

# Investigation of Change in the Computational Technique of the Sun’s Physical Ephemeris in *The Astronomical Almanac*

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## 1 PREFACE

The following USNO Technical Note was written to document results of implementing suggested changes by the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements (WGCRE). As such, it was distributed to the WGCRE prior to their finalizing the 2006 report [4]. One of this note’s points, summarized in the concluding paragraph,

“Whether the new  $W_0$  value (the ephemeris position of the Sun’s prime meridian at standard epoch J2000.0) should be altered to further minimize the differences in  $L_0$  (the heliographic longitude of sub-Earth Point) is a decision for the working group. Altering the value of  $W_0$  to  $84.176^\circ$  would have no further effect on  $P$  (the position angle of the axis) or  $B_0$  (the heliographic latitude of sub-Earth Point), but would reduce the number of differences in  $L_0$  between the two methods by more than half and make similar number of differences positive and negative.”

was the impetus used for changing the  $W_0$  from the proposed  $84.18^\circ$  value to the now accepted value of  $84.176^\circ$ . Therefore, as this study began, the authors were under the idea that the  $W_0$  would be  $84.18^\circ$  and consequently the paper reflects this value.

— Sean Urban and George Kaplan, July 13, 2007

## 2 INTRODUCTION

In Section C of *The Astronomical Almanac*, the heliographic latitude and longitude of the sub-Earth point are printed for each day of the year at 0h TT. This utilizes the Sun’s rotation parameters as given in the reports of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements [1], originally given by Carrington [2]. Prior to the 2009 edition of *The Astronomical Almanac*, neither the light-time correction nor a correction for aberration were applied to the rotation because they were presumably already in Carrington’s meridian since he refers his observations to the “apparent central meridian” [3]. Having both the light-time and aberration compensated for in the zero point of the meridian is possible only for the Sun since the Sun is essentially always the same distance from the Earth and the aberration correction is nearly constant, therefore the differences in light-time and aberration between perigee and apogee are small. This brought about a situation where the computation technique of the Sun’s rotation has always been different from that of the other Solar System bodies.

During the 2006 IAU General Assembly, at the urging of Patrick Wallace, the Working Group on Cartographic Coordinates and Rotational Elements (hereafter known simply as “the working group”) decided that computing the Sun’s rotation differently has detrimental effects for a number of reasons. First, it confuses many people which has led to erroneous computations over the years. Second, the method used previously is only valid at the Earth. Third, the accuracy is compromised because neither the light-time nor aberration is constant between the Earth and Sun since the Earth-Sun distance varies.

To make the computation consistent and applicable anywhere, the working group recommends that the  $W_0$  value for the Sun—the angle of the prime meridian at the standard epoch—be “foredated” by an amount which approximately corresponds to the light-travel time. Using this new value, the computation *must* take into account the light travel time; that is, the rotation is antedated. The combination of foredating and antedating should have little effect on the values computed for locations on or near the Earth, but would make the computation technique more consistent with the other objects and be suitable anywhere in the Solar System.

Not explicitly stated in the working group’s report, but required to bring about uniformity in the computational method between the Sun’s physical ephemeris and that of the other Solar System bodies, is the need to compensate for aberration. Whether aberration has been accounted for in Carrington’s data is subject to some debate. Similar to the light-time correction, the correction for aberration at Earth for the Sun is nearly constant. Considering Carrington’s work [2] and the common computational practices of the mid-19th century, USNO personnel believe that Carrington’s meridian contains corrections for aberration much the same way that it contains corrections for light-time error. Therefore it has been USNO’s practice *not* to explicitly compensate for aberration in the printed physical ephemeris values, i.e., heliographic latitude and longitude or position angle of the axis of rotation. This practice is different from similar computations for the planets. Since the goal of the working group’s recommendation is to compute the physical ephemerides of the Sun similarly to that of the other solar system bodies, the aberration correction should now be explicitly applied. This change will be made beginning with the 2009 edition.

This paper documents the changes in the computational technique as suggested by the working group and describes some of the effects that users can expect in the data.

