E6318 - Microwave Circuit Design

Columbia University

Spring 2006

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Outline of Lecture 1

- Course information & overview
  - Contact info, syllabus, calendar, website
- Introduction to Microwaves
  - Applications, microwave bands, …
  - Microwave circuits
- Transmission line theory
  - Telegrapher equations
  - Terminated transmission lines

Personal Info

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  - Office hours: after class, by appointment (e-mail!)

- Course: Th 4:10-6:40 PM, 1127 Mudd
- Webpage: http://www.cisl.columbia.edu/~ee6318/
- CA/TA: **Austin Chen (yc2134@columbia.edu)**

Website

- Webpage: http://www.cisl.columbia.edu/~ee6318/
- These notes, assignments and some useful files and links will be posted in a password-protected area, please do not share this password or any files downloaded from this area.
- username: ee6318
  password: 4MWcour53only
- Please check website regularly for updates on assignments, possible rescheduling of classes, etc…

Calendar

- Course: Th 4:10-6:40 PM, 1127 Mudd
  - 01/19
  - 01/26
  - 02/02
  - 02/09
  - 02/16
  - 02/23
  - 03/02
  - 03/09 **Midterm**
  - 03/16 Spring Holidays
  - 03/23
  - 03/30
  - 04/06
  - 04/13
  - 04/20
  - 04/27
  - **Final** (05/11)

Objectives of Microwave Circuit Design Course

- Learn Basic Microwave Design Principles:
  - Transmission lines & Smith-chart
  - S-parameters, Microwave networks
  - Impedance matching and tuning
  - Coupled line theory
- Study Practical Microwave Components:
  - Transmission lines, power dividers & couplers
  - Active and passive microwave devices
- Study design of some active microwave circuits
  - Amplifiers: smallband, low-noise, broadband, power
  - Non-linear circuits: oscillators, multipliers, mixers

Objectives of Microwave Circuit Design Course

- Take a look at simulation and measurement tools for microwave circuits
- Apply Microwave Design in small design-projects
  - after midterm
  - Agilent ADS
- Study transmission line effects in digital systems

Detailed syllabus

1. Transmission line theory 2 wks
   - Transmission lines, standing waves and VSWR
   - Smith chart
   - Quarterwave transformer, lossy lines
   - Transients in transmission lines
2. Microwave transmission lines in practice 1 wk
   - Effective dielectric constant, dispersion, attenuation, skin effect
   - Coaxial lines (Waveguides)
   - Microstrip and strip-lines, coplanar waveguide
3. Microwave Network Analysis 1 wk
   - S-parameter matrix and properties of S-parameters
   - Mason's signal flow rules,
   - Discontinuities in transmission lines

Detailed syllabus (2)

4. Impedance matching and tuning 1½ wks
   - Matching using lumped elements
   - Lumped microwave components (inductors, capacitors, etc…)
   - Matching using transmission lines (single and double stub, quarterwave transformer)
   - Multi-section transformers and Bode-Fano Criterion

5. Microwave Resonators ½ wk

Detailed syllabus (3)

6. Power dividers and directional couplers 1 wk
   - Basic properties 3 and 4-ports
   - T-junction and Wilkinson Power divider
   - Quadrature hybrid (branch-line coupler)
   - Coupled line directional couplers and Lange coupler
   - The 180º hybrid (rat-race)

------------------------------------ **MIDTERM** ------------------------------------

7. Noise and active microwave devices 1 wk
   - Noise in Microwave Circuits (noise temperature and noise figure)
   - Dynamic Range and Intermodulation
   - Modern Microwave Transistors: Figures of Merit, modelling, Current state-of-the-art in active devices

Detailed syllabus (4)

8. Microwave Amplifier Design — 2 ½ wks
    - Two-port Power Gains and Amplifier Stability
    - Single-Stage amplifier design for maximum gain
    - Low-noise microwave amplifiers
    - Broadband amplifiers (balanced, feedback, distributed amplifiers)
    - Multi-stage Amplifiers
    - Cascode Amplifiers
    - Power Amplifiers

9. Nonlinear Microwave Circuits — 1 ½ wks
    - Microwave Oscillators
    - Frequency multipliers
    - Mixers
    - Transmission line effects in digital systems

10. Recap lecture, Microwave design Project — 1 wk

Grading

- Homework (~weekly), including couple small design projects towards end 30%
- Midterm (written) 20%
- Final (written) 50%

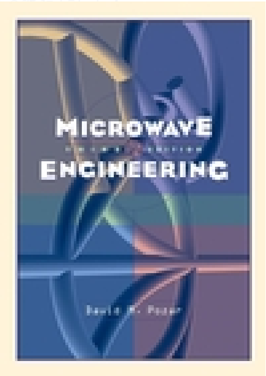
Reference book

D. Pozar: "**Microwave Engineering**", 3rd Ed., J. Wiley & Sons

While Pozar's book (typically 2 semester course) describes the mathematical derivation of EM properties in detail, this course will concentrate on the outcome of these derivations.

Reading (see website for updates!):

- **Chapter 1**: optional reading
- **Chapter 2**: completely (L1&L2)
- **Chapter 3**: only some results discussed in class (L3)
- **Chapter 4**: 1&2 optional, in class: 3, 4 & 5 (L4&5)
- **Chapter 5**: sections 1,2,3,4 & 9
- **Chapter 6**: 1&2
- **Chapter 7**: 1,2,3,5,6,7,8,9
- **Chapter 10**: 1, 2, 3 4&5 optional
- **Chapter 11**: 1,2,3,4,5 in part, complemented with notes
- **Chapter 12**: 1,2,3,4 and 6 in part, complemented with notes



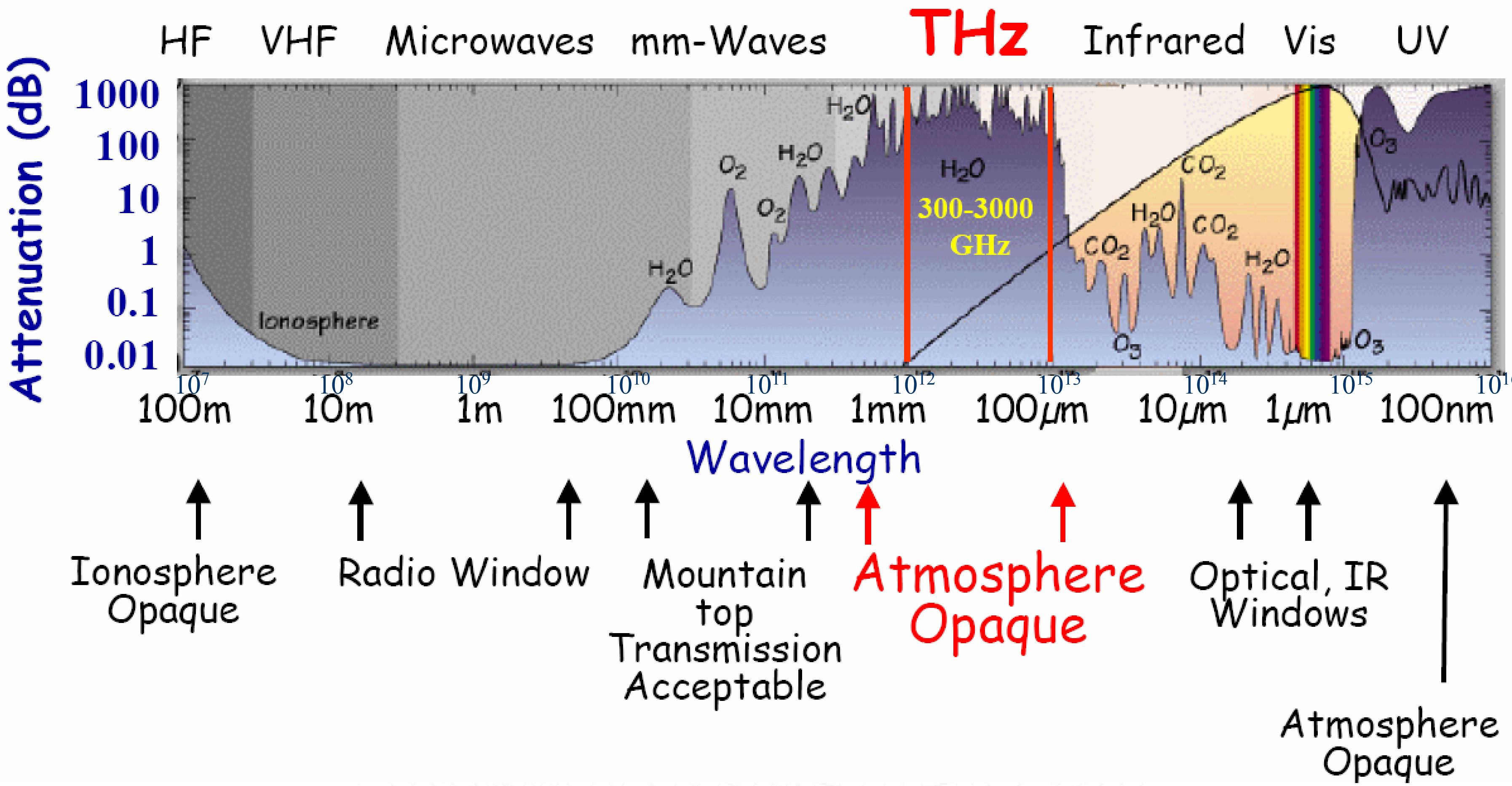
Other good reference works

- G. Gonzalez: “**Microwave Transistor Amplifiers: analysis and design**”, 2nd Ed., Prentice Hall, Inc.
- S. Y. Liao: “**Microwave Circuit Analysis and Amplifier Design**”, Prentice Hall, Inc.
- J. C. Freeman: “**Fundamentals of Microwave Transmission Lines**”, John Wiley & Sons, Inc.
- S. H. Hall: “**High-Speed Digital System Design: A Handbook of Interconnect Theory and Design Practices**”, John Wiley & Sons, Inc.
- R.J. Weber: “**Introduction to Microwave Circuits**”, IEEE Press.
- J. White: “**High Frequency Techniques**”, John Wiley & Sons, Inc.

