vec, inversevec, diag, svd from linearalgebra

ElementSets from MeshConnectivity

NeighborVerticesInFace, Faces, VertexOneRing, OrientedVertices from Neighborhoods(M)

$$
\begin{aligned}
M &: \text{TriangleMesh} \\
\bar{x}_i &\in \mathbb{R}^3 \text{ rest pos in 3D} \\
x_i &\in \mathbb{R}^2 \text{ current pos in 2D} \\
\varepsilon &\in \mathbb{R} \text{ eps} \\
\mathit{psd} &: \mathbb{R}^{p\times p} \to \mathbb{R}^{p\times p}, \text{ sparse}
\end{aligned}
$$

$$
\begin{aligned}
V, E, F &= \mathit{ElementSets}(M) \\
m_r(f) &= \begin{bmatrix} \mathit{br} - \mathit{ar} & \mathit{cr} - \mathit{ar} \end{bmatrix} \\
&\quad \text{where} \\
&\quad\quad f \in F \\
&\quad\quad a, b, c = \mathit{OrientedVertices}(f) \\
&\quad\quad n = (\bar{x}_b - \bar{x}_a) \times (\bar{x}_c - \bar{x}_a) \\
&\quad\quad \mathit{b1} = \frac{\bar{x}_b - \bar{x}_a}{\|\bar{x}_b - \bar{x}_a\|} \\
&\quad\quad \mathit{b2} = \frac{n \times \mathit{b1}}{\|n \times \mathit{b1}\|} \\
&\quad\quad \mathit{ar} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\
&\quad\quad \mathit{br} = \begin{pmatrix} (\bar{x}_b - \bar{x}_a) \cdot \mathit{b1} \\ 0 \end{pmatrix} \\
&\quad\quad \mathit{cr} = \begin{pmatrix} (\bar{x}_c - \bar{x}_a) \cdot \mathit{b1} \\ (\bar{x}_c - \bar{x}_a) \cdot \mathit{b2} \end{pmatrix} \\
S(f, x) &= \begin{cases} \infty & \text{if } |m| \le 0 \\ A\left(\|J\|^2 + \|J^{-1}\|^2\right) & \text{otherwise} \end{cases} \\
&\quad \text{where} \\
&\quad\quad f \in F \\
&\quad\quad x_i \in \mathbb{R}^2 \\
&\quad\quad a, b, c = \mathit{OrientedVertices}(f) \\
&\quad\quad m = \begin{bmatrix} x_b - x_a & x_c - x_a \end{bmatrix} \\
&\quad\quad A = \tfrac{1}{2}|m_r(f)| \\
&\quad\quad J = m m_r(f)^{-1} \\
e &= \sum_{i \in F} S(i, x) \\
H &= \sum_{i \in F} \mathit{psd}\left(\frac{\partial^2 S(i, x)}{\partial x^2}\right) \\
G &= \frac{\partial e}{\partial x}
\end{aligned}
$$