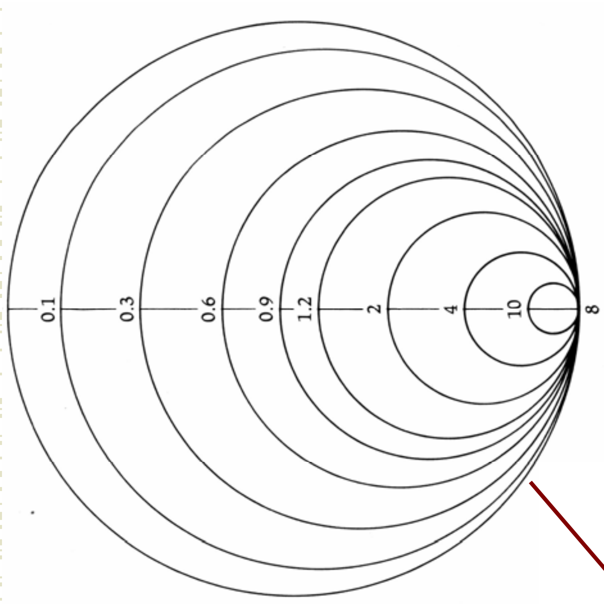


$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

$$\text{center} \; : \left(1, \frac{1}{x_L}\right), \;\; \text{radius} \; : \frac{1}{x_L}$$

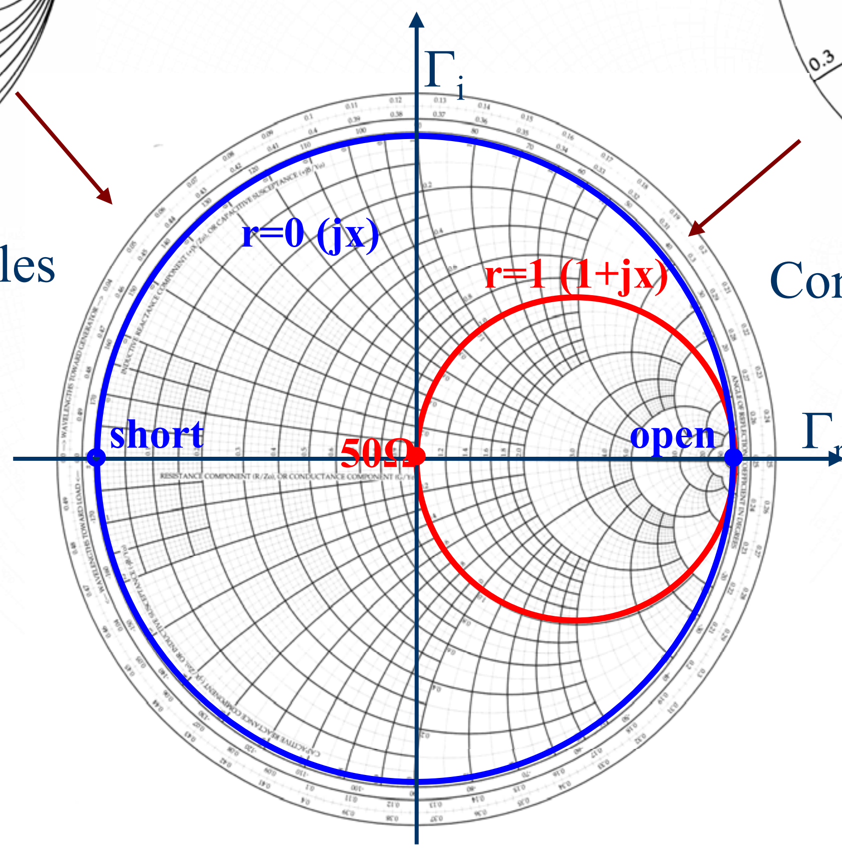
- x>0 : upper half : inductive
  x<0 : lower half : capacitive
- Radius ↓ as x ↑
- All circles pass through (Γ_r =1, Γ_i =0)