Microwaves

- Microwave range:
  - 300 MHz ≤ frequency ≤ 300 GHz
  - 100 cm ≤ λ ≤ 0.1 cm
- IEEE Microwave bands:
  - L: 1-2 GHz — λ: 30-15 cm
  - S: 2-4 GHz — λ: 15-7.5 cm
  - C: 4-8 GHz — λ: 7.5-3.75 cm
  - X: 8-12.5 GHz — λ: 3.75-2.4 cm
  - Ku:12-18 GHz — λ: 2.4-1.67 cm
  - K: 18-26.5 GHz — λ: 1.67-1.13 cm
  - Ka:26-40 GHz — λ: 1.13-0.75 cm
  - mm-waves: 40-300 GHz — λ: 7.5-1 mm
  - mm-waves further divided in U (Q), V, W, D, F, G waveguide bands
  - sub-mm-wave (THz):>300 GHz — λ: 1-0.1 mm

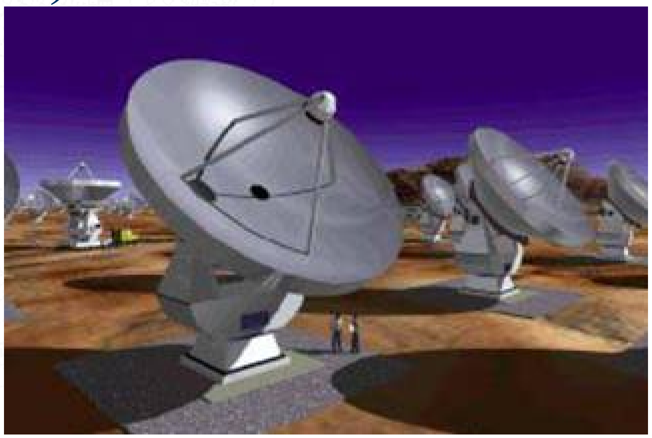
The electromagnetic spectrum



Typical telecom applications in .1-100 GHz range (µwave, mm-wave) and in optical range (~200 THz)

The need for higher frequencies!!!

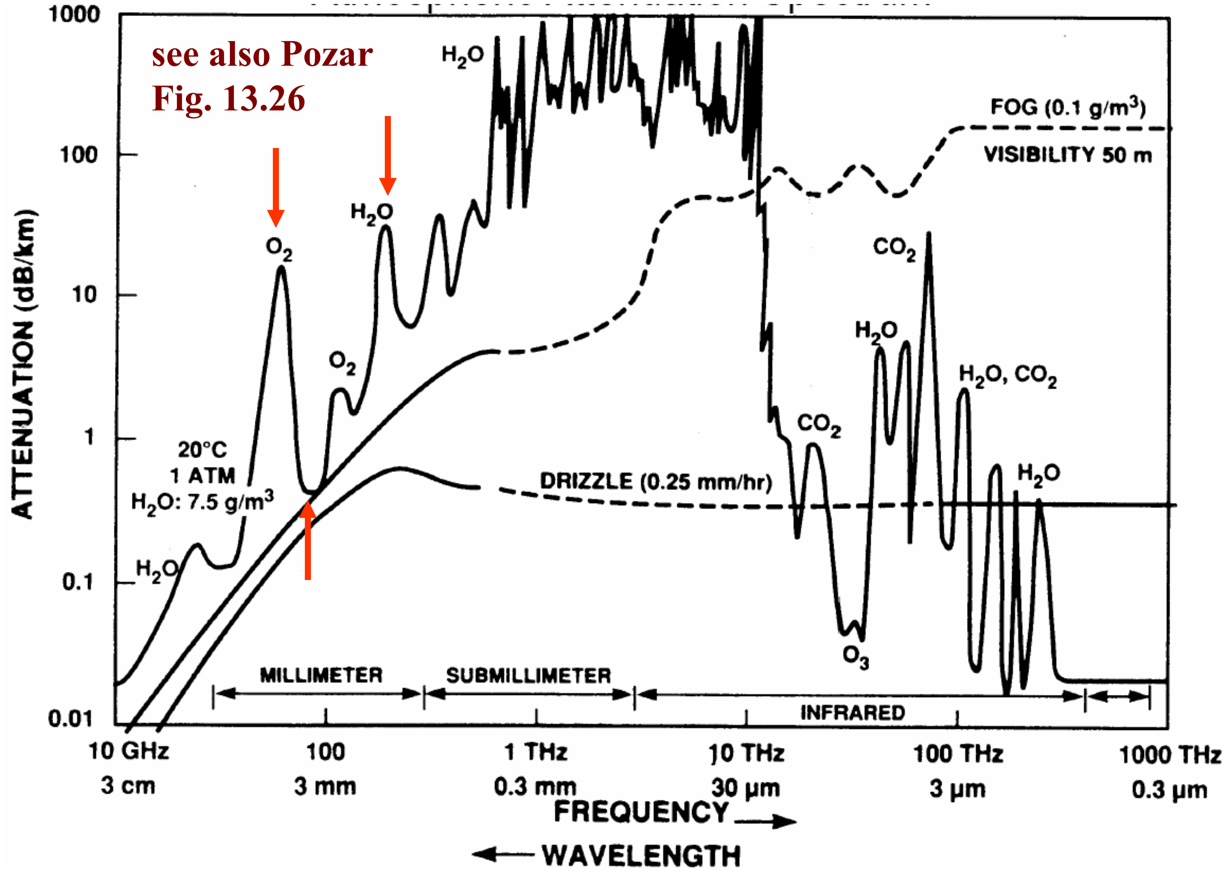
- The higher the frequency, the more information can be sent over same fractional bandwidth (600 MHz: 1% BW=1 TV channel, at 60 GHz: 1% =100 TV channels, at 200 THz, ….)
- The higher the frequency, the smaller wavelength and wavelength depending structures (antennas, radar resolution, waveguides, etc…)
- At higher frequency, specific molecular spectra can be used to our advantage (radio-astronomy, secure LAN, T-ray imaging)



But some drawbacks for telecom…

- The higher the frequency, the smaller (read more accurate and therefore expensive) things get…
- This definitely applies to the electronics and test equipment!
- The higher the frequency, the more loss (ohmic, dielectric, molecular absorption etc…) atmospheric transmission will incur
- Data transmission at microwave frequencies is prone to signal degradation due to multiple reflections, Doppler, etc…

Atmospheric attenuation vs. frequency



- Molecular resonance peaks at 22 & 183 G (H_2O), at 60 & 120G (O_2)
- Windows at 35, 94 and 140 GHz (high-resolution radar)
- MM-wave lossy, but still better then optical in fog!!

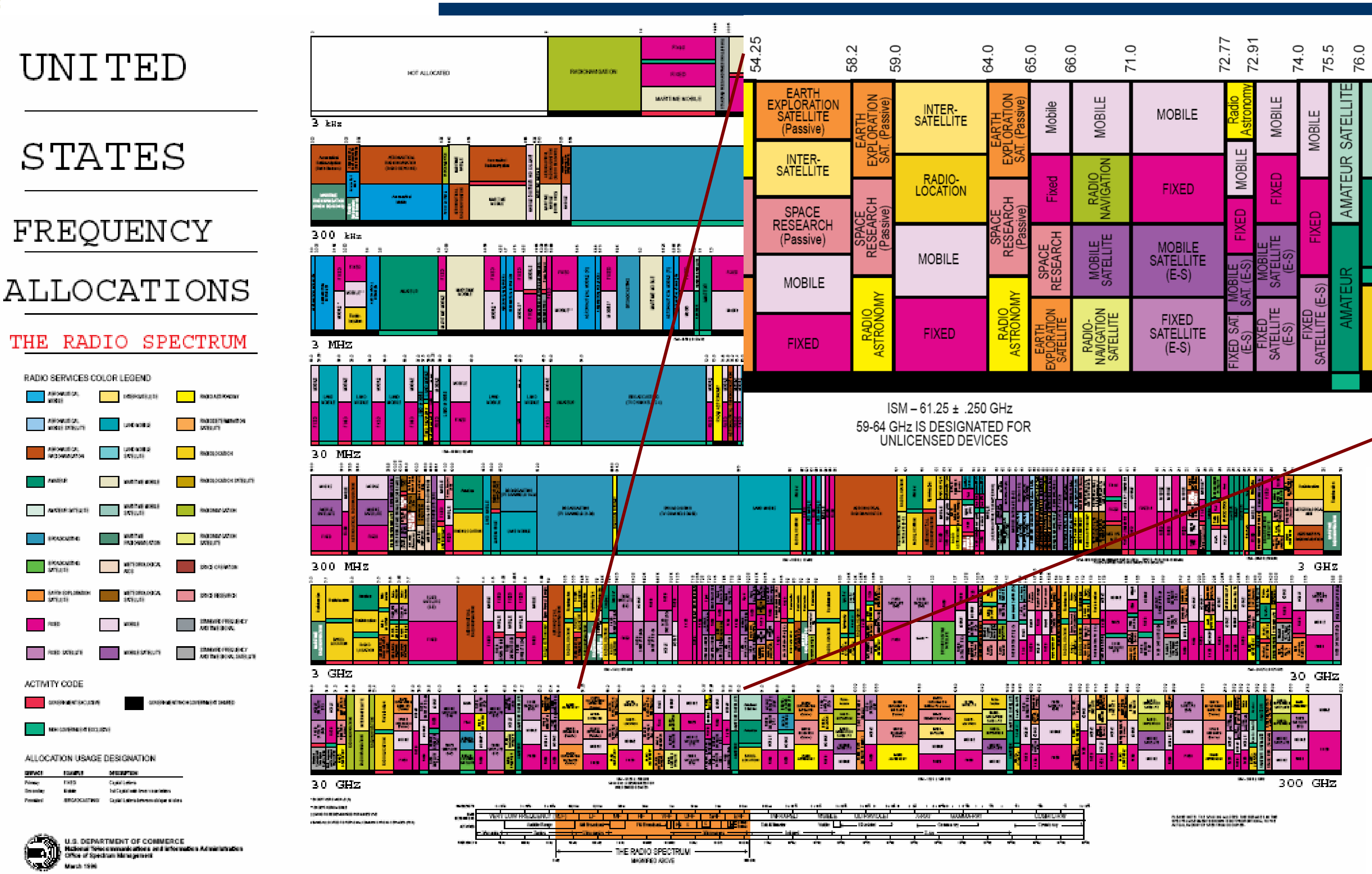
Applications of Microwave Circuit Design

