Regression Models and Simulation for Problem 3

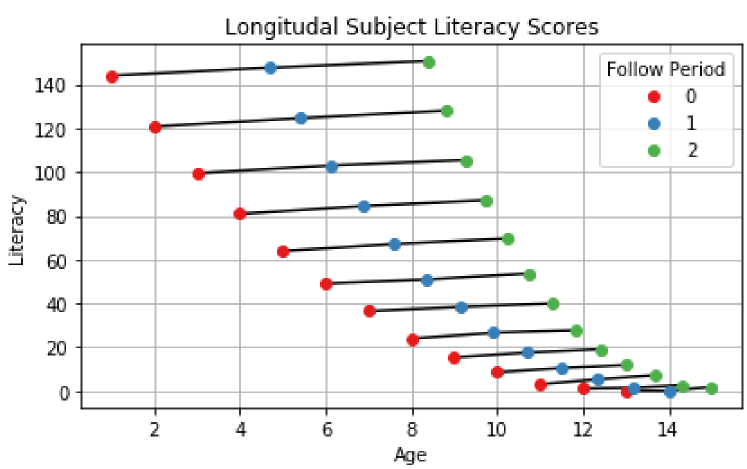
In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
from scipy import linalg
from scipy import stats
```

In [2]:

```
computer_data = pd.read_csv('./computer-data-hw1.csv')

fig = plt.figure(figsize=(6, 3.75))
ax = fig.gca()
ax.grid(True)

for i, subject in enumerate(computer_data['subj'].unique()):
    sub_data = computer_data[computer_data['subj'] == subject]
    ax.plot(sub_data['total.age'], sub_data['literacy'].values, '-k')

for i, follow_per in enumerate(computer_data['follow.per'].unique()):
    sub_data = computer_data[computer_data['follow.per'] == follow_per]
    ax.plot(sub_data['total.age'], sub_data['literacy'],
            'o', color=plt.cm.Set1(i), label='{}'.format(follow_per))

ax.set_xlabel('Age')
ax.set_ylabel('Literacy')
ax.set_title('Longitudal Subject Literacy Scores')
ax.legend(title='Follow Period')
fig.tight_layout()
fig.savefig('p3_data.pdf', bbox_inches='tight')

computer_data.head(n=10)
computer_data[computer_data['follow.per'] == 1].describe()
```

Out[2]:

	subj	base.age	delta.age	total.age	literacy	follow.per
count	13.00000	13.00000	13.000000	13.000000	13.000000	13.0
mean	7.00000	7.00000	2.218935	9.218935	52.103420	1.0
std	3.89444	3.89444	0.879002	3.019735	49.541677	0.0
min	1.00000	1.00000	1.000000	4.698225	0.120457	1.0
25%	4.00000	4.00000	1.514793	6.863905	10.463912	1.0
50%	7.00000	7.00000	2.136095	9.136095	38.377584	1.0
75%	10.00000	10.00000	2.863905	11.514793	84.414795	1.0
max	13.00000	13.00000	3.698225	14.000000	147.606099	1.0



In [3]:

```
def fit_ols(X, y):
    gram_matrix = X.T.dot(X)
    gram_matrix_inv = linalg.cho_solve(linalg.cho_factor(gram_matrix), np.eye(len(
gram_matrix)))
    beta_hat = gram_matrix_inv.dot(X.T).dot(y)
    sigma_2_hat = np.sum(np.square(y - X.dot(beta_hat)))/(len(y) - len(beta_hat))
    return beta_hat, gram_matrix_inv*sigma_2_hat, sigma_2_hat
```

In [4]:

```
def fit_original(data):
    X = np.column_stack((np.ones(len(data)), data[['base.age', 'delta.age']].value
s))
    y = data['literacy'].values
    return fit_ols(X, y)

fit_original(computer_data)
```

Out[4]:

```
(array([133.02690855, -11.9680251 ,   1.24721578]),
 array([[35.38965932, -3.08971504, -4.28560487],
        [-3.08971504,  0.36085295,  0.25406082],
        [-4.28560487,  0.25406082,  1.12990206]]),
 165.83485669258215)
```

In [5]:

```
def fit_alternative(data):
    sub_data = data[data['follow.per'] == 0][['subj', 'literacy']]
    sub_data = pd.merge(data, sub_data.rename(columns={'literacy': 'base.literacy'
}))
    sub_data = sub_data[sub_data['follow.per'] != 0]
    X = sub_data[['delta.age']].values
    y = sub_data['literacy'].values - sub_data['base.literacy'].values
    return fit_ols(X, y)

fit_alternative(computer_data)
```

Out[5]:

```
(array([1.02185154]), array([[0.00105099]]), 0.3850802073163594)
```

Simulations

In [6]:
```python
def make_covariates(n):
    X = []
    for i in range(n):
        base_age = stats.uniform.rvs(0, 15)
        X.append([i, base_age, 0, 0])

        for j in range(2):
            delta_age = stats.norm.rvs(3 + j*3, scale=0.5)
            X.append([i, base_age, delta_age, j + 1])

    return pd.DataFrame(X, columns=['subj', 'base.age', 'delta.age', 'follow.per'
])

def make_independent_covariates(n):
    subj = {}
    X = []
    for i in range(n):
        base_age = stats.uniform.rvs(0, 15)
        X.append([i, base_age, 0, 0])
        subj[i] = {'follow_per': 0, 'base_age': base_age}

    for j in range(n*2):
        i = stats.randint.rvs(low=0, high=n)
        subj[i]['follow_per'] += 1
        X.append([i, subj[i]['base_age'], stats.uniform.rvs(0, 10), subj[i]['follo
w_per']])

    return pd.DataFrame(X, columns=['subj', 'base.age', 'delta.age', 'follow.per'
])

def make_response(data, f):
    data = data.copy()
    data['literacy'] = stats.norm.rvs(f(data['base.age']) + 2*data['delta.age'], s
cale=1)
    return data
```

In [7]:
```python
np.random.seed(2020)
simulated_data = make_covariates(64)
```

Both Assumptions Violated

In [8]:
```python
beta_hat_original_estimates = []
beta_hat_alternative_estimates = []

for i in range(100):
    response_data = make_response(simulated_data, lambda x: 200*np.exp(-x))
    beta_hat, beta_hat_variance, sigma_2_hat = fit_original(response_data)
    beta_hat_original_estimates.append(beta_hat)

    beta_hat, beta_hat_variance, sigma_2_hat = fit_alternative(response_data)
    beta_hat_alternative_estimates.append(beta_hat)


np.mean(beta_hat_original_estimates, 0), np.mean(beta_hat_alternative_estimates, 0
)
```

Out[8]:
```
(array([31.96619861, -3.20229314,  2.16353098]), array([2.00158068]))
```

Linear f but Dependent Covariates

Only the Part (b) condition is now violated.

In [9]:
```
beta_hat_original_estimates = []
beta_hat_alternative_estimates = []

for i in range(100):
    response_data = make_response(simulated_data, lambda x: 200 - 12*x)
    beta_hat, beta_hat_variance, sigma_2_hat = fit_original(response_data)
    beta_hat_original_estimates.append(beta_hat)

    beta_hat, beta_hat_variance, sigma_2_hat = fit_alternative(response_data)
    beta_hat_alternative_estimates.append(beta_hat)


np.mean(beta_hat_original_estimates, 0), np.mean(beta_hat_alternative_estimates, 0
)
```

Out[9]:
```
(array([200.00853446, -11.9993114 ,   1.99779186]), array([1.99509547]))
```

Non-linear f but Independent Covariates

Only the Part (a) condition is violated now.

In [10]:
```
beta_hat_original_estimates = []
beta_hat_alternative_estimates = []

np.random.seed(2019)
for i in range(1024):
    simulated_data = make_independent_covariates(64)
    response_data = make_response(simulated_data, lambda x: 200*np.exp(-x))
    beta_hat, beta_hat_variance, sigma_2_hat = fit_original(response_data)
    beta_hat_original_estimates.append(beta_hat)

    beta_hat, beta_hat_variance, sigma_2_hat = fit_alternative(response_data)
    beta_hat_alternative_estimates.append(beta_hat)

np.mean(beta_hat_original_estimates, 0), np.mean(beta_hat_alternative_estimates, 0
)
```

Out[10]:
```
(array([47.92338712, -4.63105486,  1.99852915]), array([2.00052851]))
```

Standard Error Simulations

In [11]:

```
beta_hat_estimates = []
beta_hat_l_variances = []
is_covered = []
is_covered_sandwich = []

np.random.seed(2019)
for i in range(2048):
    simulated_data = make_independent_covariates(64)
    response_data = make_response(simulated_data, lambda x: 200*np.exp(-x))
    beta_hat, beta_hat_variance, sigma_2_hat = fit_original(response_data)

    X = np.column_stack((np.ones(len(response_data)), response_data[['base.age',
'delta.age']].values))
    y = response_data['literacy']

    gram_inverse = beta_hat_variance/sigma_2_hat
    sandwich_variance = X.T.dot(np.diag(np.square(y - X.dot(beta_hat)))).dot(X)
    sandwich_variance = gram_inverse.dot(sandwich_variance).dot(gram_inverse)

    beta_hat_estimates.append(beta_hat)
    beta_hat_l_variances.append(beta_hat_variance[2,2])
    is_covered.append(np.abs(beta_hat[2] - 2) <= stats.norm.ppf(0.975)*np.sqrt(bet
a_hat_variance[2,2]))
    is_covered_sandwich.append(np.abs(beta_hat[2] - 2) <= stats.norm.ppf(0.975)*np
.sqrt(sandwich_variance[2,2]))

np.sum(is_covered)/len(is_covered), np.sum(is_covered_sandwich)/len(is_covered_san
dwich)
```

Out[11]:

```
(0.98095703125, 0.9775390625)
```