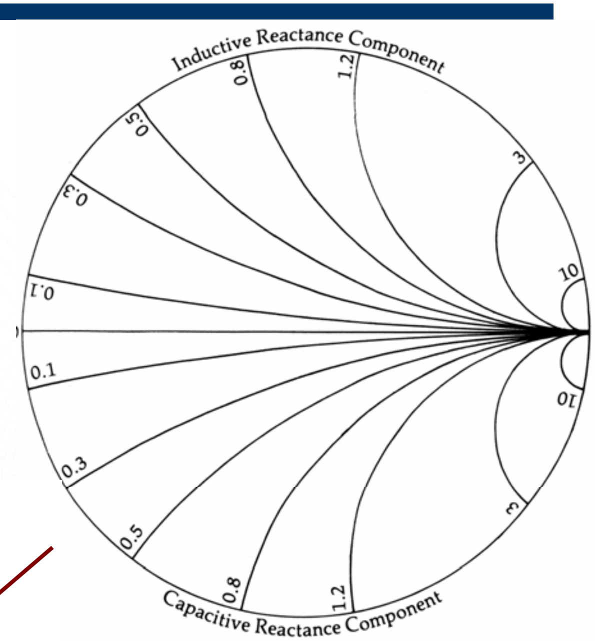
Construction Z-chart



Constant resistance circles

Smith chart: mapping complex impedance plane to complex reflection plane

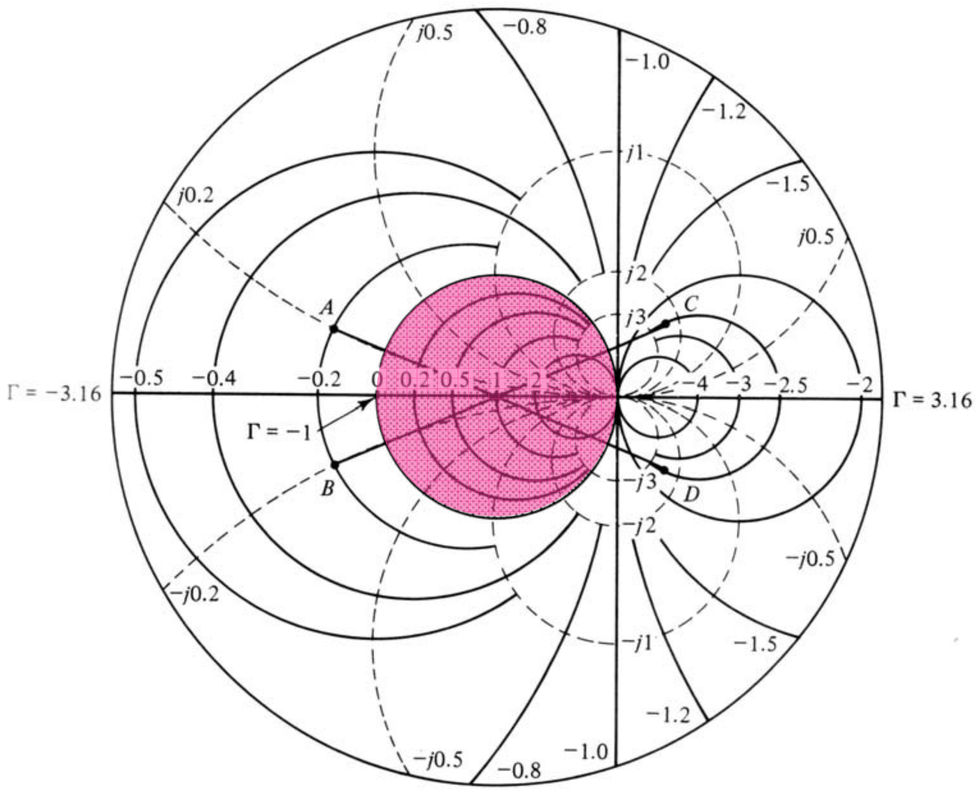




Constant reactance circles

norm. impedance:
z=r+jx
r: resistance
x: reactance

The Compressed Smith Chart

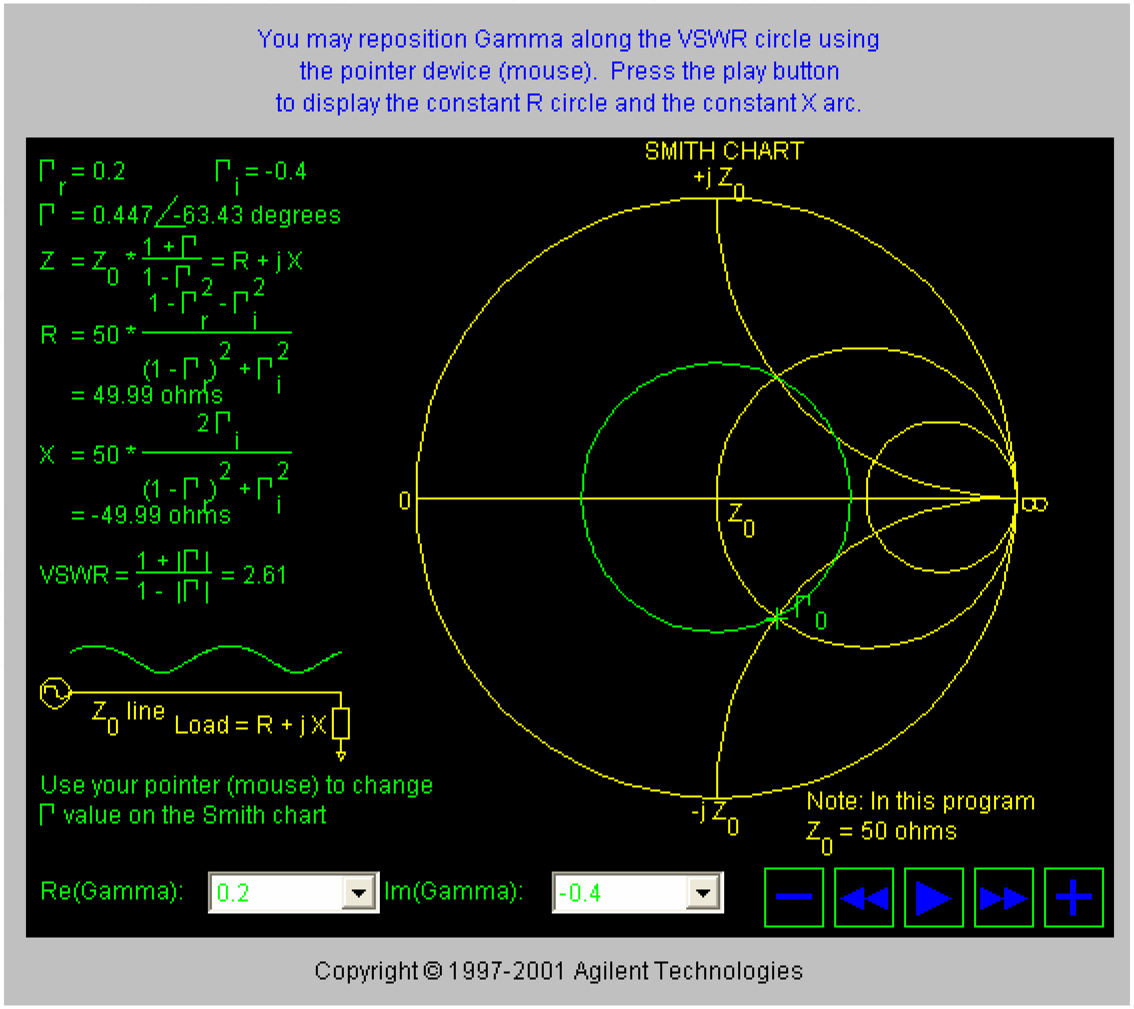


- For negative load resistance Γ-circles outside Chart
- The extended "compressed" Smith Chart adds region with return gain up to –10 dB
- Useful for the visualization of negative resistances used in oscillator or reflection-type amplifier design

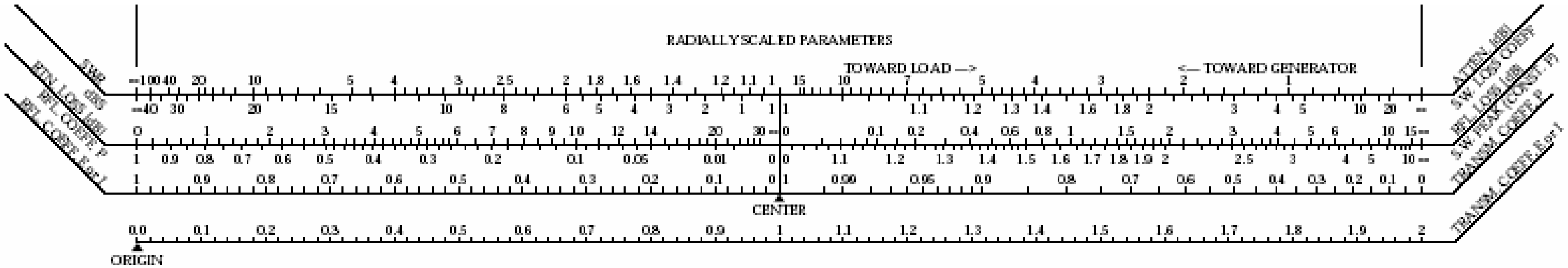
From S.Y. Liao:"Microwave Circuit Analysis and Amplifier Design", Prentice-Hall

The automated Smith Chart

- An animated (Java applet) Smith chart can be found in the Agilent Educators - RF Corner



Radially Scaled Parameters Smith Chart



- Scale on bottom Smith Chart allows for given impedance to graphically derive radially scaled microwave parameters such as VSWR, return loss, etc...
- Procedure:
  - Add impedance on Smith Chart (normalize!!) using r & x-circles
  - Take magnitude of reflection coefficient (Compass)
  - Use scales to read corresponding Voltage Reflection Coefficient (in magnitude), SWR, Power Reflected (%), Return Loss (dB), Power Transmitted (%), etc...

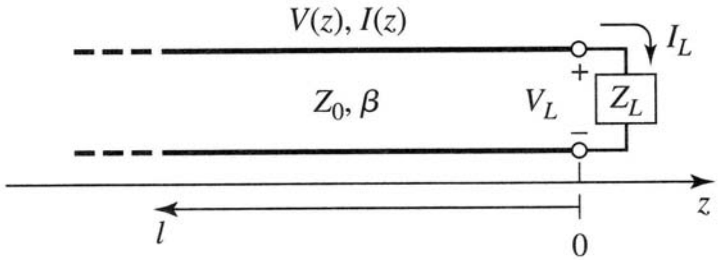
Solving TL impedance equation on Smith Chart

- From equation for Γ at distance l from load (2.42)

$$\Gamma(l) = \Gamma(0)e^{-2j\beta l}$$

- So, $\Gamma(l)$ only has different phase angle, $\Gamma(l)$ can obtained by plotting $\Gamma(0)$ on Smith Chart and rotating clockwise an amount of $2\beta l$ on circle around the center the chart
- To facilitate rotation, Smith Chart has scale around periphery calibrated in electrical wavelengths, ranging from 0 to 0.5 wavelength (or 0 to 360˚)
- TL length of $\lambda/2$: $2\beta l=2\pi$, no change
- TL length of $\lambda/4$: $2\beta l=\pi$, so rotation over 180˚ or equivalently imaging across center Smith Center
- !! (re)-normalization dependent on Z_0

Example of basic Smith Chart Operation

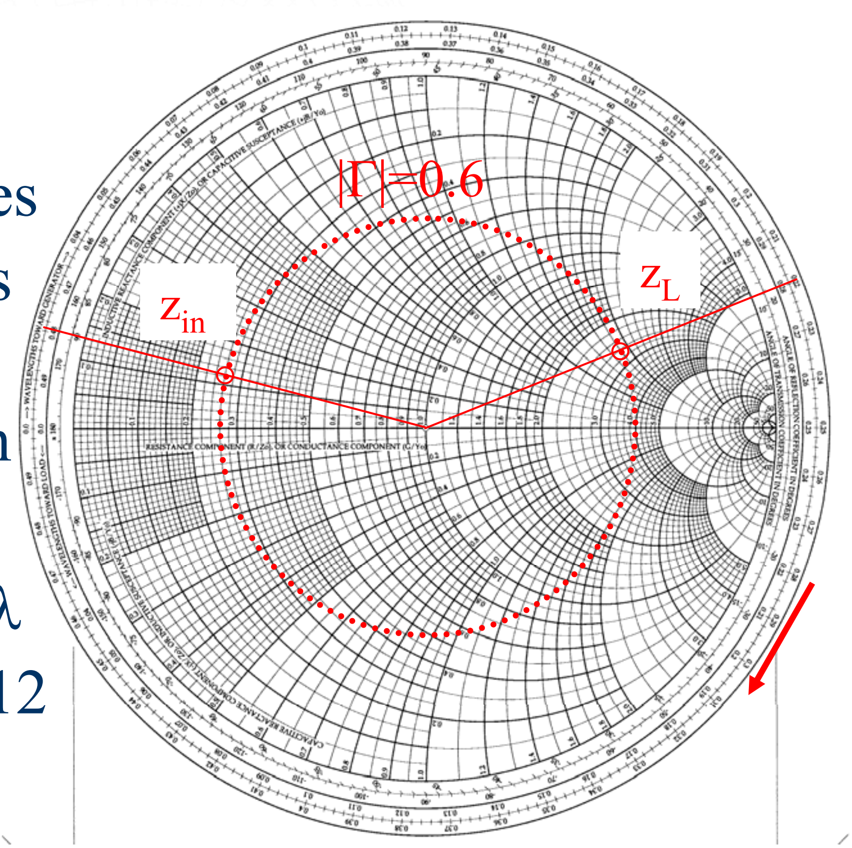


- $Z_L = 130 + j\,90\ \Omega$
- TL: $Z_0 = 50\ \Omega$, $l = 0.3\lambda$
- $\Gamma(0)$, $\Gamma(l)$, Z_{in}, SWR, RL?