- Telecommunications
  - Wireless: cellular, WLAN, pt-to-pt link, satellite
  - Wireline: optical (OC-768), Gigabit-ethernet, …
- High-speed VLSI design
  - On-chip interconnects
  - Packaging, high-speed bus
- Radar / Remote Sensing
- Radio-astronomy
- Global Positioning Satellite (GPS)

High-frequencies put in perspective….

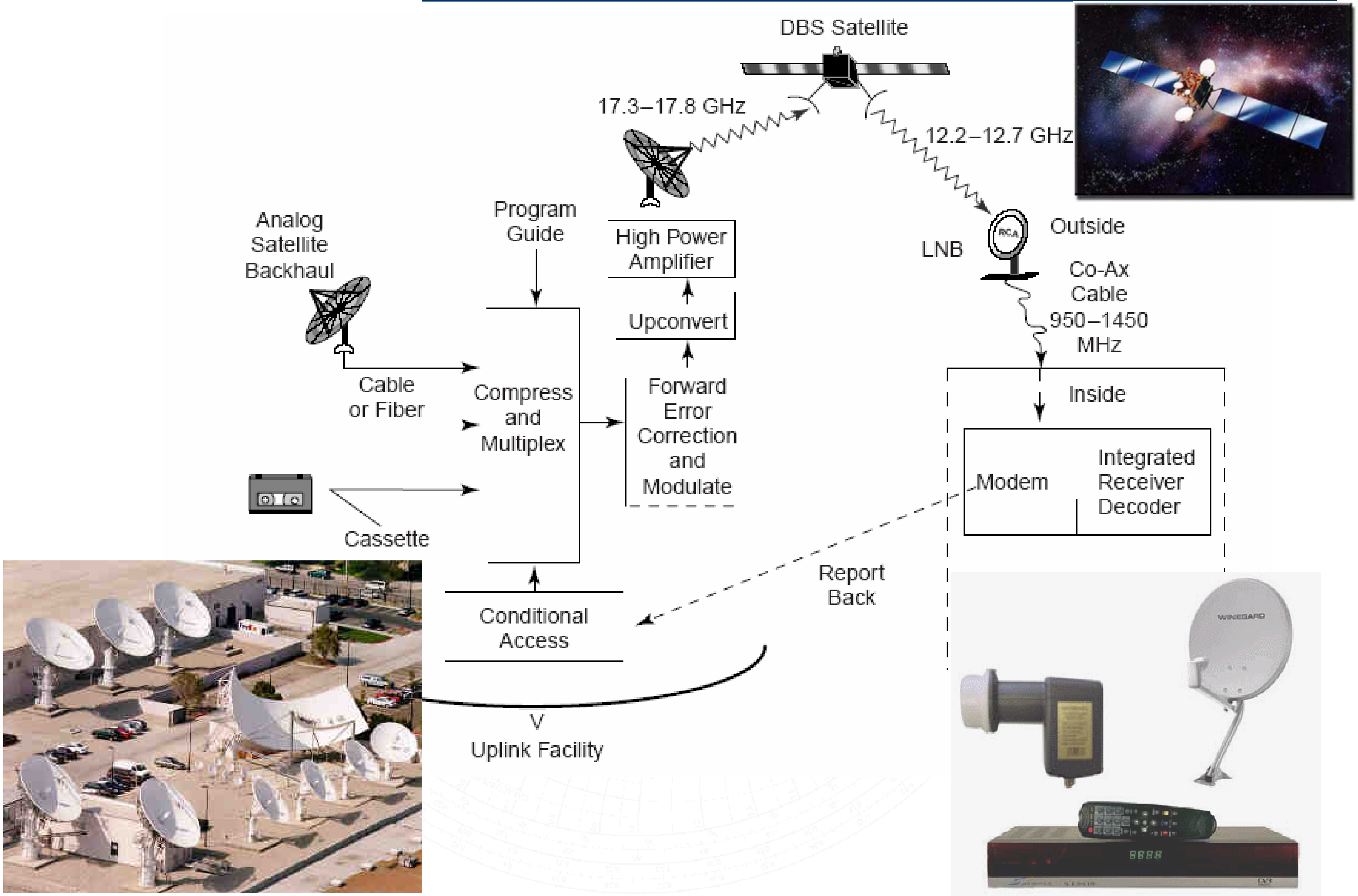
- AM Radio: 2 MHz
- Intel 8086 clock speed: 4.7 & 8 MHz
- FM Radio: 85-105 MHz
- Cellular Phone: ~1 & 2 GHz
- Fastest Pentium clock speed: 3.8 GHz
- WLAN (802.11): 2.4 GHz & 5.8 GHz
- DBS (Direct Broadcast Satellite): 12 GHz
- AICC: 24 & 77 GHz
- Gigabit LAN: 60 GHz
- High-resolution Radar: 94 & 140 GHz
- Fastest Bell-Labs electronic circuit: 270 GHz
- Optical carrier frequency: ~200 THz, modulation: 100 Gb/s

United States Frequency Allocations*

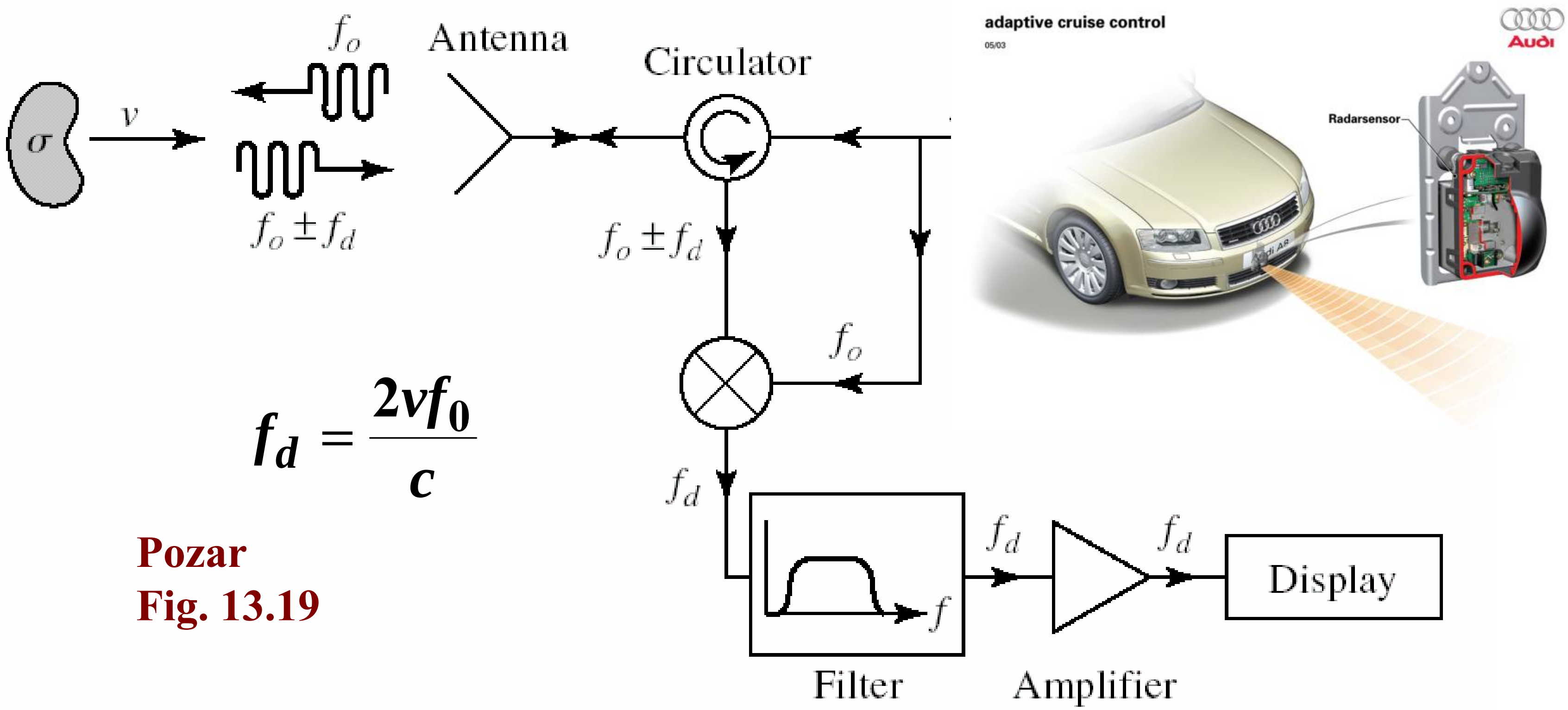


* See link on website

Example Microwave System: Direct Broadcast Satellite



Example 2: Doppler Radar System (77 GHz)



$$f_d = \frac{2vf_0}{c}$$

Pozar
Fig. 13.19

- Returned signal shifted in frequency according to speed
- Combination with pulsed (pulse-doppler radar) gives both range and velocity