### 3 CHANGES SUGGESTED BY THE WORKING GROUP

Using the 2000 working group report [1], the solar prime meridian is computed by using

$$W = 84.10^\circ + 14.1844000 \times d \quad (1)$$

where  $84.10^\circ$ , referred here as  $W_0$ , is the ephemeris position of the Sun’s prime meridian at standard epoch J2000.0; 14.1844000 is the daily rotation rate (degrees/day); and  $d$  is the date of interest, in days of Barycentric Dynamical Time (TDB), since the standard epoch. The 2007 report of the working group [4] recommends using

$$W = 84.18^\circ + 14.1844000 \times (d - \tau) \quad (2)$$

where the  $W_0$  value has changed and  $\tau$  is the light-time between Sun center and the geocenter. The change of  $0.08^\circ$  in  $W_0$  is to compensate for the rotation during the light-time of 1 au. Thus, if the Earth’s orbit were circular and the change in  $W_0$  were exactly the rotation rate over 1 au/c  $\approx 499$  seconds, the results of the new method would be identical to the old. (The actual value for light-time varies by 1.7% over the course of the year.)

Not addressed in the working group’s report is aberration. As explained in the introduction, prior to the 2009 edition of *The Astronomical Almanac* the correction for aberration has not been made explicitly in

the printed values of the heliographic latitude and longitude nor the position angle of the axis. It has been concluded that aberration was already in Carrington's rotational parameters.

A new computational method has been implemented in *The Astronomical Almanac* production code. The latest code uses the new  $W_0$  value for the Sun's prime meridian while also explicitly compensating for the light-time. The new code also explicitly applies aberration in computing the heliographic coordinates of the sub-Earth point and position angle of the axis. The next section analyzes the effect on this implementation on the data in the printed book.

## 4 ANALYSIS AND DISCUSSION

Using the new technique, all values for *The Astronomical Almanac* pages C4 through C23 were computed. Data for the semi-randomly selected years 2004, 2007 and 2010 were investigated. As expected, only those values for the heliographic latitude and longitude of the sub-Earth point and position angle of the axis changed. None of the printed values differed by more than  $0.01^\circ$ , the precision given in the book for each of these quantities.

Upon analyzing the differences, it was concluded that the precision of  $0.01^\circ$  was not sufficient to completely see the effects of the programming changes. Therefore, the data for 2007 were run with precision of  $0.00001^\circ$  (36 mas) using the old method, the new rotation method with no aberration included, aberration only, and the new rotation plus aberration.

### 4.1 Position Angle of the Axis, $P$

The position angle of axis,  $P$ , is the position angle of the Sun's north pole as would be seen by an observer at the geocenter. This angle is measured eastward from north on the celestial sphere, where north is defined by the great circle on the celestial sphere passing through the true celestial north pole of date and the center of the Sun's apparent disk; see Figure 7.12.3 in [5]. Figure 1 shows  $P$  for 2007, and details the changes using the new method. The dot-dash curve gives  $P$  in degrees, the dotted line ( $y$  value equals zero) gives the change in  $P$  due solely to the new rotation computation, and the solid curve gives the change in  $P$  due to aberration, in arcseconds. Since the change due to the new rotation is zero, the solid curve also gives the total change in  $P$  between the pre-2009 method and the new method.

Initially, Fig. 1 might look a little odd since one might expect  $P$  to show a signature of the obliquity, that is, a  $23.5^\circ$  oscillation with extrema at the equinoxes and null values at the solstices. However, it is important to note that the Sun's polar axis is not orthogonal to the ecliptic. To clarify the changes and to ensure that the program is working as expected, it is useful to set the Sun's pole orthogonal to the ecliptic. Figure 2 shows the same data as Fig. 1, but with this change. The dot-dash line in Fig. 2 shows the effect of obliquity, exactly as expected.

The dotted curves in Figs. 1 and 2 are flat and zero. This is as expected since a change in rotation should have no effect on the position of the pole.