- Normalized load impedance:

$$z_L = Z_L / Z_0 = 2.60 + j1.80$$

- Plotted on Smith chart using circles
- Radius circle (compass!) on scales gives $|\Gamma|$, SWR, RL
- Angle from radial line intersection
- Reference load on WTG: 0.22λ
- Moving down 0.3λ: 0.52λ or 0.02λ
- Radial line intersects @ 0.26+j.0.12
- Renormalized: $Z_{in} = 13 + j.6\ \Omega$



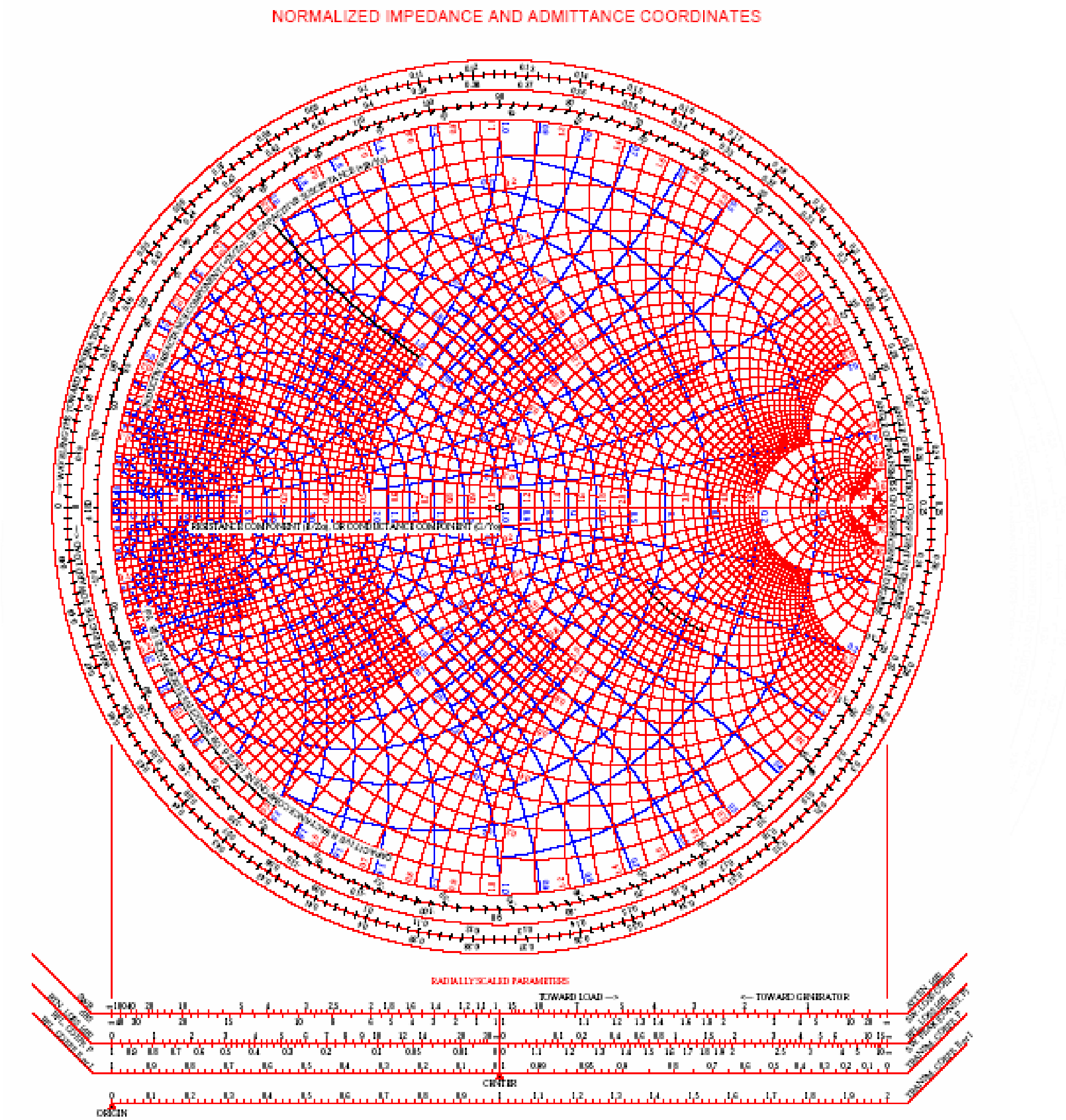
Combined Impedance-Admittance Smith Chart

- Smith Chart can also be used for normalized admittance + conversion between impedance & admittance
- In normalized form impedance load z_L after $\lambda/4$ line:

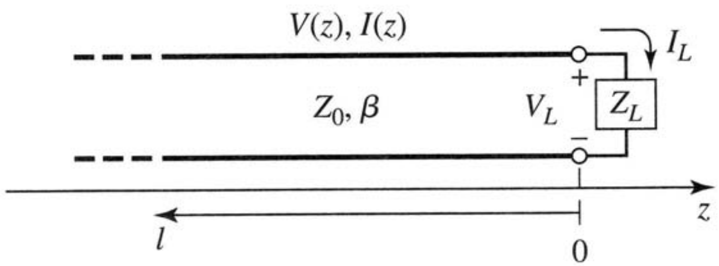
$$z_{in} = 1/z_L$$

- So converting from normalized impedance to admittance can be done by $\lambda/4$ transformation or rotating chart 180˚
- Same Smith chart used for impedance or admittance calculations
- Procedure less confusing using superposition of two scales: one regular and one rotated 180˚ usually different colored

Combined Impedance-Admittance Smith Chart



Example of Admittance Smith Chart Operation

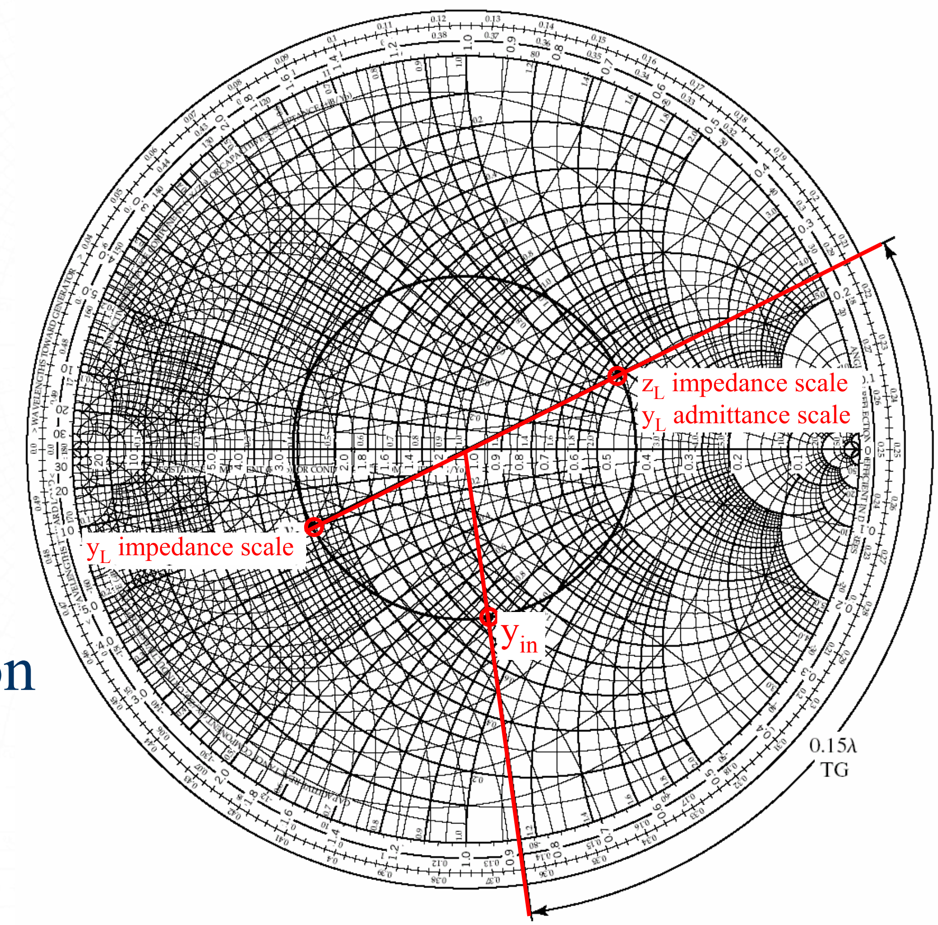


- Z_L=100+j 50 Ω
- TL: Z_0= 50 Ω, l = 0.15λ
- Y_L, Y_{in}?

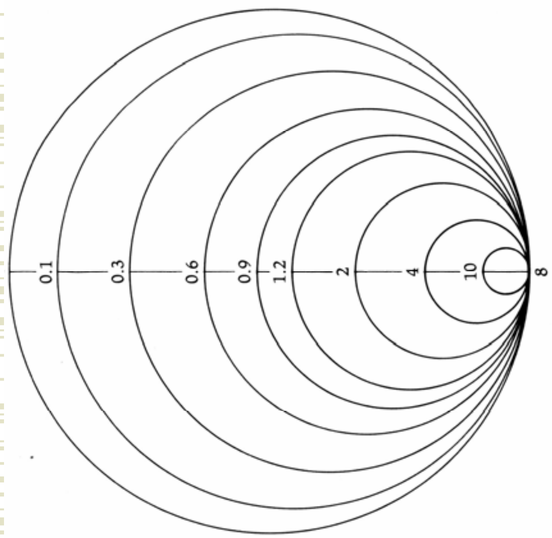
- Normalized load impedance:

$$z_L = Z_L / Z_0 = 2 + j1$$

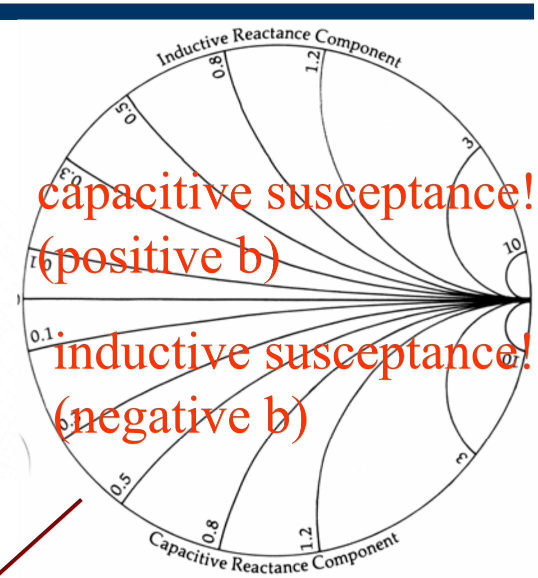
- z_L plotted on Smith chart
- y_L read on admittance scale
- Or determined on impedance scale after imaging
- y after 0.15λ line: rotation TG
- y_{in} determined from intersection SWR circle, read on y-scale
- Renormalized: $Y=y/Z_0$