Microwave versus analog/digital design

Microwave:

- Design optimizes power-flow
- Performance close to limits active devices in frequency, noise, power added efficiency
- Typically smallband (AC)
- Very few active devices
- Which dissipate lots of power
- Combined with large reactive passives (TL's, inductors,…)
- Resulting in relatively large circuit size
- Mostly single-ended design

Analog/Digital:

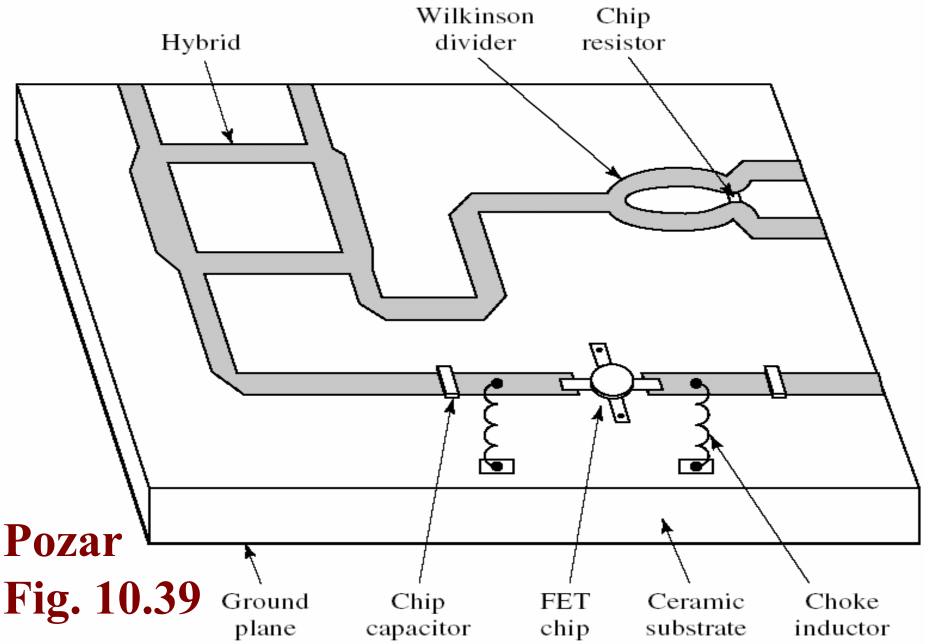
- Voltage/current
- Performance at single frequency typically far from fundamental limit
- Broadband operation (DC)
- K's (analog)/M's (digital)
- Each device low power
- With small resistors (or only FETs)
- Relatively small even with M's of FETs
- Very often differential

Why using transmission lines at high-speed?

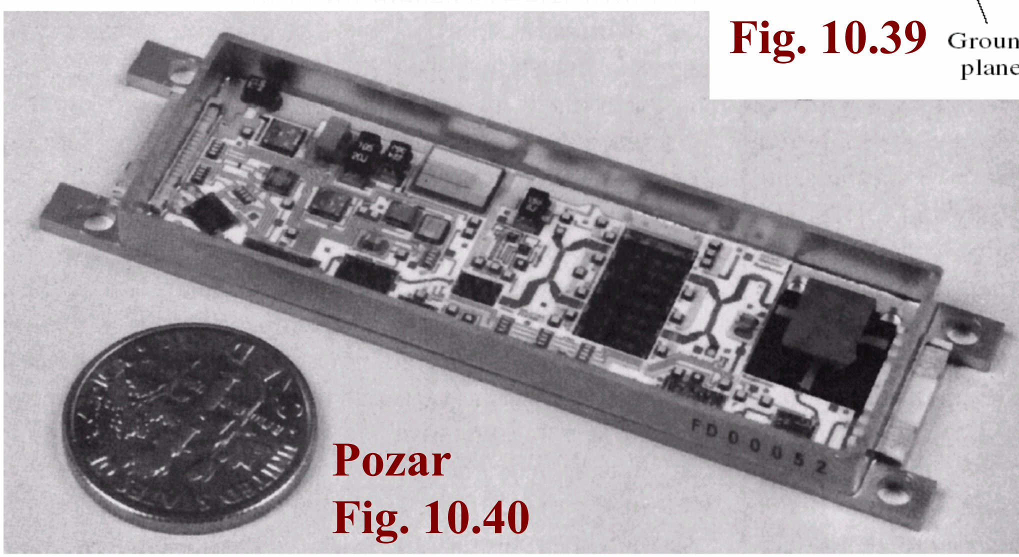
- Make longer connections which:
  - Are low-loss: low **ohmic** or **dielectric** losses + minimal **radiation**
  - Have well-known characteristics allowing to avoid degradation in analog or digital systems due to reflections, ringing, limited bandwidth, …
- Use the transmission lines as circuit element:
  - Reactive matching element to optimize power transfer, maximize gain of active elements, etc…
  - Frequency dependent properties: resonators, filters
  - Multiports with interesting power combining properties: couplers, hybrid-T’s. etc…

Microwave integrated circuits: hybrid

Layout hybid microwave
integrated circuit (MIC)
in this case microstrip topology



Pozar
Fig. 10.39

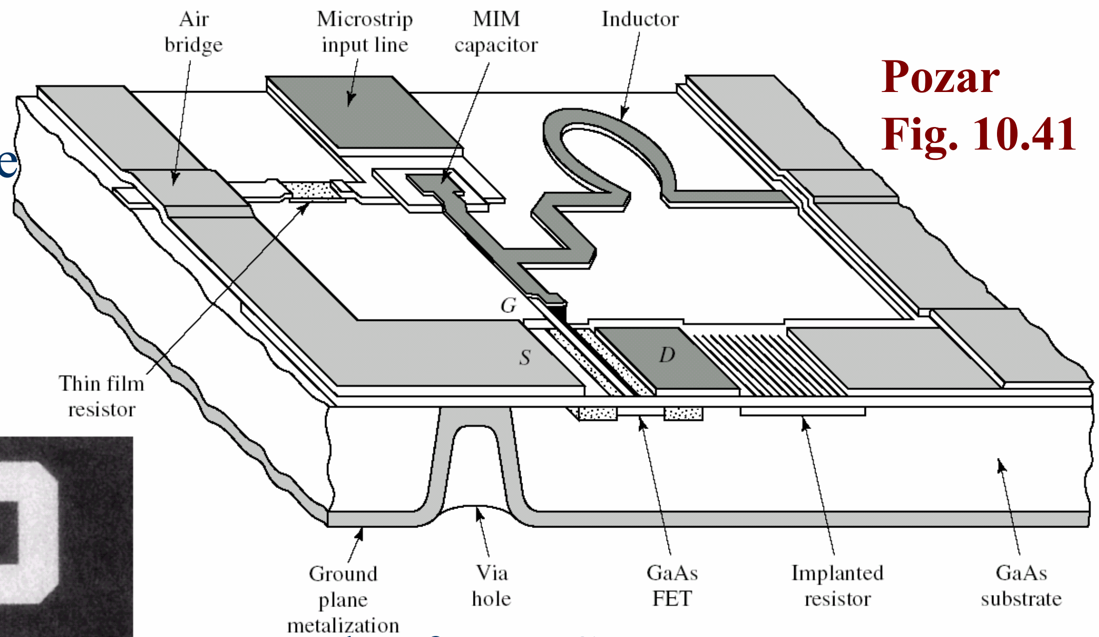


Pozar
Fig. 10.40

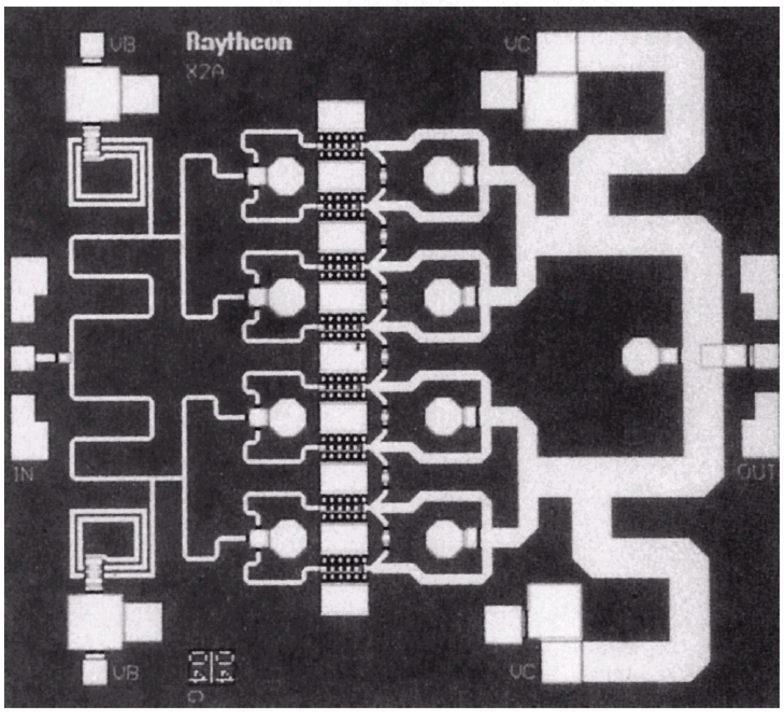
Radar T/R module, contains
phase shifters, amplifiers,
switches,couplers, circulator..