The solid curves represent the effect of aberration in arcseconds, and are clearly neither flat nor zero. Aberration effectively shifts the ecliptic longitude of the Sun,  $\lambda$ , westward on the sky (decreasing the value of  $\lambda$ ). Like the light-time correction, the aberrational shift results in a slight change of viewing angle. The dot-dash curve in Fig. 1 can be considered to show  $P$  as a function of  $\lambda$  (plus a constant), since the latter increases by about one degree per day. So the curve can be used to provide a good estimate of how much  $P$  will change at different times of the year, if we subtract 20.5 arcseconds from  $\lambda$  (20.5 arcseconds being the mean aberration; the true amount of aberration varies from 20.14 arcseconds at aphelion to 20.85 arcseconds

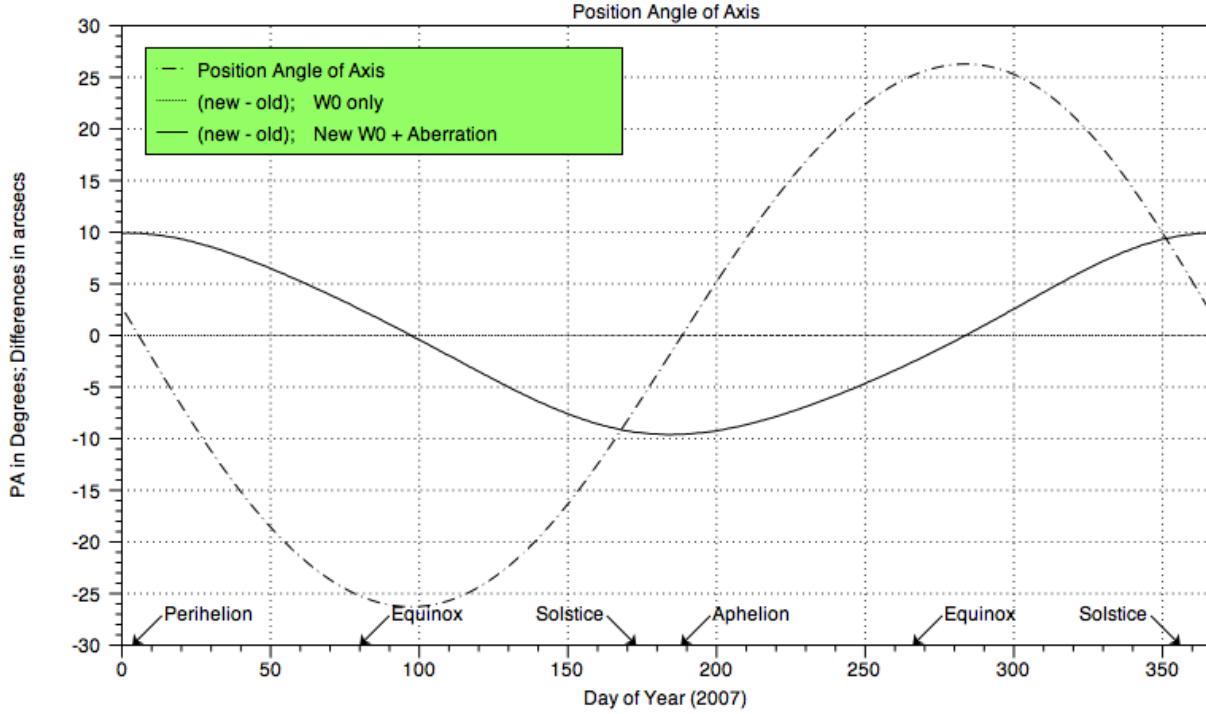


Figure 1:  $P$  using pre-2009 method, new rotation only, and new rotation plus aberration.

at perihelion). During the first part of the year,  $P$  is decreasing rapidly, about  $0.5^\circ/\text{day}$ , while the ecliptic longitude is increasing by a little more than  $1^\circ/\text{day}$ ; so the ratio of the change in  $P$  to the change in longitude is about  $-0.5$ . The effect of aberration is to decrease the ecliptic longitude by 20.5 arcseconds, so one would expect a change of about +10 arcseconds in  $P$  at this time of year, exactly what is shown in the solid curve. The sinusoid with an amplitude of about 10 arcseconds reflects the changing function of  $P$  with longitude throughout the year, multiplied by the actual change in longitude due to aberration. When the rate of change in  $P$  is near zero (at the minimum and maximum of the dot-dash curve), the difference due to aberration is also zero; when the rate of change is at maximum or minimum (when the dot-dash curve crosses zero), so is the change due to aberration. This is exactly what one would expect.

