vec, inversevec, diag, svd from linearalgebra

ElementSets from MeshConnectivity

NeighborVerticesInFace, Faces, Vertices, VertexOneRing, OrientedVertices from TetrahderonNeighborhoods(M)

$$
\begin{aligned}
&M: \text{ CellMesh}\\
&\bar{x}_i \in \mathbb{R}^3 \text{ rest pos}\\
&x_i \in \mathbb{R}^3 \text{ current pos}\\
&bx_j \in \mathbb{Z}, \text{ index boundary indices}\\
&bp_j \in \mathbb{R}^3 \text{ boundary positions}\\
&w \in \mathbb{R} \text{ penalty}\\
&\varepsilon \in \mathbb{R} \text{ eps}\\
&psd : \mathbb{R}^{p\times p} \to \mathbb{R}^{p\times p}, \text{ sparse}
\end{aligned}
$$

$$
\begin{aligned}
V, E, F, C &= ElementSets(M)\\
vol_{i,j,k,l} &= \tfrac{1}{6}\left|\begin{bmatrix}\bar{x}_j-\bar{x}_i & \bar{x}_k-\bar{x}_i & \bar{x}_l-\bar{x}_i\end{bmatrix}\right| \text{ where } i,j,k,l \in V\\
m_r(s) &= \begin{bmatrix}\bar{x}_b-\bar{x}_a & \bar{x}_c-\bar{x}_a & \bar{x}_d-\bar{x}_a\end{bmatrix}\\
&\text{where}\\
&\quad s \in C\\
&\quad a,b,c,d = OrientedVertices(s)\\
S(s,x) &= \begin{cases}\infty & \text{if } |m| \le 0\\ vol_{a,b,c,d}\left(\|J\|^2 + \left\|J^{-1}\right\|^2\right) & \text{otherwise}\end{cases}\\
&\text{where}\\
&\quad s \in C\\
&\quad x_i \in \mathbb{R}^3\\
&\quad a,b,c,d = OrientedVertices(s)\\
&\quad m = \begin{bmatrix}x_b-x_a & x_c-x_a & x_d-x_a\end{bmatrix}\\
&\quad J = mm_r(s)^{-1}\\
EXPS(s,x) &= \begin{cases}\infty & \text{if } |m| \le 0\\ vol_{a,b,c,d}e^{\|J\|^2+\left\|J^{-1}\right\|^2} & \text{otherwise}\end{cases}\\
&\text{where}\\
&\quad s \in C\\
&\quad x_i \in \mathbb{R}^3\\
&\quad a,b,c,d = OrientedVertices(s)\\
&\quad m = \begin{bmatrix}x_b-x_a & x_c-x_a & x_d-x_a\end{bmatrix}\\
&\quad J = mm_r(s)^{-1}\\
AMIPS(s,x) &= \begin{cases}\infty & \text{if } |m| \le 0\\ vol_{a,b,c,d}e^{\frac{1}{2}\left(\frac{\|J\|^2}{|J|}+\frac{1}{2}\left(|J|+|J^{-1}|\right)\right)} & \text{otherwise}\end{cases}\\
&\text{where}\\
&\quad s \in C\\
&\quad x_i \in \mathbb{R}^3\\
&\quad a,b,c,d = OrientedVertices(s)\\
&\quad m = \begin{bmatrix}x_b-x_a & x_c-x_a & x_d-x_a\end{bmatrix}\\
&\quad J = mm_r(s)^{-1}\\
CAMIPS(s,x) &= \begin{cases}\infty & \text{if } |m| \le 0\\ vol_{a,b,c,d}\left(\frac{\|J\|^2}{|J|^{\frac{2}{3}}}\right) & \text{otherwise}\end{cases}\\
&\text{where}\\
&\quad s \in C\\
&\quad x_i \in \mathbb{R}^3\\
&\quad a,b,c,d = OrientedVertices(s)\\
&\quad m = \begin{bmatrix}x_b-x_a & x_c-x_a & x_d-x_a\end{bmatrix}\\
&\quad J = mm_r(s)^{-1}\\
E2 &= w\sum_j \left\|bp_j - x_{bx_j}\right\|^2\\
e &= \sum_{i\in C} S_i(x) + E2\\
G &= \frac{\partial e}{\partial x}\\
H &= \sum_{i\in C} psd\left(\frac{\partial^2 S_i(x)}{\partial x^2}\right) + psd\left(\frac{\partial^2 E2}{\partial x^2}\right)
\end{aligned}
$$