vec, inversevec, diag, svd from linearalgebra

ElementSets from MeshConnectivity

NeighborVerticesInFace, Faces, Vertices, VertexOneRing, OrientedVertices from TetrahderonNeighborhoods(M)

$$
\begin{aligned}
M &: \text{ TetrahedralMesh} \\
\bar{x}_i &\in \mathbb{R}^3 \text{ rest pos} \\
x_i &\in \mathbb{R}^3 \text{ current pos} \\
bx_j &\in \mathbb{Z}, \text{ index boundary indices} \\
bp_j &\in \mathbb{R}^3 \text{ boundary positions} \\
w &\in \mathbb{R} \text{ penalty} \\
\varepsilon &\in \mathbb{R} \text{ eps}
\end{aligned}
$$

$$
\begin{aligned}
V, E, F, C &= \mathit{ElementSets}(M) \\
vol_{i,j,k,l} &= \tfrac{1}{6} \left| \begin{bmatrix} \bar{x}_j - \bar{x}_i & \bar{x}_k - \bar{x}_i & \bar{x}_l - \bar{x}_i \end{bmatrix} \right| \text{ where } i, j, k, l \in V \\
psd(x) &= u \ \ \mathrm{diag}(ps) \ \ v^T \\
&\quad \text{where} \\
&\qquad x \in \mathbb{R}^{p \times p} \\
&\quad u, sigma, v = \mathrm{svd}(x) \\
&\qquad ps_i = \begin{cases} sigma_i & \text{if } sigma_i > 0 \\ 0 & \text{otherwise} \end{cases} \\
S(s, x) &= vol_{a,b,c,d} \ \left(\|\mathcal{J}\|^2 + \left\|\mathcal{J}^{-1}\right\|^2 \right) \\
&\quad \text{where} \\
&\qquad s \in C \\
&\qquad x_i \in \mathbb{R}^3 \\
&\quad a, b, c, d = \mathit{Vertices}(s) \\
&\qquad \mathcal{J} = \begin{bmatrix} x_b - x_a & x_c - x_a & x_d - x_a \end{bmatrix} \ \begin{bmatrix} \bar{x}_b - \bar{x}_a & \bar{x}_c - \bar{x}_a & \bar{x}_d - \bar{x}_a \end{bmatrix}^{-1} \\
E2 &= w \ \ \sum_j \left\| bp_j - x_{bx_j} \right\| \\
e &= \sum_{i \in C} S(i, x) + E2 \\
G &= \frac{\partial e}{\partial x} \\
H &= \sum_{i \in C} psd\left(\frac{\partial^2 S(i, x)}{\partial x^2} \right) + psd\left(\frac{\partial^2 E2}{\partial x^2} \right) \\
d &= H^{-1}(-G) \\
y &= \begin{cases} \mathrm{vec}^{-1}{}_x(\mathrm{vec}(x) + 0.1 \ \ d) & \text{if } \sqrt{-d \cdot G} > \varepsilon \\ x & \text{otherwise} \end{cases}
\end{aligned}
$$