

vec, inversevec, diag, svd from linearalgebra

ElementSets from MeshConnectivity

NeighborVerticesInFace, Faces, VertexOneRing, OrientedVertices from Neighborhoods(M)

M : TriangleMesh

$\bar{x}_i \in \mathbb{R}^3$ rest pos in 3D

$x_i \in \mathbb{R}^2$ current pos in 2D

$\varepsilon \in \mathbb{R}$ eps

$psd : \mathbb{R}^{p \times p} \rightarrow \mathbb{R}^{p \times p}$, sparse

$V, E, F = ElementSets(M)$

$mr(f) = \begin{bmatrix} br - ar & cr - ar \end{bmatrix}$

where

$f \in F$

$a, b, c = OrientedVertices(f)$

$n = (\bar{x}_b - \bar{x}_a) \times (\bar{x}_c - \bar{x}_a)$

$b1 = \frac{\bar{x}_b - \bar{x}_a}{\|\bar{x}_b - \bar{x}_a\|}$

$b2 = \frac{n \times b1}{\|n \times b1\|}$

$ar = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$br = \begin{pmatrix} (\bar{x}_b - \bar{x}_a) \cdot b1 \\ 0 \end{pmatrix}$

$cr = \begin{pmatrix} (\bar{x}_c - \bar{x}_a) \cdot b1 \\ (\bar{x}_c - \bar{x}_a) \cdot b2 \end{pmatrix}$

$S(f, x) = \begin{cases} \infty & \text{if } |m| \leq 0 \\ A \left(\|\mathcal{J}\|^2 + \|\mathcal{J}^{-1}\|^2 \right) & \text{otherwise} \end{cases}$

where

$f \in F$

$x_i \in \mathbb{R}^2$

$a, b, c = OrientedVertices(f)$

$m = \begin{bmatrix} x_b - x_a & x_c - x_a \end{bmatrix}$

$A = \frac{1}{2} |mr(f)|$

$\mathcal{J} = m \; mr(f)^{-1}$

$e = \sum_{i \in F} S(i, x)$

$H = \sum_{i \in F} psd \left(\frac{\partial^2 S(i, x)}{\partial x^2} \right)$

$G = \frac{\partial e}{\partial x}$