Mathematical Biology

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The course “Mathematical Modeling” has a dual purpose: on one hand, to introduce students to some basic mathematical models in various areas of biology (demography, ecology, infectious diseases, enzyme reactions, physiology, molecular networks); on the other hand, to provide fundamental knowledge in the analysis and numerical simulation of ordinary and partial differential equations.

Specifically, the first part of the course is dedicated to modeling using ordinary differential equations and introduces various analytical techniques (linearization, equilibria and their stability, bifurcation, regular and singular perturbations).

- Overview of ordinary differential equations (ODEs): Solution of linear equations; equilibria and linearized stability; phase plane, limit cycles; numerical schemes for solving ODEs.
- One- or two-dimensional models in demography, ecology, epidemiology, and immunology. Non-dimensionalization of variables and parameters.
- Slow-fast systems, enzyme reaction models and their simplification using perturbative methods.
- Bifurcation of equilibria and application to predator-prey systems and molecular networks. Simplified models of important biological phenomena, such as the cell cycle and glucose-insulin oscillations.
- Excitable systems: Hodgkin-Huxley equations (overview) and FitzHugh-Nagumo equations.
- Parameter estimation for differential models.

In the second part, partial differential equation models and some techniques for constructing or approximating solutions will be studied. Additionally, some of the most interesting phenomena of reaction-diffusion equations (traveling wave solutions, Turing mechanism) will be presented in a biological context (morphogenesis).

- Dynamical systems on networks. Examples in epidemiology.
- Introduction to partial differential equations (PDEs): Solutions by separation of variables. Fourier series. The heat equation and Brownian motion. Eigenfunctions of the Laplacian. Numerical approximation.
- Skellam and Fisher equations: Waveform solutions; stationary solutions of the boundary value problem.

- Stability of stationary solutions of reaction-diffusion systems and Turing's mechanism for morphogenesis. Conditions for its validity and examples. Chemotaxis: The Keller-Segel model.