#### 4.2 Heliographic Latitude of sub-Earth Point, $B_0$

Figure 3 shows the heliographic latitude of the sub-Earth point,  $B_0$ , for 2007, and details the changes using the new method. Similar to Fig. 1, the dot-dash curve gives  $B_0$  in degrees, the dotted line ( $y$  value equals zero) gives the change in  $B_0$  due solely to the new rotation computation, and the solid curve gives the change in  $B_0$  due to aberration, in arcseconds. Since the change due to the new rotation is zero, the solid curve also gives the total change in  $B_0$  between the pre-2009 method and the new method.

As with  $P$ , one can separate the effects of the tilt of the Sun's axis with respect to the ecliptic. In fact, when the Sun's axis is set orthogonal to the ecliptic, all curves are zero (not shown), exactly as one would

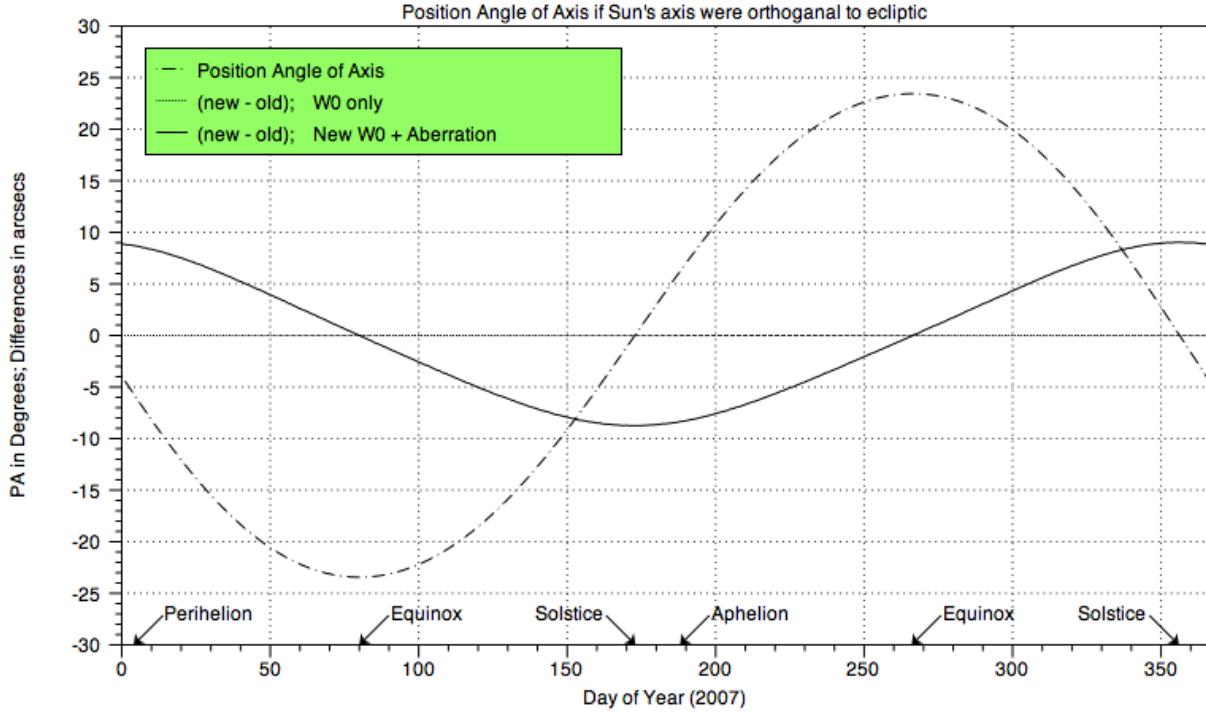


Figure 2: Same as Fig. 1 but with Sun's pole set orthogonal to the ecliptic.

expect. Since the Sun's axis is tilted by  $7.25^\circ$ , we expect the effect of aberration to move the latitude at most by  $20.5 \text{ arcseconds} \times \sin 7.25^\circ$ , or about 2.6 arcseconds, and be seasonally dependent. This is exactly what is seen. Also as expected and explained in Fig. 1, the maximum changes occur when the rate of change of  $B_0$  is largest and are zero when the rate of change of  $B_0$  is zero.