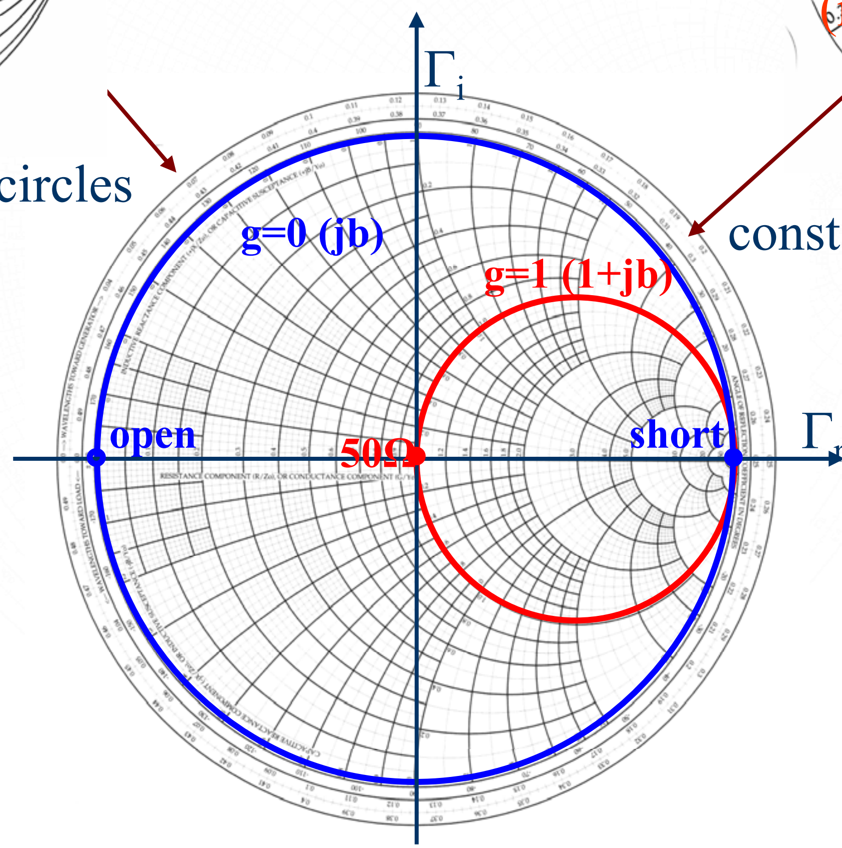
The Z-chart as admittance chart….



Use: first imaging to go from z_L to y_L or when y_L given



constant conductance circles



constant susceptance circles

norm. admittance:
y=g+jb
g: conductance
b: susceptance

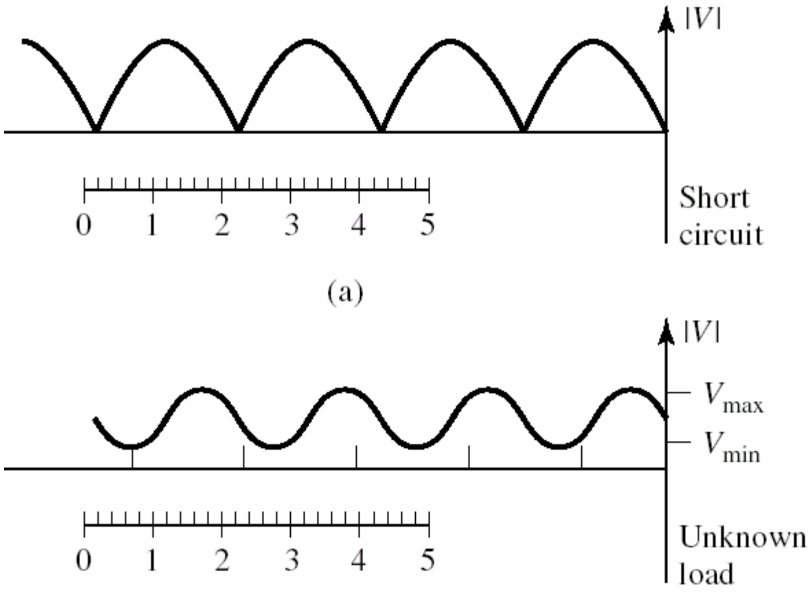
Smith chart application: The Slotted Line

- Slotted line: TL which allows direct sampling E-field amplitude
- Allows determination complex Z_L from: SWR and distance first voltage minimum from load
- First amplitude of Γ from SWR:

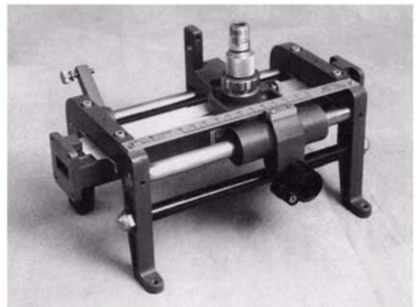
$$\mathbf{SWR} = \frac{1+|\Gamma|}{1-|\Gamma|} \Rightarrow |\Gamma| = \frac{\mathbf{SWR}-1}{\mathbf{SWR}+1}$$

- From: $|V(z)| = |V_o^+|\left|1+|\Gamma| e^{j(\theta-2\beta l)}\right|$ with θ phase angle Γ we know the minimum occurs when: $e^{j(\theta-2\beta l)} = -1$ or with l_{min} distance first minimum from load: $\theta = \pi + 2\beta l_{\mathbf{min}}$
- From this,we can see any minimum (nλ/2) is good
- Z_L from: $Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$

Example slotted line impedance measurement



- Short circuit:
  - min. at 0.2, 2.2, 4.2 cm
- Unknown load:
  - SWR=1.5,
  - min. at 0.72, 2.72 & 4.72 cm



- Minima repeat each $\lambda/2 \Rightarrow \lambda$=4 cm
- Minima short circuit reveals location load, any minima OK
- Taking min. at 4.2 cm next min. away from load 2.72 cm gives l_{min} = 4.2-2.72 =1.48 cm = 0.37 λ results in:

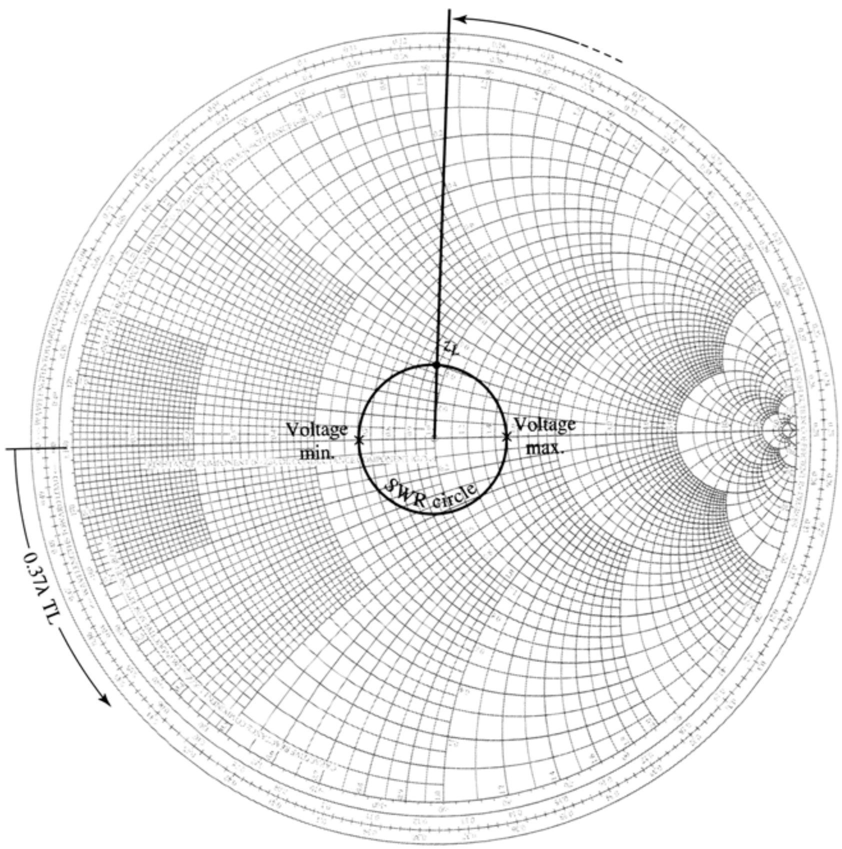
$$|\Gamma| = \frac{1.5-1}{1.5+1} = 0.2 \qquad \theta = \pi + \frac{2.2\pi}{4} \cdot 1.48 = 86.4°$$

$$\Gamma = 0.2e^{j86.4°} = 0.0126 + j0.1996$$

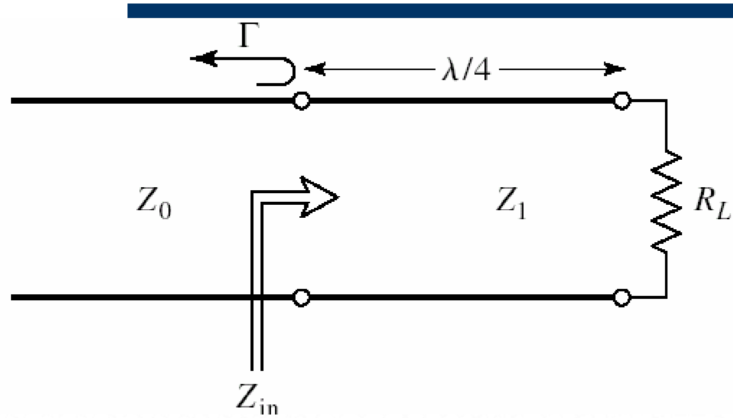
$$Z_L = 50\frac{1+\Gamma}{1-\Gamma} = 47.3 + j19.4\,\Omega$$