Microwave integrated circuits: monolithic

Layout monolithic microwave integrated circuit (MMIC) again microstrip topology



Pozar Fig. 10.41



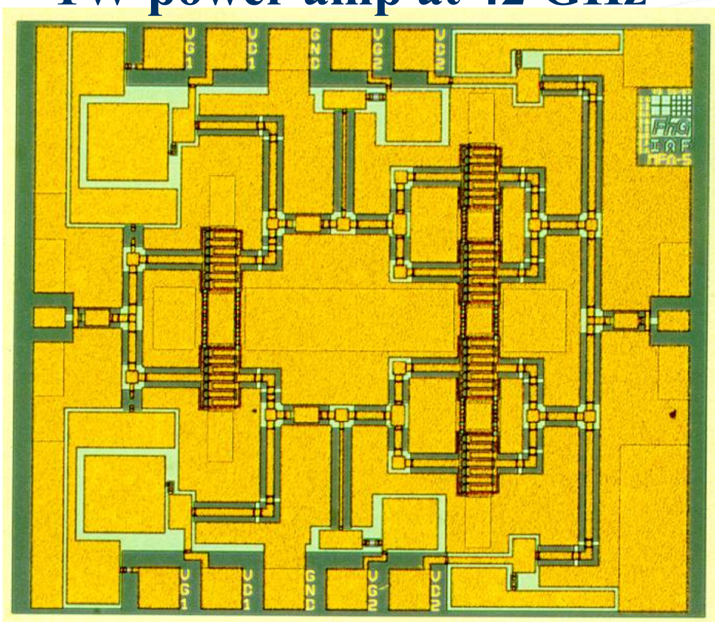
Example of MMIC:
Integrated X-band power amplifier

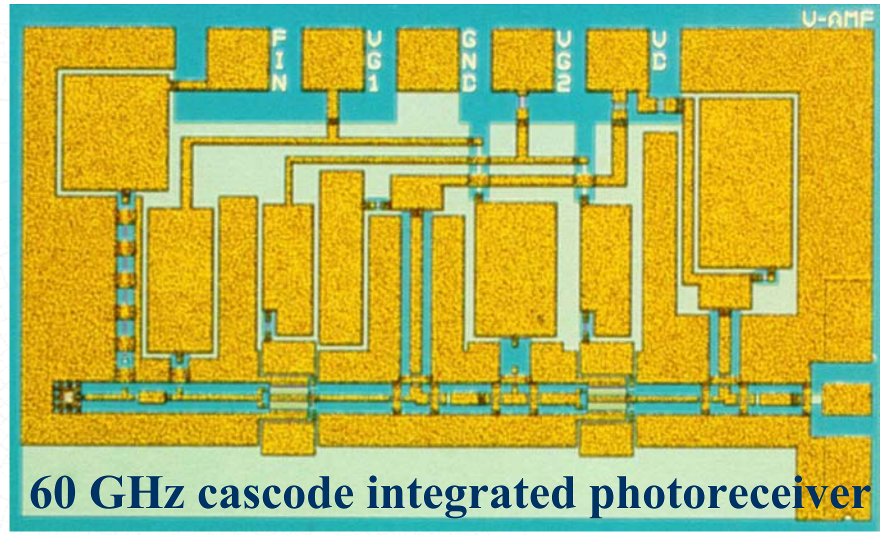
Multiple HBT's combined to deliver 5W

Pozar Fig. 10.42

Some more recent MMIC examples….

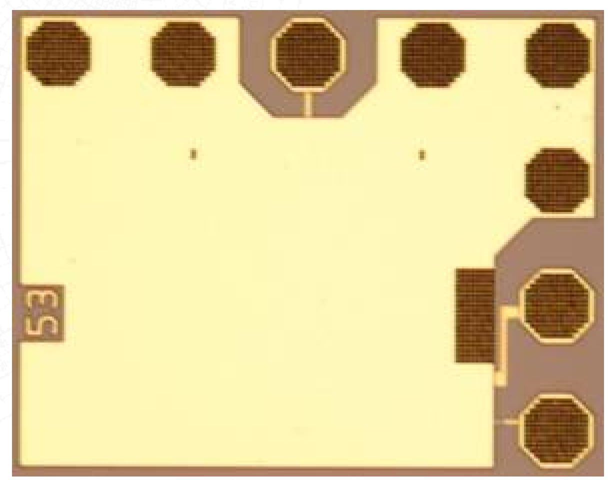
1W power amp at 42 GHz



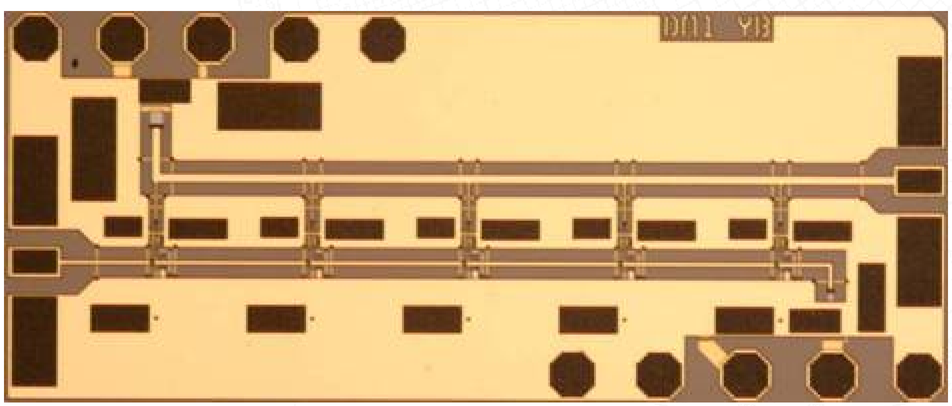


60 GHz cascode integrated photoreceiver

270 GHz InP HBT push-push VCO



>110 GHz BW InP HBT amplifier



Hierarchy of Microwave Engineering

- **Electromagnetics Theory**
  - Gauss', Ampere's and Faraday's law
  - Maxwell Equations

↓ uniform, TEM

- **Distributed Circuits**
  - Transmission lines, Telegraph Equations
  - Smith Chart, S-parameters, etc…

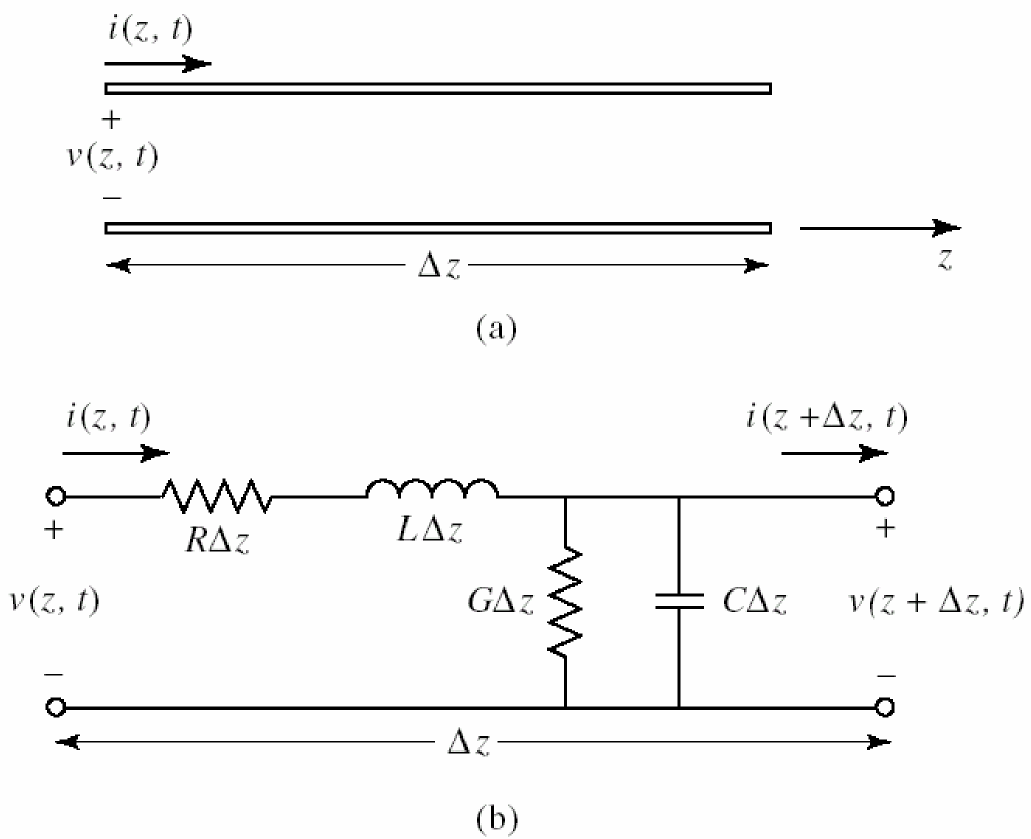
↓ dimensions <<λ

- **Lumped Circuits**
  - Ohm's and Kirchoff's Law
  - R, L, C

Transmission line theory: lumped-element circuit

- Circuit theory: physical dimensions << electrical wavelength
- Transmission lines: size fraction wavelength or larger
- As a result voltages and current vary in both magnitude and phase along distributed transmission line
- TL schematically represented as 2-wire line (TEM propagation: at least 2 conductors). Short piece of TL modelled with per unit length quantities:
  - R: series resistance in Ω/m
  - L: series inductance in H/m
  - G: shunt conductance in S/m
  - C: shunt capacitance in F/m
- Finite length of TL: cascade of multiple sections