### 4.3 Heliographic Longitude of sub-Earth Point, $L_0$

The changes in the heliographic longitude of the sub-Earth point,  $L_0$ , are plotted in Fig. 4 for 2007. The dotted curve gives the change in  $L_0$  due solely to the new rotation computation; the dashed curve gives the change in  $L_0$  due solely to aberration; the solid curve gives the total change between the pre-2009 method and the new method.

It is clear in Fig. 4 that none of the curves are centered at zero. This is the result we would expect if the  $W_0$  value were not set exactly to compensate for the Sun's rotation over the light-time for 1 au and the aberration correction. In fact, this is the case.

Neglecting the effects of aberration for a moment, the Sun's rotation rate of 14.1844 degrees/day translates to 0.08192 degrees in 499s; 499s being the rounded average light-time at 1 au; however, the new  $W_0$  value of  $84.18^\circ$  has only been changed by  $0.08^\circ$ . Therefore we expect a bias from the new rotation computation of  $0.00192^\circ$ , or about 7 arcseconds. This is exactly where the dotted curve is centered. The oscillation in the dotted curve is due to light-time being smaller at perihelion and larger at aphelion. With the light-time

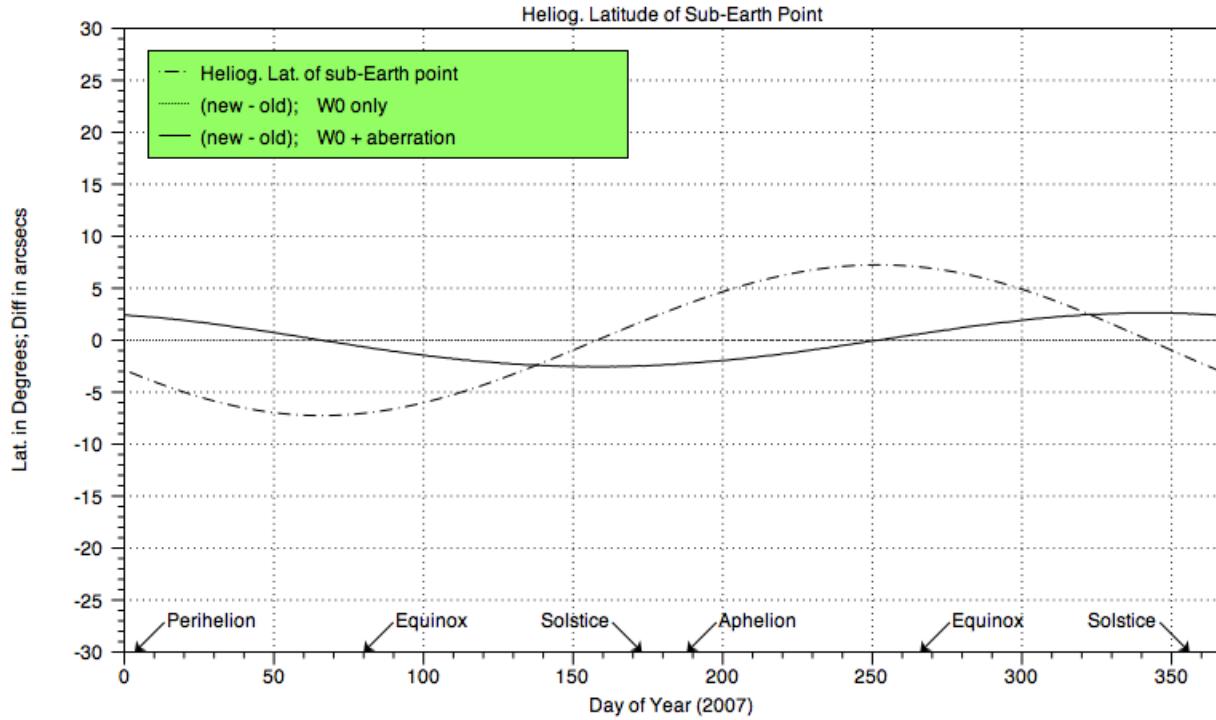


Figure 3:  $B_0$  using pre-2009 method, new rotation only, and new rotation plus aberration.

varying by about 16.7 seconds between the two extremes, we expect variations of about 11 arcseconds solely based on the fluctuating distance between Earth and Sun throughout the year. This is seen in the dotted curve.