Smith Chart solution to slotted line problem

- First: SWR circle is plotted: unknown load on this circle
- Voltage minimum determined (left intersection real axis)
- Load is 0.37λ away, determined by 0.37λ rotation toward load



The λ/4 transformer: impedance viewpoint



- λ/4 transformer can be used to match real impedance to TL

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan \beta l}{Z_1 + jR_L \tan \beta l}$$

- For βl=(2π/ λ)(λ/4)=π/2 this becomes:

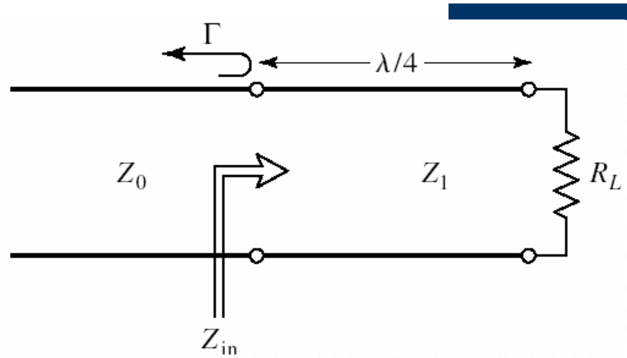
$$Z_{in} = Z_1^2 / R_L$$

- To match impedance Z_{in}=Z_o

$$Z_1 = \sqrt{Z_0 R_L}$$

- Result: SWR at feedline=0 at frequency where l= λ/4

Example of λ/4 transformer



- $R_L = 100\Omega$, $Z_0 = 50\Omega$
- Z_1?
- Plot $|\Gamma|$ versus norm. freq. f/f_0 with f_0 freq. where $l = \lambda/4$

$$Z_1 = \sqrt{Z_0 R_L} = \sqrt{50.100} = 70\ \Omega$$

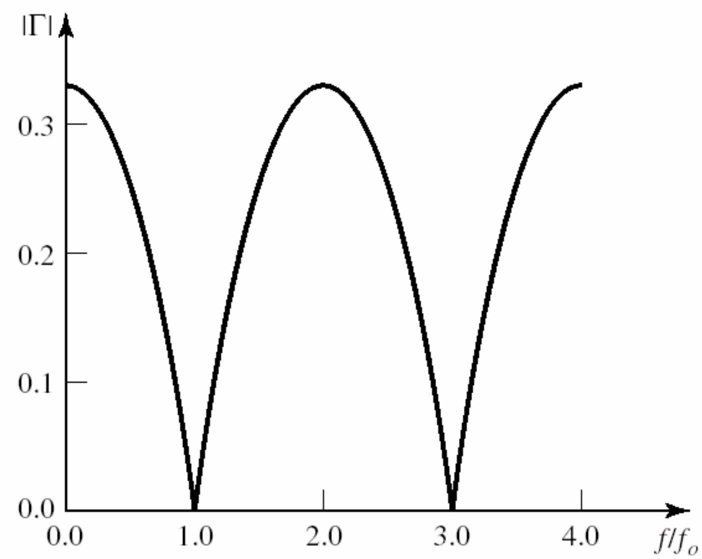
$$|\Gamma| = \left| \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \right| \quad \text{with} \quad Z_{in} = Z_1 \frac{R_L + jZ_1 \tan \beta l}{Z_1 + jR_L \tan \beta l}$$

rewrite βl afo f/f_0:

$$\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_0}{4}\right)$$

$$= \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4 f_0}\right) = \frac{\pi f}{2 f_0}$$

So if f↑, electrical length ↑



The λ/4 transformer: multiple reflection viewpoint

SWR=0 at Z_0 feedline does not imply there are no standing waves on Z_1 λ/4 section !! In fact multiple reflections do occur:

- Γ= total reflection
- Γ_1= partial refl. wave incident on load Z_1 from line Z_0
- Γ_2= partial refl. wave incident on load Z_0 from line Z_1
- Γ_3= partial refl. wave incident on load R_L from line Z_1
- T_1= partial transmission wave from line Z_0 in line Z_1
- T_2= partial transmission wave from line Z_1 in line Z_0

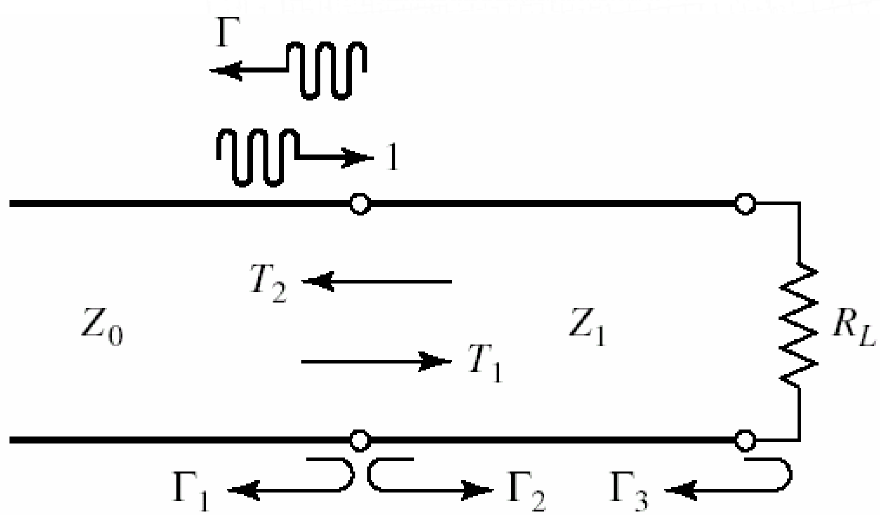
$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

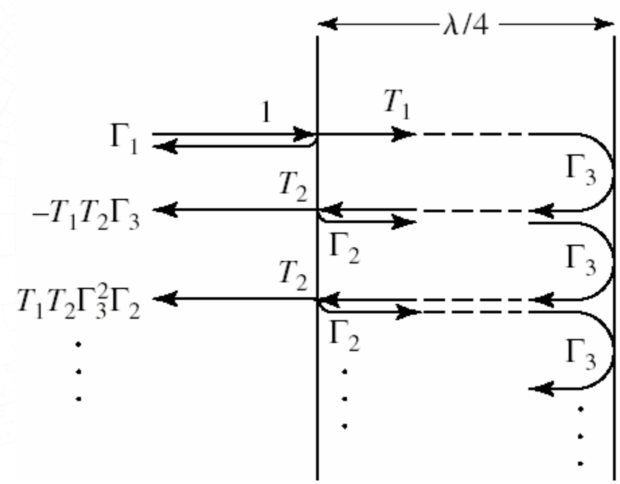
$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1}$$

$$T_1 = \frac{2Z_1}{Z_1 + Z_0}$$

$$T_2 = \frac{2Z_0}{Z_1 + Z_0}$$

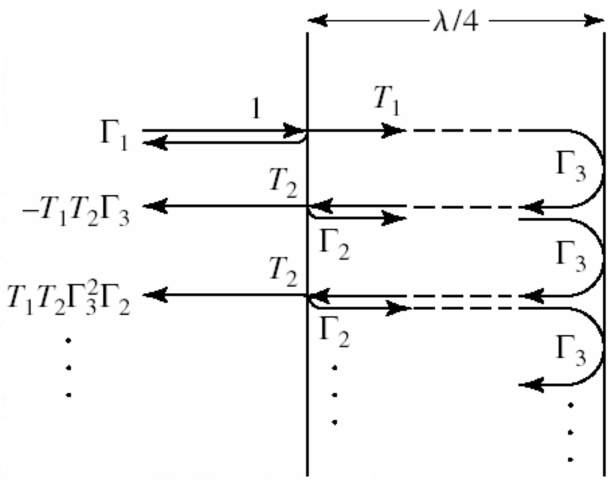




Calculation Γ of λ/4 transformer

Γ can be calculated as sum of infinite number of partial reflections of bouncing waves, each with phase difference of 180˚ (2x λ/4)

$$\Gamma = \Gamma_1 - T_1\Gamma_3 T_2 + T_1\Gamma_3\Gamma_2\Gamma_3 T_2 - \ldots$$
$$= \Gamma_1 - T_1\Gamma_3 T_2 \sum_{n=0}^{\infty} (-\Gamma_2\Gamma_3)^n$$



For $|\Gamma_2|<1$& $|\Gamma_3|<1$, we can use:

$$\sum_{n=0}^{\infty} (x)^n = \frac{1}{1-x} \quad \text{for } |x| < 1$$

$$\Gamma = \Gamma_1 - \frac{T_1T_2\Gamma_3}{1+\Gamma_2\Gamma_3} = \frac{\Gamma_1 + \Gamma_1\Gamma_2\Gamma_3 - T_1T_2\Gamma_3}{1+\Gamma_2\Gamma_3}$$

Numerator becomes:

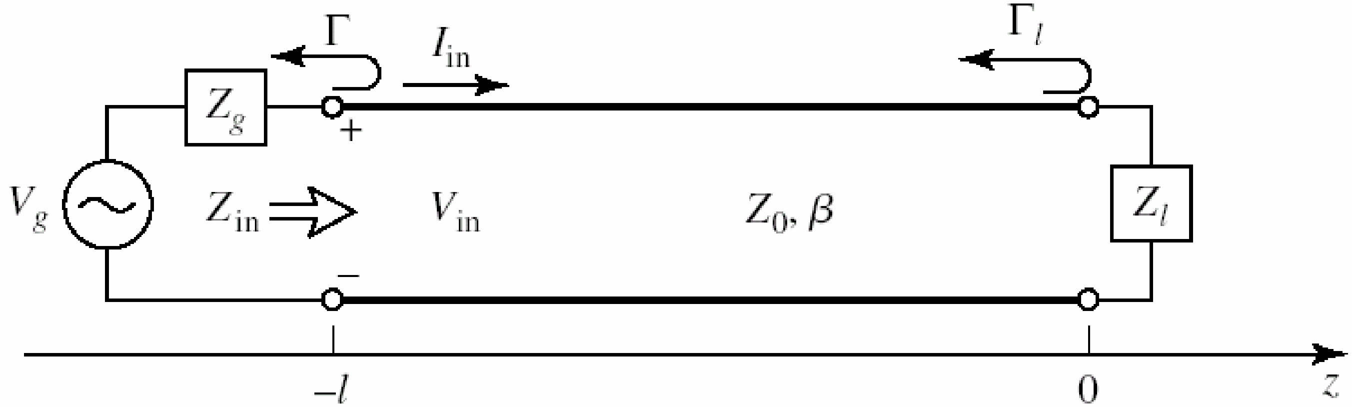
$$\Gamma_1 - \Gamma_3\left(\Gamma_1^2 + T_1T_2\right) = \Gamma_1 - \Gamma_3\left[\frac{(Z_1-Z_0)^2}{(Z_1+Z_0)^2} + \frac{4Z_1Z_0}{(Z_1+Z_0)^2}\right]$$

$$= \Gamma_1 - \Gamma_3 = \frac{2\left(Z_1^2 - Z_0R_L\right)}{(Z_1+Z_0)(R_L+Z_1)}$$

$= 0$ for : $Z_1 = \sqrt{Z_0 R_L}$

Generator and load mismatches

In general, generator not necessarily matched to terminated transmission line, maximum power transfer may require standing waves on TL, analysis here using impedance transformation



Looking into terminated TL from generator end:

$$Z_{in} = Z_0 \frac{1 + \Gamma_l e^{-2j\beta l}}{1 - \Gamma_l e^{-2j\beta l}} = Z_0 \frac{Z_l + jZ_0 \tan \beta l}{Z_0 + jZ_l \tan \beta l}$$

With: $\Gamma_l = \dfrac{Z_l - Z_0}{Z_l + Z_0}$

$$V(z) = V_o^+ \left[e^{-j\beta z} + \Gamma e^{+j\beta z} \right]$$

Generator and load mismatches

At the generator (z=-l) the voltage can be written as:

$$V(l) = V_g \frac{Z_{in}}{Z_{in} + Z_g} = V_o^+ \left[e^{+j\beta l} + \Gamma_l e^{-j\beta l} \right]$$

$$V_o^+ = V_g \frac{Z_{in}}{Z_{in} + Z_g} \frac{1}{e^{+j\beta l} + \Gamma_l e^{-j\beta l}}$$

using: $Z_{in} = Z_0 \dfrac{1 + \Gamma_l e^{-2j\beta l}}{1 - \Gamma_l e^{-2j\beta l}}$ After some calculation…

$$V_o^+ = V_g \frac{Z_0}{Z_0 + Z_g} \frac{e^{-j\beta l}}{\left(1 - \Gamma_l \Gamma_g e^{-2j\beta l}\right)}$$

Relation incident wave TL and generator voltage

with: $\Gamma_g = \dfrac{Z_g - Z_0}{Z_g + Z_0}$ multiple reflections between load-generator

SWR on line: $SWR = \dfrac{1 + |\Gamma_l|}{1 - |\Gamma_l|}$

Generator and load mismatches: matching

Power delivered to load:

$$P = \frac{1}{2}\mathrm{Re}\left\{V_{in} I_{in}^{*}\right\} = \frac{1}{2}\left|V_{in}\right|^{2} \mathrm{Re}\left\{\frac{1}{Z_{in}}\right\} = \frac{1}{2}\left|V_{g}\right|^{2}\left|\frac{Z_{in}}{Z_{in}+Z_{g}}\right|^{2} \mathrm{Re}\left\{\frac{1}{Z_{in}}\right\}$$

with: $Z_{in} = R_{in} + jX_{in}$ and $Z_{g} = R_{g} + jX_{g}$

Power becomes: $P = \frac{1}{2}\left|V_{g}\right|^{2} \frac{R_{in}}{\left(R_{in}+R_{g}\right)^{2} + \left(X_{in}+X_{g}\right)^{2}}$

For load matched to line: $Z_{in}=Z_0$ $\quad P = \frac{1}{2}\left|V_{g}\right|^{2} \frac{Z_{0}}{\left(Z_{0}+R_{g}\right)^{2} + X_{g}^{2}}$

For generator matched to loaded line: $Z_{in}=Z_g$ $\quad \Gamma = \frac{Z_{in}-Z_{g}}{Z_{in}+Z_{g}} = 0$

$$P = \frac{1}{2}\left|V_{g}\right|^{2} \frac{R_{g}}{4\left(R_{g}^{2}+X_{g}^{2}\right)}$$ still not optimal power transfer

Generator & load mismatch: conjugate matching

With Z_g fixed what is optimal Z_{in} for maximum power delivered to load?? If Z_{in} is known we can determine Z_l using Smith Chart!

$$P = \frac{1}{2}\left|V_g\right|^2 \frac{R_{in}}{\left(R_{in}+R_g\right)^2+\left(X_{in}+X_g\right)^2}$$

$$\frac{\partial P}{\partial R_{in}} = 0 \rightarrow \frac{1}{\left(R_{in}+R_g\right)^2+\left(X_{in}+X_g\right)^2} + \frac{-2R_{in}\left(R_{in}+R_g\right)}{\left[\left(R_{in}+R_g\right)^2+\left(X_{in}+X_g\right)^2\right]^2} = 0$$

$$R_g^{\,2} - R_{in}^{\,2} + \left(X_{in}+X_g\right)^2 = 0$$

$$\frac{\partial P}{\partial X_{in}} = 0 \rightarrow \frac{-2X_{in}\left(X_{in}+X_g\right)}{\left[\left(R_{in}+R_g\right)^2+\left(X_{in}+X_g\right)^2\right]^2} = 0 \quad \text{or} \quad 2X_{in}\left(X_{in}+X_g\right) = 0$$

Solving simultaneously: $R_{in} = R_g \quad \& \quad X_{in} = -X_g \quad \text{or} \quad Z_{in} = Z_g^{*}$

This is conjugate matching: $P = \frac{1}{2}\left|V_g\right|^2 \frac{1}{4R_g}$

Γ, Γ_g, Γ_l are not necessarily 0: reflection may add up at load

Not highest efficiency: half power lost in Z_g

Lossy Transmission Lines – The low-loss line

So far, most of derivations done for lossless case, loss important for cases of attenuation along a transmission line or Q resonator.

Often loss TL small (reason to use TL after all…) allows simplification of TL parameters, for instance for γ:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{(j\omega L)(j\omega C)\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \cancel{\frac{RG}{\omega^2 LC}}}$$ if R<<ωL & G<< ωC

$$= j\omega\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right]$$ using Taylor expansion $\sqrt{1+x} = 1 + \frac{x}{2} + \ldots$

$$\boxed{\begin{aligned}\alpha &\cong \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) \\ \beta &\cong \omega\sqrt{LC}\end{aligned}}$$

Z_0 for lossless line, but also

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \cong \sqrt{\frac{L}{C}}}$$

β and Z_0 low-loss case can be approximated by lossless case

Lossy TL's – The distortionless TL

General expression for γ implies that β may become a non-linear function of frequency. This results in frequency dependence for the phase velocity v_p, so for wideband signals different frequency components might travel at different velocities causing dispersion and distortion of signal.