Transmission line theory: lumped-element circuit



Voltage and current definitions for incremental length of transmission line

Pozar Fig. 2.1

Lumped-element equivalent circuit

- KVL: $$v(z,t) - R\Delta z \cdot i(z,t) - L\Delta z \cdot \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$
- KCL: $$i(z,t) - G\Delta z \cdot v(z+\Delta z,t) - C\Delta z \cdot \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

Transmission line theory: wave equations

- Dividing by Δz and taking limit for Δz→0

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t}$$

time-domain form TL or telegrapher equations

- For sinusoidal steady-state (cosine-phasors):

$$\frac{dV(z)}{dz} = -(R + j\omega L) \cdot I(z)$$

$$\frac{dI(z)}{dz} = -(G + j\omega C) \cdot V(z)$$

- Solved simultaneously gives wave eq. V(z) & I(z)

$$\frac{d^2V(z)}{dz^2} - \gamma^2 \cdot V(z) = 0 \qquad \frac{d^2I(z)}{dz^2} - \gamma^2 \cdot I(z) = 0$$

γ is complex propagation constant $\boxed{\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}}$

Traveling wave solutions

- Traveling wave solutions to last equation can be found:

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$$
$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$$

V, I= sum of 2 waves, one in pos. z direction one in negative

- Applying $\frac{dV(z)}{dz} = -(R + j\omega L) \cdot I(z)$

$$I(z) = \frac{\gamma}{R + j\omega L}\left[V_o^+ e^{-\gamma z} - V_o^- e^{+\gamma z}\right]$$

With Z_0: characteristic impedance, relates V and I on line

$$\boxed{Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \frac{V_o^+}{I_0^+} = -\frac{V_o^-}{I_0^-}}$$

V_o^+&I_o^+ are in phase if Z_0 real

This becomes: $I(z) = \frac{V_o^+}{Z_0} e^{-\gamma z} - \frac{V_o^-}{Z_0} e^{+\gamma z}$

Traveling wave solutions in time domain

- Converting back in time domain:

$$V(z) = \left|V_o^+\right| \cos\left(\omega t - \beta z + \phi^+\right) e^{-\alpha z} + \left|V_o^-\right| \cos\left(\omega t + \beta z + \phi^-\right) e^{\alpha z}$$

for fixed point on wave (ωt-βz=cte) z has to increase if t increases, so wave traveling in pos. z direction

- From this we can find wavelength on line:

[ωt-βz]-[ωt-β(z+λ)]=2π for distance between 2 reference points

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

Phase velocity on line (velocity fixed phase point on wave):

$$\boxed{v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{\omega t - \text{constant}\, t}{\beta}\right) = \frac{\omega}{\beta} = \lambda \cdot f}$$

$$\boxed{v_p = 1/\sqrt{\mu_0 \varepsilon_0 \varepsilon_r} = c/\sqrt{\varepsilon_r}}$$

$$\downarrow$$

$$\boxed{\lambda = \frac{c \cdot f}{\sqrt{\varepsilon_r}}}$$

TEM (will see in Lecture 3)
c: velocity of light free-space: 2.998×10^8 m/s
ε_r: dielectric constant (effective for quasi-TEM)

In case of lossless line:

- Many practical cases loss very small and can be neglected

$$\boxed{\gamma = j\beta = j\omega\sqrt{LC}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

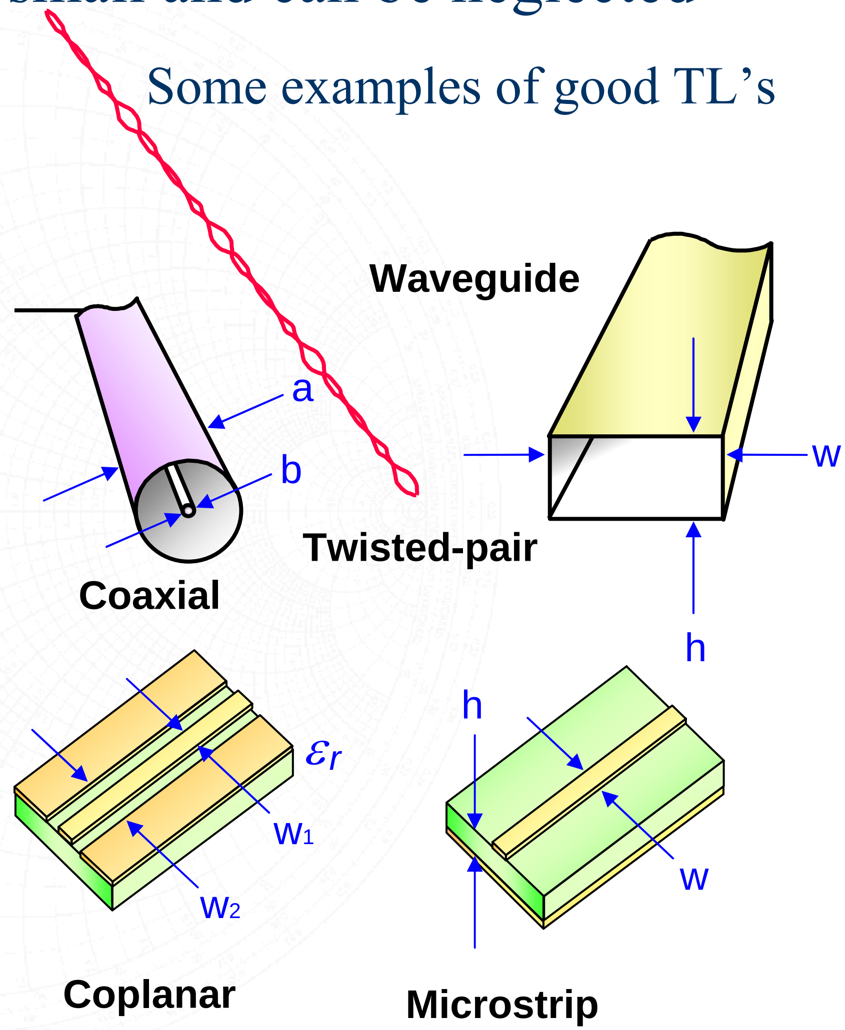
$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}}$$

$$\boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}}$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^+}{Z_0} e^{+j\beta z}$$

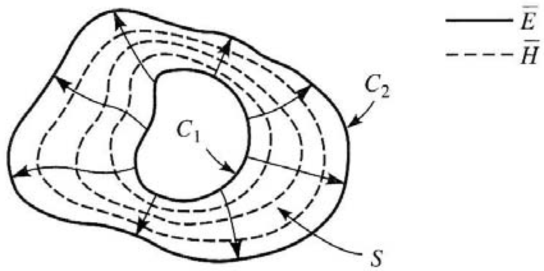
Some examples of good TL's



When do we care about transmission lines?

- Analog/microwave applications:
  - Lumped if dimensions (l) << $\lambda/10$
  - Back-of-the-envelope: $\lambda = \frac{c \cdot f}{\sqrt{\varepsilon_r}}$, so for ε_r=9 (Alumina board)
    - 1 GHz (ε_r=9): $\lambda=3.10^8/3.10^9$=0.1 m, l < 10 mm
    - 10 GHz (ε_r=9): $\lambda=3.10^8/3.10^{10}$=10 mm, l < 1 mm
    - 30 GHz (free-space): $\lambda=3.10^8/3.10^{10}$=10 mm, l < 1 mm
    - 300 GHz (free-space): $\lambda=3.10^8/3.10^{11}$=1 mm, l < .1 mm
- Digital Applications (see Agilent TL Fundamentals):
  - Lumped only if Td <T_r (T_r/2.5) ("flight-time=L/v_p" along TL < rise (or fall) time digital signal (/2.5))
  - TL important if total series R<5*Z_0, lossless if R<Z_0/5
  - Back-of-the-envelope:
    - 1.25 Gb/s backplane: Tr~250ps on board with ε_r=4 ($v_p=3.10^8/\sqrt{4}$ m/s or 150 mm/ns), so max. length 37.5 (15) mm
    - 4 GHz Pentium: Tr~100ps on low-K Si (ε_r=2.25, vp=200 mm/ns), so max. length 20 (8) mm
    - For clock, lumped approximation for analog case!