The dashed curve shows the differences using the pre-2009 rotation computation but explicitly correcting for aberration. As expected, a nearly constant offset of  $-20.5$  arcseconds is seen, with oscillations under 1 arcsecond mostly due to varying velocity as the Earth moves through perihelion and aphelion.

The solid curve represents the total change in  $L_0$  due to the new computational method. It is clear that the new rotation zero point does not completely compensate for the new method; on average it is off by 13 arcseconds. Whether this is an acceptable amount is a topic that the working group should decide. An alternative would be to alter the new  $W_0$  value by  $0.004^\circ$  to  $84.176^\circ$ . This would bring the new method closer to the pre-2009 method, but may be unnecessary since the differences between any of these methods is one unit in the least significant digit printed in *The Astronomical Almanac*. Figure 5 shows the effect of new method using a  $W_0$  value of  $84.176^\circ$ .

## 5 Summary

The goal of the working group's recommendation for the new computational technique for Sun's physical ephemeris is to compute the Sun's data in a similar fashion to that of other Solar System bodies while

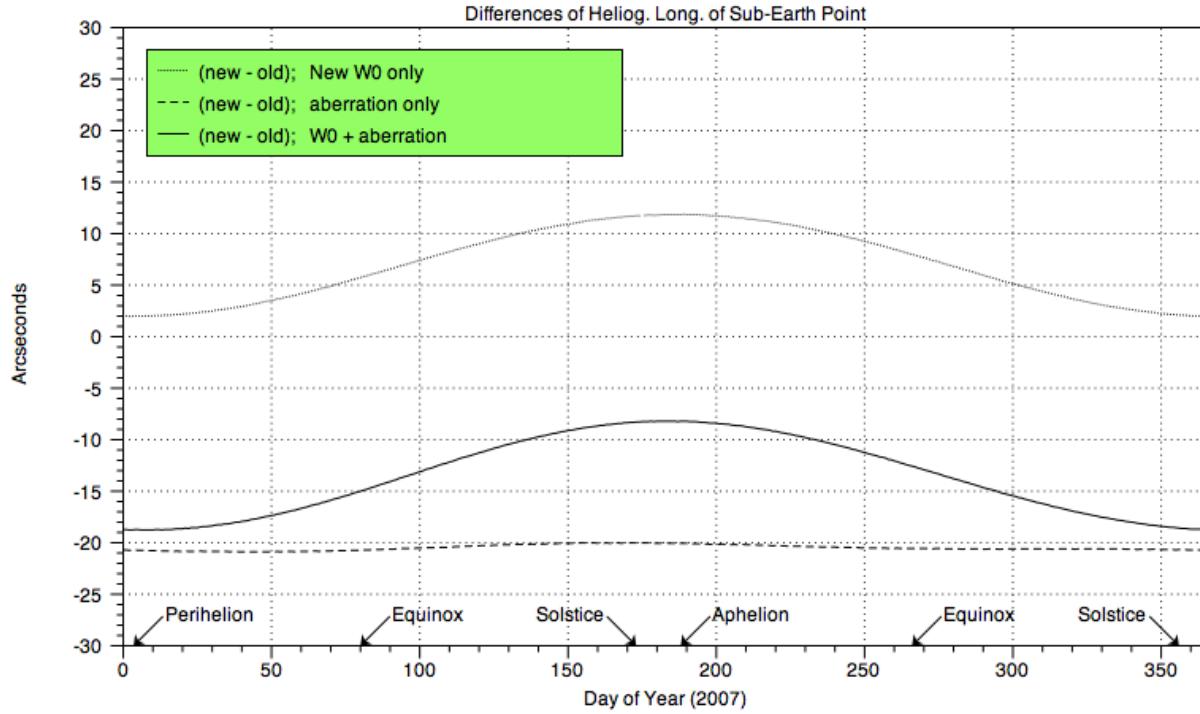


Figure 4: Differences in  $L_0$  between the pre-2009 method and new rotation only, aberration only, and new rotation plus aberration.

maintaining consistency with data computed previously. To do this, the Sun's prime meridian at the standard epoch has been shifted, but this is mostly compensated for in the light-time correction. Aberration also needs to be taken into account explicitly, as is common practice for other Solar System bodies.