Special case are lines with linear phase factor: distortionless, TL parameters satisfy: $\frac{R}{L} = \frac{G}{C}$

$$\gamma = j\omega\sqrt{LC}\sqrt{1 - 2j\frac{R}{\omega L} - \left(\frac{R}{\omega L}\right)^2} = j\omega\sqrt{LC}\left(1 - j\frac{R}{\omega L}\right) = \frac{R}{Z_0} + j\omega\sqrt{LC}$$

Result is line with β a linear function of frequency + attenuation is frequency independent: no distortion on pulses

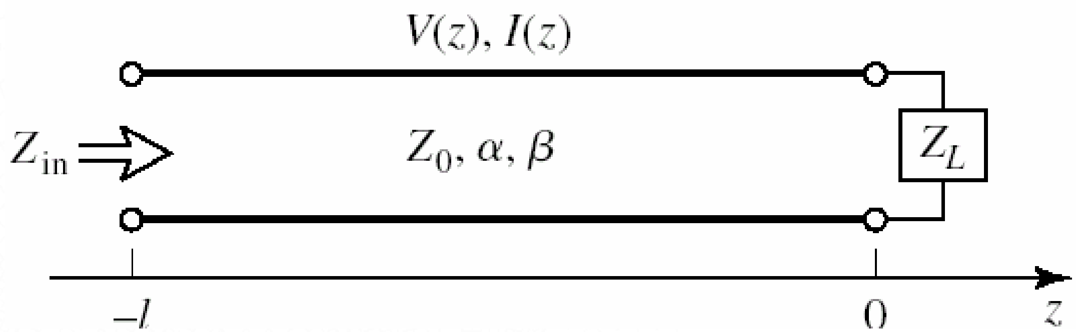
R/L=G/C often requires adding series inductance

The terminated lossy transmission line

For lossy line with ~ real Z_0, expressions for voltage and current:

$$V(z) = V_o^+\left[e^{-\gamma z} + \Gamma e^{+\gamma z}\right]$$

$$I(z) = \frac{V_o^+}{Z_0}\left[e^{-\gamma z} - \Gamma e^{+\gamma z}\right]$$



Reflection at distance l from load:

with Γ reflection coefficient load, V_o^+ incident amplitude at z=0

So, reflection coefficient is typically reduced by attenuation

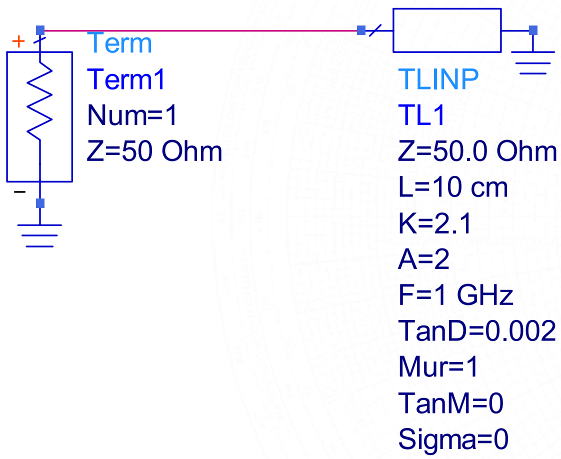
Z_{in} at distance l from load (in negative z-direction):

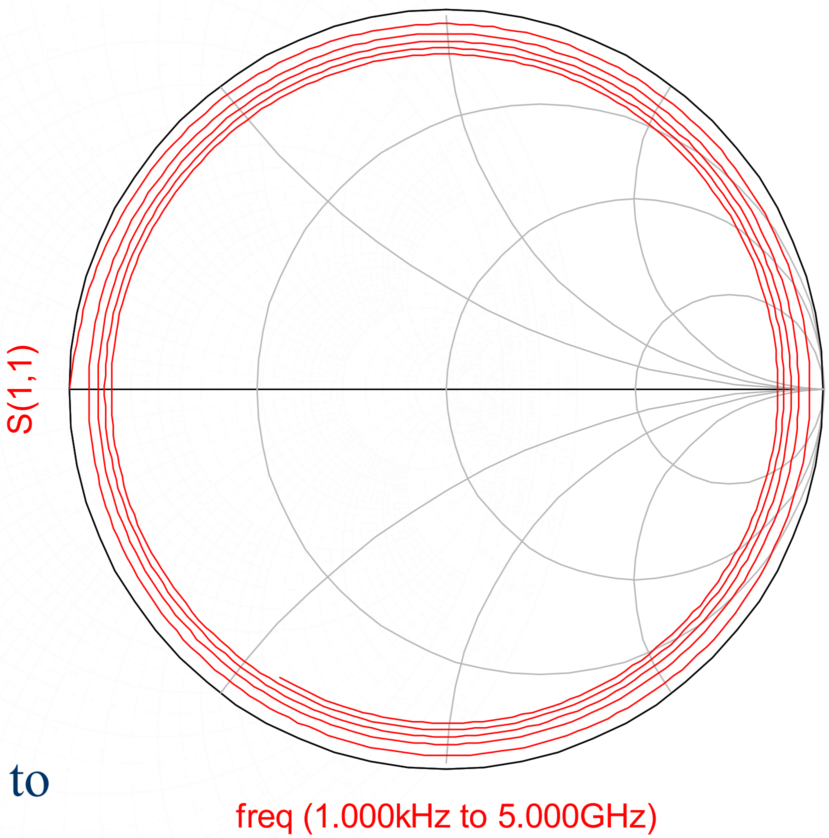
$$Z_{in} = \frac{V(-l)}{I(-l)} = Z_0 \frac{1 + \Gamma e^{-2\gamma l}}{1 - \Gamma e^{-2\gamma l}} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

Example of lossy TL terminated by short

$$\Gamma(l) = \Gamma e^{-2j\beta l} e^{-2\alpha l} = \Gamma e^{-2\gamma l}$$

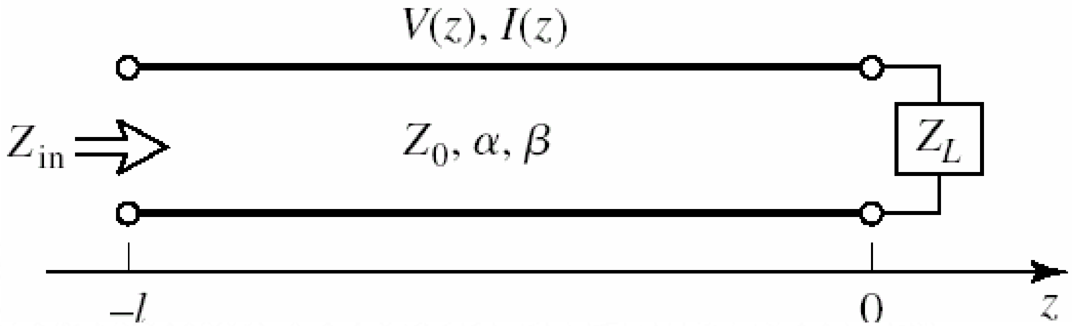
ADS schematic lossy line terminated by short





Result as function of frequency is circle spiraling towards center due to increased loss at higher frequency

Power flow terminated lossy transmission line



Power delivered to input terminated line:

$$P_{in} = \frac{1}{2}\mathrm{Re}\left\{V(-l)I^*(-l)\right\} = \frac{\left|V_0^+\right|^2}{2Z_0}\left[e^{2\alpha l} - |\Gamma|^2 e^{-2\alpha l}\right] = \frac{\left|V_0^+\right|^2}{2Z_0}\left[1 - |\Gamma(l)|^2\right]e^{2\alpha l}$$

Power delivered to load:

$$P_L = \frac{1}{2}\mathrm{Re}\left\{V(0)I^*(0)\right\} = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1 - |\Gamma|^2\right)$$

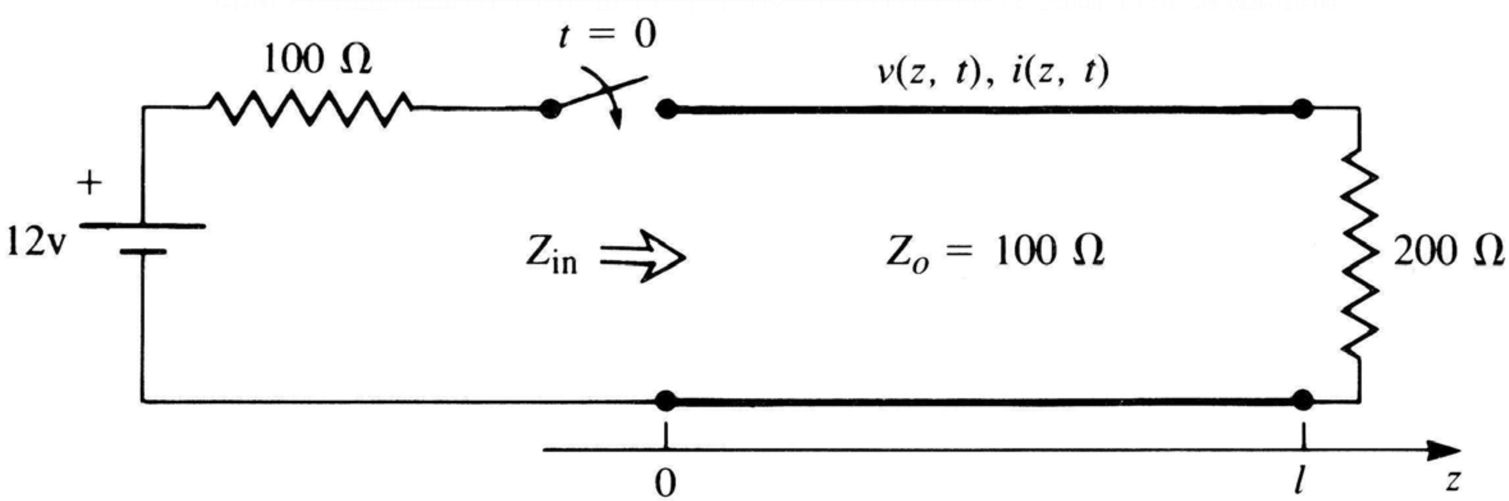
Power lost in line:

incident reflected

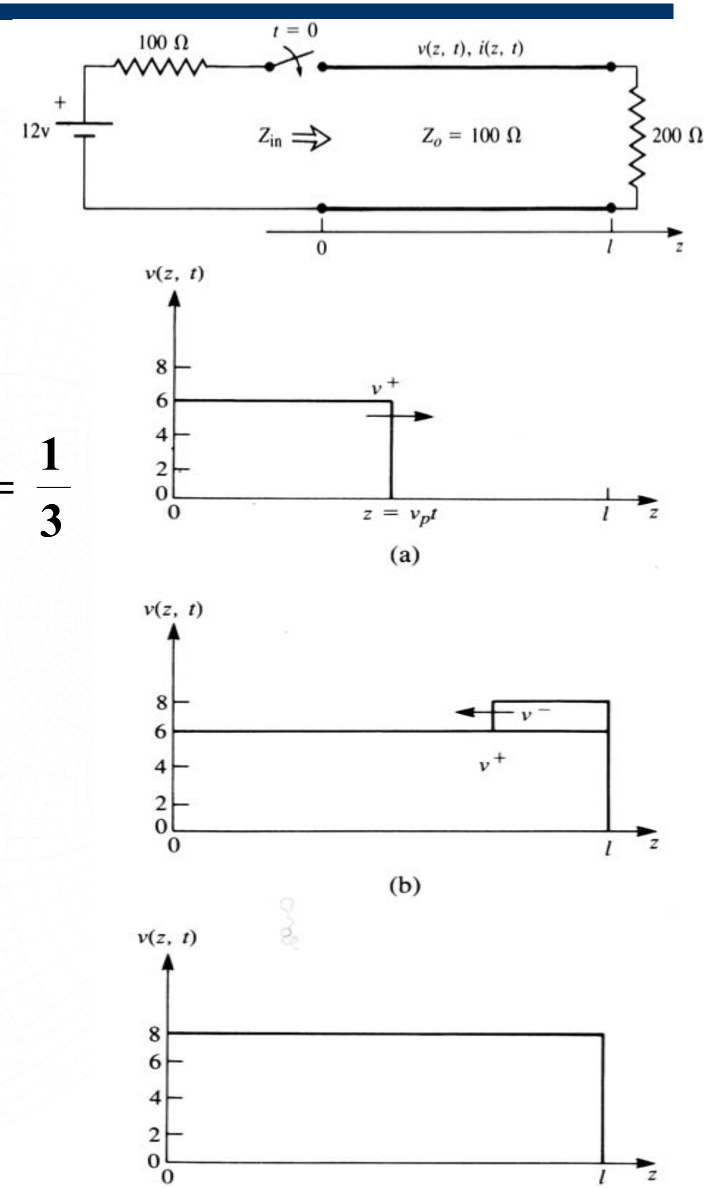
$$P_{loss} = P_{in} - P_L = \frac{\left|V_0^+\right|^2}{2Z_0}\left(\left(e^{2\alpha l} - 1\right) + |\Gamma|^2\left(1 - e^{-2\alpha l}\right)\right)$$

Transients in Transmission Lines

- So far all calculations for sinusoidal signal at one fixed frequency, satisfactory for lot of microwave applications, but not for short pulse transmission
- Intuitive or rigorous (Laplace transform) approach for transient solution
- Example: 12-V DC source switched on at t=0



Transients in Transmission Lines: example



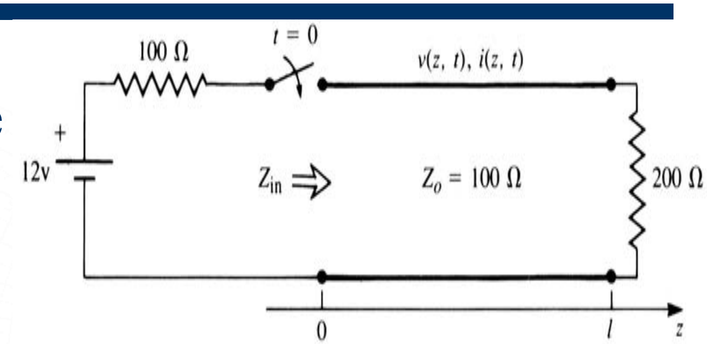
- ! At t=0, the generator does not see yet the 200 Ω load at end TL, looks at Z_0 of line, initially voltage division: 12V*(100/200) = 6V propagates with velocity v_p
- At $t=l/v_p$, pulse reaches load and gets partly reflected with coefficient of: $\Gamma_l = \dfrac{200 - 100}{200 + 100} = \dfrac{1}{3}$
- Superposition incident and reflected voltage:

$$v^+ + v^- = 6 + 6\Gamma_l = 8 \ \text{V}$$

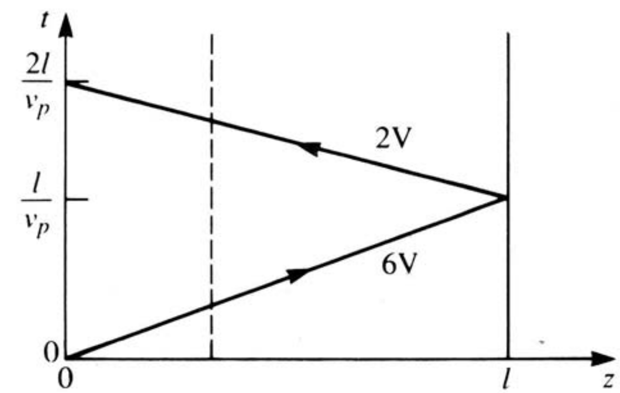
- At $t=2l/v_p$, pulse has made round trip absorbed by generator: 8V everywhere as can be expected from DC voltage division with 100 Ω generator and 200 Ω load:

$$12 \frac{200}{200 + 100} = 8 \ \text{V}$$

Transients in TL's: bounce diagram (FYI)



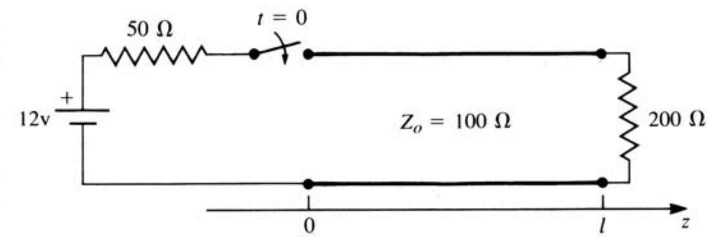
- Bounce diagram: way of viewing pulse propagating in time and position
  - X-axis: position along TL
  - Y-axis time
- Total voltage: draw line at position from t=0..t_0, total voltage adding voltages each component wave present (waves which intersect vertical line)



Transients in TL's: multiple reflection

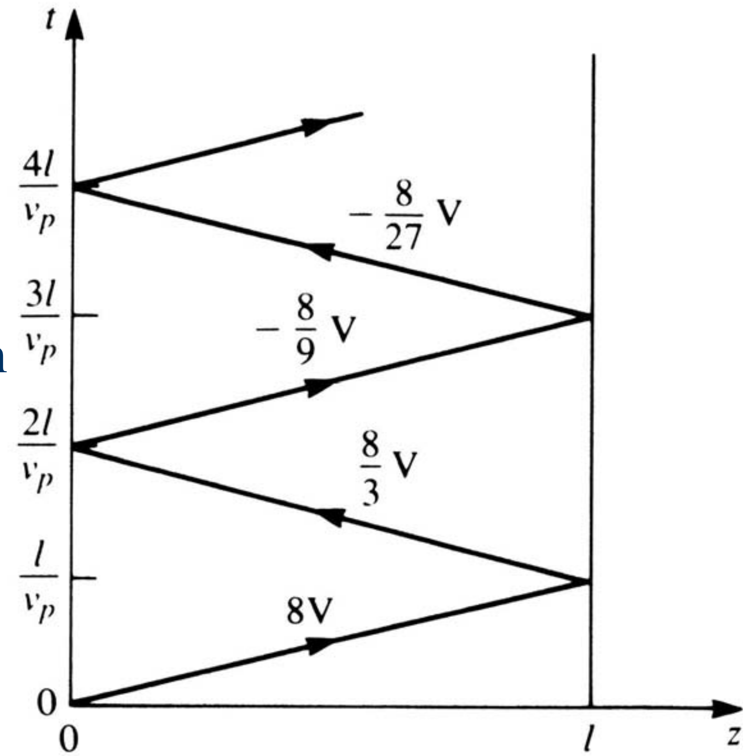
- Also generator mismatched to line, multiple reflections occur at load and generator

$$\Gamma_l = \frac{200 - 100}{200 + 100} = \frac{1}{3} \qquad \Gamma_g = \frac{50 - 100}{50 + 100} = -\frac{1}{3}$$



- At $t=l/v_p$, pulse reaches load and gets partly reflected with coefficient of 1/3
- At $t=2l/v_p$, pulse has made round trip gets reflected by generator:
- Finally, will settle at 9.6V everywhere as can be expected from DC voltage division with 50 Ω generator and 200 Ω load:

$$12\,\frac{200}{200 + 50} = 9.6\ \ \mathrm{V}$$



Transient and steady–state behaviour demo

- A very nice program to **visualize transients in transmission lines,** called Bounce can be found on the website of Professor C.W. Trueman at Concordia University, Montreal. Please download the program bounce.exe on your PC and take a look at the examples. We'll do small demo of this program.
- Interesting set-ups:
  - Step-function along transmission line
  - Sine-wave on line with mismatched load: standing waves
  - Sine-wave on quarterwave-length transformer: internal reflections, but toal reflection dies out
  - Sine-wave in open or shorted transmission line with mismatched (low-impedance) generator: resonant behaviour!

Homework & next lecture!!

- Pozar, "Microwave Engineering" (3rd Ed.!)
  - 2.2
  - 2.8
  - 2.11
  - 2.19
- Due date: hand in before next Class (2/2)
- Next week, we'll discuss practical transmission lines