Field analysis of Transmission Lines



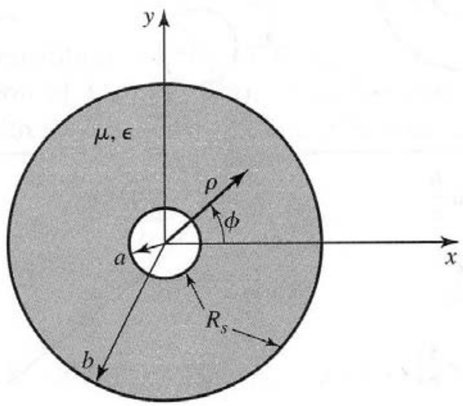
- Transmission line equations can also be derived from Maxwell equations – see 2.2
- Transmission line parameters from E and H fields

$$L = \frac{\mu}{|I_o|^2}\int_S \overline{H}\cdot\overline{H}^* ds \ \ \text{H/m} \qquad R = \frac{R_s}{|I_o|^2}\int_{C_1+C_2} \overline{H}\cdot\overline{H}^* dl \ \ \Omega\text{/m}$$

$$C = \frac{\varepsilon}{|V_o|^2}\int_S \overline{E}\cdot\overline{E}^* ds \ \ \text{F/m} \qquad G = \frac{\omega\varepsilon''}{|V_o|^2}\int_S \overline{E}\cdot\overline{E}^* ds \ \ S\text{/m}$$

More on physical transmission lines in Lecture 3

Field analysis of Transmission Lines (coax)



$$\bar{E} = \frac{V_o \hat{\rho}}{\rho \ln b/a} e^{-\gamma z},$$

$$\bar{H} = \frac{I_o \hat{\phi}}{2\pi\rho} e^{-\gamma z},$$

$$L = \frac{\mu}{(2\pi)^2} \int_{\phi=\phi}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho \, d\rho \, d\phi = \frac{\mu}{2\pi} \ln b/a \quad \text{H/m},$$

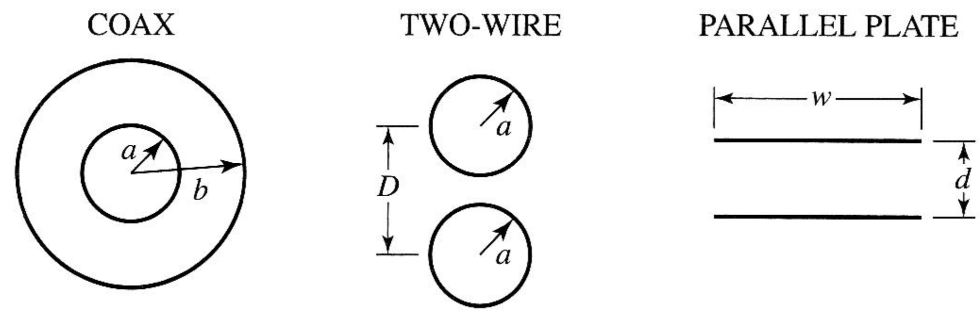
$$C = \frac{\epsilon'}{(\ln b/a)^2} \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho \, d\rho \, d\phi = \frac{2\pi\epsilon'}{\ln b/a} \quad \text{F/m},$$

$$R = \frac{R_s}{(2\pi)^2} \left\{ \int_{\phi=0}^{2\pi} \frac{1}{a^2} a \, d\phi + \int_{\phi=0}^{2\pi} \frac{1}{b^2} b \, d\phi \right\} = \frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right) \quad \Omega/\text{m},$$

$$G = \frac{\omega\epsilon''}{(\ln b/a)^2} \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho \, d\rho \, d\phi = \frac{2\pi\omega\epsilon''}{\ln b/a} \quad \text{S/m}.$$

Parameters for some common lines

TABLE 2.1 Transmission Line Parameters for Some Common Lines

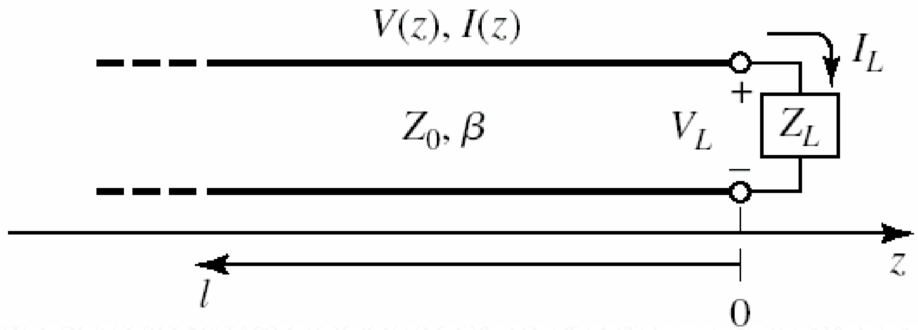


	COAX	TWO-WIRE	PARALLEL PLATE
L	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu d}{w}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$	$\frac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\frac{\epsilon' w}{d}$
R	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$	$\frac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\frac{\omega\epsilon'' w}{d}$

- **R_S is the surface resistivity related with skin-depth δ_s**
- **permittivity $\varepsilon = \varepsilon' - j\,\varepsilon'' = \varepsilon'(1 - j\tan\delta)$ lossy dielectric fill**
- **permeability $\mu = \mu_0\,\mu_r$**

$$\boxed{R_s = \frac{1}{\sigma\delta_s} = \sqrt{\frac{\omega\mu}{2\sigma}}}$$

Terminated lossless line



$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{+j\beta z}$$

$$V(z) = V_o^+ \left[e^{-j\beta z} + \Gamma e^{+j\beta z} \right]$$

$$I(z) = \frac{V_o^+}{Z_0} \left[e^{-j\beta z} - \Gamma e^{+j\beta z} \right]$$

I and V superposition of Incident and reflected waves

At load: $$Z_L = \frac{V(0)}{I(0)} = Z_0 \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-}$$

Solving for V_o^-: $$\boxed{V_o^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_o^+}$$

Voltage reflection coefficient Γ: $$\boxed{\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$\Gamma = 0$ if $Z_L = Z_0$ or load matched to line

Time-averaged power flow along TL

$$P_{av} = \frac{1}{2}\mathrm{Re}\left[V(z)I(z)^*\right] = \frac{1}{2}\frac{\left|V_o^+\right|^2}{Z_0}\left(1 - |\Gamma|^2\right)$$

Power delivered in load by TL **Incident power** **Reflected power**

For matched generator (no double reflections):

- Average power flow constant each point of line
- Total power in load=incident-reflected power
- Γ=0: maximum power delivered to load, Γ=1: no power

Fraction of incident power absorbed by load missing from signal returned to generator: return loss (RL)

$$\boxed{RL = -20 \log |\Gamma| \quad \mathrm{dB}}$$

- Matched load (|Γ|=0): RL=∞ dB
- Total reflection (|Γ|=1): RL = 0 dB

Voltage Standing Waves and VSWR

- Matched load ($|\Gamma|=0$) $\Rightarrow$ $|V(z)|= |V_o^+|$ so constant
- For $|\Gamma|\neq 0$ $\Rightarrow$ standing waves ($|V(z)|$ not constant)

$$\left|V(z)\right| = \left|V_o^+\left(e^{-j\beta z} + \Gamma e^{+j\beta z}\right)\right| = \left|V_o^+\right|\left(1 + |\Gamma| e^{j(\theta - 2\beta l)}\right)$$

where l=-z the positive distance away from load and
θ is the phase of Γ; $\Gamma=|\Gamma|e^{j\theta}$

- Voltage magnitude oscillates with z along line, maximum and minimum values when phase term $e^{j(\theta-2\beta l)}=\pm 1$:

$$V_{\max,\min} = \left|V_o^+\right|\left(1 \pm |\Gamma|\right)$$

- Measure of mismatch of load is standing wave ratio or voltage standing wave ratio

$$SWR = VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Standing Waves and Impedance

- $\boxed{|V(z)| = |V_o^+| \left(1 + |\Gamma| e^{j(\theta - 2\beta l)}\right)}$ so distance between successive max. (or min.) is λ/2, distance between a max. and a min is λ/4
- In general Γ and Z_{in} varying with position along line:

$$\Gamma(l) = \frac{V_o^-}{V_o^+} e^{-2j\beta l} = \Gamma(0) e^{-2j\beta l}$$

$$V(z) = V_o^+ \left[e^{-j\beta z} + \Gamma e^{+j\beta z}\right]$$

$$I(z) = \frac{V_o^+}{Z_0} \left[e^{-j\beta z} - \Gamma e^{+j\beta z}\right]$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

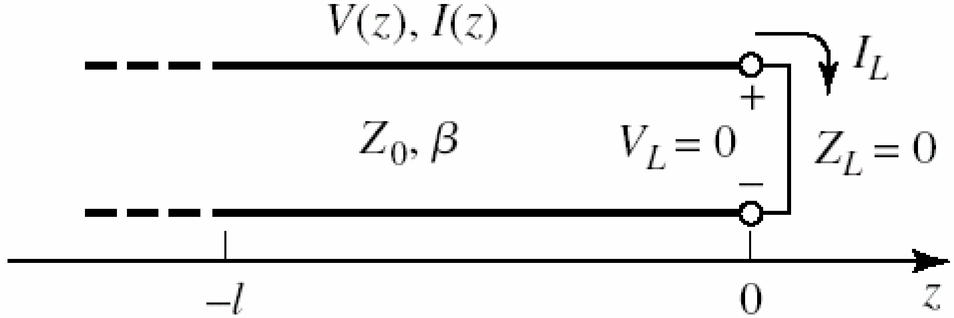
$$Z_{in} = \frac{V(-l)}{I(-l)} = Z_0 \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} = Z_0 \frac{1 + \Gamma e^{-2j\beta l}}{1 - \Gamma e^{-2j\beta l}} = Z_0 \frac{1 + \Gamma(l)}{1 - \Gamma(l)}$$

