

$$1. m(x) = \frac{1}{n} \sum_{i=1}^n x_i \quad x = a + bX$$

$$m(x) = \frac{1}{n} \sum_{i=1}^n (a + bX_i)$$

$$m(x) = \frac{1}{n} \sum_{i=1}^n a + \frac{1}{n} \sum_{i=1}^n bX_i$$

$$m(x) = \frac{na}{n} + b \left(\frac{1}{n} \sum_{i=1}^n x_i \right)$$

$$m(x) = a + bm(x)$$

$$2. \text{cov}(X, Y) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(y))$$

$$m(y) = a + bm(y)$$

$$\text{cov}(x, y) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(y))$$

$$\text{cov}(x, y) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(a + by_i - (a + bm(y))) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(by_i - bm(y))$$

$$\text{cov}(x, a + by) = b \text{cov}(x, y)$$

$$= b \text{cov}(x, y)$$

$$3. s^2 = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2$$

$$\text{cov}(x, x) = s^2 = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2$$

$$x = a + bX$$

$$m(x) = \frac{1}{N} \sum_{i=1}^N (a + bX_i)$$

$$s_x^2 = \frac{1}{N} \sum_{i=1}^N (x - m(y))^2$$

$$m(x) = a + bm(x)$$

$$s_x^2 = \frac{1}{N} \sum_{i=1}^N (a + bX - (a + bm(x)))^2$$

$$= \frac{1}{N} \sum_{i=1}^N (bX - bm(x))^2$$

$$= b^2 \left(\frac{1}{N} \sum_{i=1}^N (X - m(x))^2 \right)$$

$$s_x^2 = b^2 s^2$$

$$\text{cov}(a + bX, a + bX) = b^2 s^2$$