The effects of these changes on *The Astronomical Almanac* is minimal. Previously published values of  $P$ ,  $B_0$ , and  $L_0$  will match the new computations within one unit in the least significant printed digit ( $0.01^\circ$ ). The changes in  $P$  and  $B_0$  are due solely to aberration, and the differences are both positive and negative. The differences in  $L_0$  are due to a combination of effects, mainly that the new  $W_0$  doesn't completely compensate for the average light-time plus aberration. There is an additional effect that light-time varies over the course of the year and a much smaller effect that aberration is not constant over the year. The result is that about 35% of the time, the differences (new technique – old technique) will be  $-0.01$  while the rest of the time the printed values will match.

Whether the new  $W_0$  value should be altered to further minimize the differences in  $L_0$  is a decision for the working group. Altering the value of  $W_0$  to  $84.176^\circ$  would have no further effect on  $P$  or  $B_0$ , but would reduce the number of differences in  $L_0$  between the two methods by more than half and make similar number of differences positive and negative.

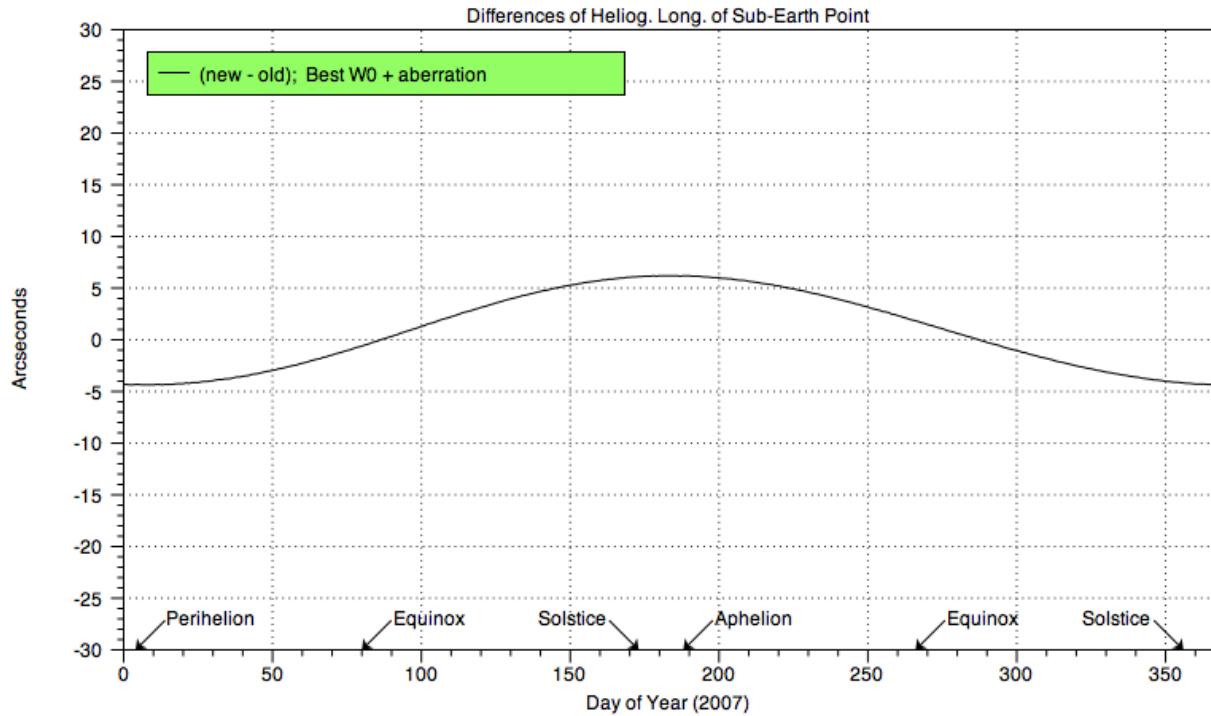


Figure 5: Differences in  $L_0$  between the pre-2009 method and new method but using a  $W_0$  value of  $84.176^\circ$  instead of the currently recommended value of  $84.18^\circ$ . (Author's note: Due to this paper and this figure in particular, the WGCRE decided to adopt the  $84.176^\circ$  value for  $W_0$ .)

## References

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- [5] Explanatory Supplement to the Astronomical Almanac, 1992, ed P. K. Seidelmann, p. 392