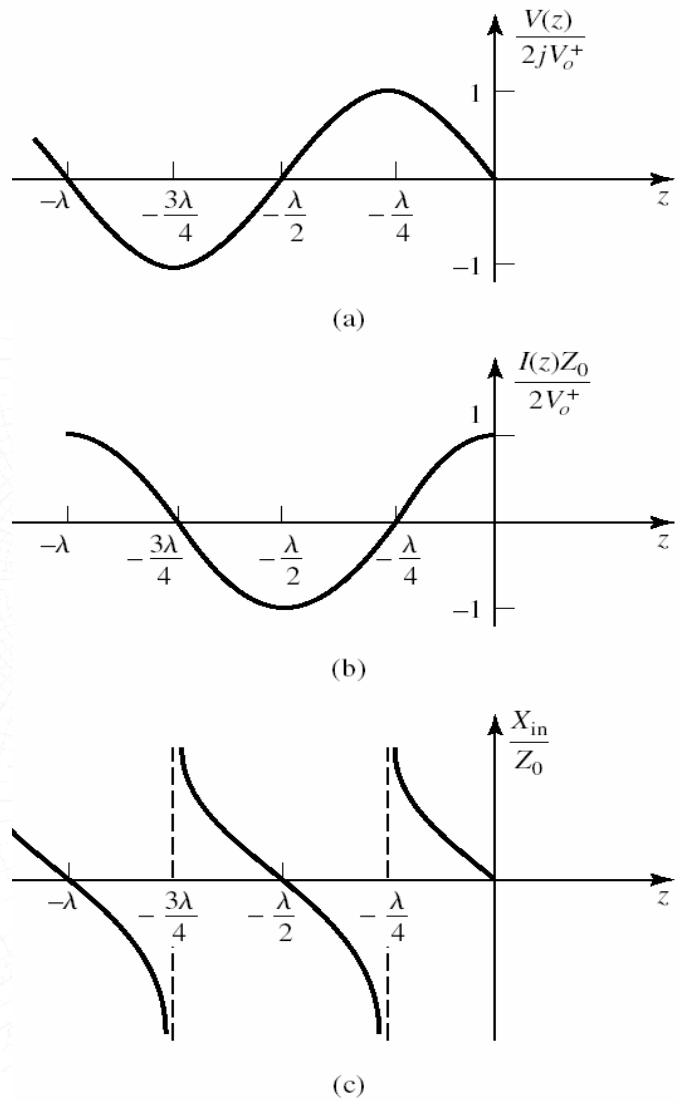
$$Z_{in} = Z_0 \frac{(Z_L + Z_0)e^{j\beta l} + (Z_L - Z_0)e^{-j\beta l}}{(Z_L + Z_0)e^{j\beta l} - (Z_L - Z_0)e^{-j\beta l}} = Z_0 \frac{Z_L \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jZ_L \sin \beta l}$$

$$\boxed{Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}}$$

Transmission line equation

Special terminations (1): short circuit





- if Z_L=0, then Γ= -1, so

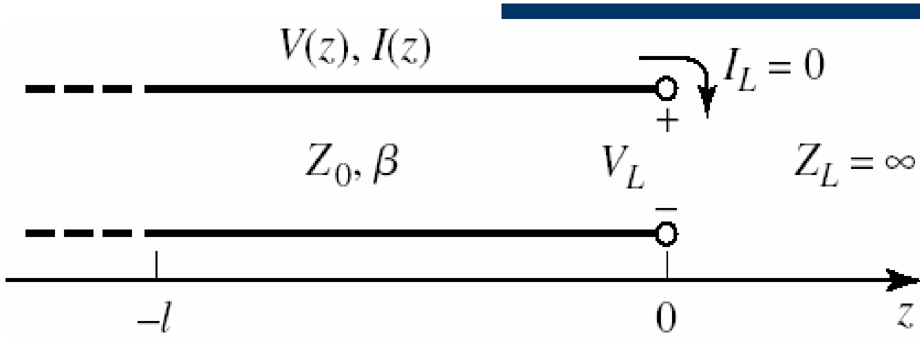
$$V(z) = V_o^+ \left[e^{-j\beta z} - e^{+j\beta z}\right] = -2jV_o^+ \sin \beta z$$

$$I(z) = \frac{V_o^+}{Z_0}\left[e^{-j\beta z} + e^{+j\beta z}\right] = \frac{2V_o^+}{Z_0} \cos \beta z$$

$$Z_{in} = \frac{V(-l)}{I(-l)} = jZ_0 \tan \beta l$$

- So impedance purely imaginary
- For l=0: Z_{in}=0 but for l=λ/4: $Z_{in}= \infty$
- Impedance is periodic with λ/2

Special terminations (2): open circuit



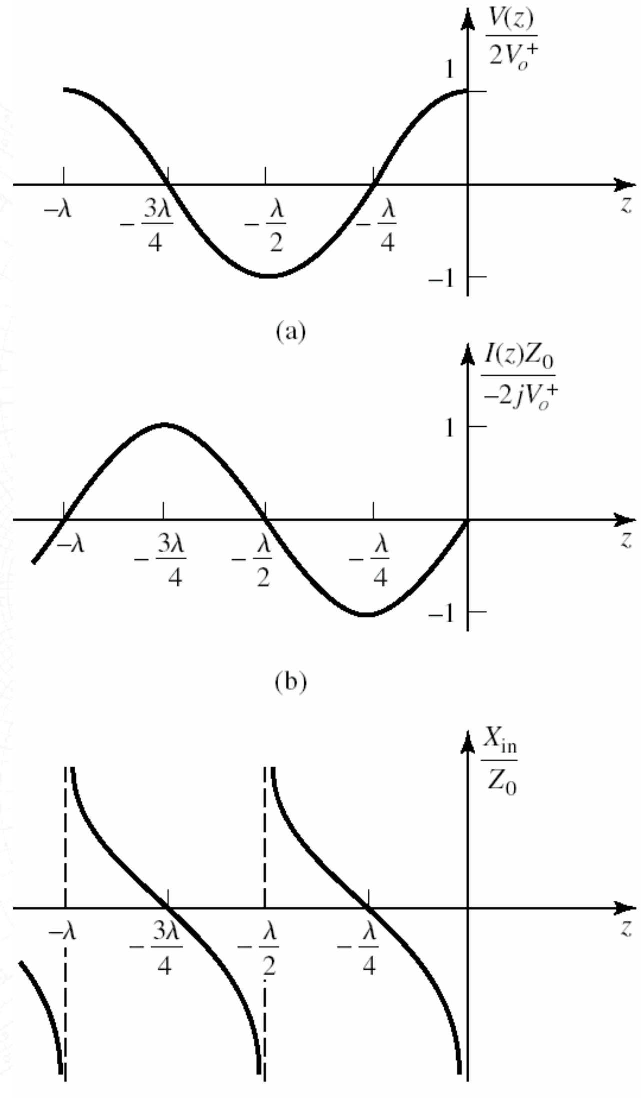
- if $Z_L=\infty$, then $\Gamma= +1$, so

$$V(z) = V_o^+\left[e^{-j\beta z} + e^{+j\beta z}\right] = 2V_o^+ \cos \beta z$$

$$I(z) = \frac{V_o^+}{Z_0}\left[e^{-j\beta z} - e^{+j\beta z}\right] = -\frac{2jV_o^+}{Z_0} \sin \beta z$$

$$Z_{in} = \frac{V(-l)}{I(-l)} = -jZ_0 \cot \beta l$$

- So impedance purely imaginary
- For $l=0$: $Z_{in}= \infty$ but for $l=\lambda/4$: $Z_{in}=0$
- Impedance is periodic with $\lambda/2$



(a)

(b)

(c)

Special lengths of transmission line

- A half-wavelength line ($l=n\lambda/2$) does not alter or transform the load impedance, regardless of Z_0

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \quad \Rightarrow \quad Z_{in}(\lambda/2) = Z_L$$

- A quarter-wavelength line ($l= \lambda/4+n\lambda/2$) transforms the impedance, according to characteristic impedance of line

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = \frac{Z_0^2}{Z_L}$$

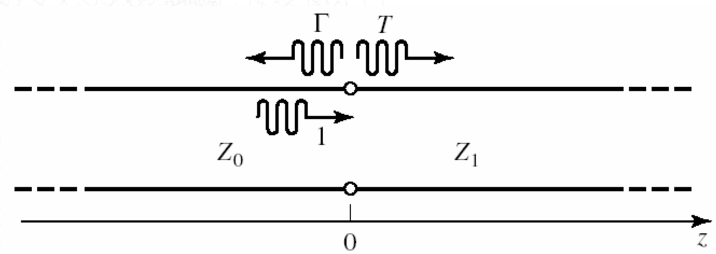
- This is called quarter-wave transformer, to be studied more in detail next lecture (2.5)

Termination with matched transmission line

- Line with characteristic impedance Z_0 feeds line with characteristic impedance Z_1, if this TL is infinitely long or terminated with Z_1, then $Z_L=Z_1$, so

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

- Not all incident wave reflected, part transmitted onto line 2 with voltage amplitude T



$$V(z) = V_o^+ \left[e^{-j\beta z} + \Gamma e^{+j\beta z} \right], \quad z < 0$$

$$V(z) = V_o^+ T e^{-j\beta z}, \quad z > 0$$

Equating both at z=0 gives $T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$

- Transmission coefficient expressed as insertion loss

$$\boxed{IL = -20 \log|T| \quad \text{dB}}$$

Recap & next lecture

- In this lecture, we reviewed fundamental properties of transmission lines:
  - TL's support two waves (traveling in positive and negative direction)
  - Relation voltage & current for each of these waves: characteristic line impedance Z_0
  - Relation voltage incoming & reflected wave: voltage reflection coefficient Γ
  - Total voltage (current) on line is sum (difference) of both waves
  - Due to changing phase difference between waves both current and voltage and impedance will change along direction of line when moving away from load, will result in voltage standing waves
- Is explained intuitively in the Agilent TL Fundamentals Course (see link website), will take brief look next lecture, please review this program!
- Next lecture we will study the Smith Chart, quarterwave-length transformers (also from multiple reflection point), transient on TL's, etc…