ElementSets from MeshConnectivity

NeighborVerticesInFace, Faces, VertexOneRing from Neighborhoods(M)

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\begin{aligned}
M &: \text{TriangleMesh} \\
x_i &\in \mathbb{R}^3 \\
\\
V, E, F &= \textit{ElementSets}(M) \\
\textit{VertexNormal}(i) &= \frac{w}{\|w\|} \\
&\text{where} \\
&\quad i \in V \\
&\quad w = \sum_{f \in \textit{Faces}(i)} \left(x_j - x_i\right) \times \left(x_k - x_i\right) \qquad \text{where } j, k = \textit{NeighborVerticesInFace}(f, i) \\
\textit{CalcNorm}(i, v, n, \sigma_c, \sigma_s) &= w_c \cdot w_s \\
&\text{where} \\
&\quad i, v \in \mathbb{Z}, \text{ vertices} \\
&\quad \sigma_c, \sigma_s \in \mathbb{R} \\
&\quad n \in \mathbb{R}^3 \\
&\quad t = \|x_i - x_v\| \\
&\quad h = \langle n, x_v - x_i \rangle \\
&\quad w_c = e^{-\frac{t^2}{2{\sigma_c}^2}} \\
&\quad w_s = e^{-\frac{h^2}{2{\sigma_s}^2}} \\
\textit{CalcS}(i, v, n, \sigma_c, \sigma_s) &= \textit{CalcNorm}(i, v, n, \sigma_c, \sigma_s) \cdot h \\
&\text{where} \\
&\quad i, v \in \mathbb{Z}, \text{ vertices} \\
&\quad \sigma_c, \sigma_s \in \mathbb{R} \\
&\quad n \in \mathbb{R}^3 \\
&\quad h = \langle n, x_v - x_i \rangle \\
\textit{DenoisePoint}(i) &= x_i + n \cdot \left(\frac{s}{\textit{norm}}\right) \\
&\text{where} \\
&\quad i \in V \\
&\quad n = \textit{VertexNormal}(i) \\
&\quad \sigma_c = \textit{CalcSigmaC}(i) \\
&\quad \textit{neighbors} = \textit{AdaptiveVertexNeighbor}(i, \{i\}, \sigma_c) \\
&\quad \sigma_s = \textit{CalcSigmaS}(i, \textit{neighbors}) \\
&\quad s = \sum_{v \in \textit{neighbors}} \textit{CalcS}(i, v, n, \sigma_c, \sigma_s) \\
&\quad \textit{norm} = \sum_{v \in \textit{neighbors}} \textit{CalcNorm}(i, v, n, \sigma_c, \sigma_s) \\
\textit{CalcSigmaC}(i) &= \min(\{\|x_i - x_v\| \mid v \in \textit{VertexOneRing}(i)\}) \text{ where } i \in V \\
\textit{CalcSigmaS}(i, N) &= \begin{cases} \sqrt{\textit{offset}} + 10^{-12} & \text{if } \sqrt{\textit{offset}} < 10^{-12} \\ \sqrt{\textit{offset}} & \text{otherwise} \end{cases} \\
&\text{where} \\
&\quad i \in V \\
&\quad N \subset V \\
&\quad n = \textit{VertexNormal}(i) \\
&\quad \textit{avg} = \sum_{v \in N} \frac{t}{|N|} \qquad \text{where } t = \sqrt{\left((x_v - x_i) \cdot n\right)^2} \\
&\quad \textit{sqs} = \sum_{v \in N} \left(t - \textit{avg}\right)^2 \qquad \text{where } t = \sqrt{\left((x_v - x_i) \cdot n\right)^2} \\
&\quad \textit{offset} = \frac{\textit{sqs}}{|N|} \\
\textit{AdaptiveVertexNeighbor}(i, n, \sigma) &= \begin{cases} n & \text{if } |n| = |\textit{target}| \\ \textit{AdaptiveVertexNeighbor}(i, \textit{target}, \sigma) & \text{otherwise} \end{cases} \\
&\text{where} \\
&\quad i \in V \\
&\quad \sigma \in \mathbb{R} \\
&\quad n \subset V \\
&\quad \textit{target} = \{v \mid v \in \textit{VertexOneRing}(n), \|x_i - x_v\| < 2\sigma\}
\end{aligned}
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