vec, inversevec, diag, svd from linearalgebra

ElementSets from MeshConnectivity

NeighborVerticesInFace, Faces, Vertices, VertexOneRing, OrientedVertices from TetrahderonNeighborhoods(M)

$$
\begin{aligned}
M &: \text{ TetrahedralMesh} \\
\bar{x}_i &\in \mathbb{R}^3 \text{ rest pos} \\
x_i &\in \mathbb{R}^3 \text{ current pos} \\
\mathit{bx}_j &\in \mathbb{Z},\ \text{index boundary indices} \\
\mathit{bp}_j &\in \mathbb{R}^3 \text{ boundary positions} \\
w &\in \mathbb{R} \text{ penalty} \\
\varepsilon &\in \mathbb{R} \text{ eps} \\
\mathit{psd} &: \mathbb{R}^{p\times p} \to \mathbb{R}^{p\times p},\ \text{sparse}
\end{aligned}
$$

$$
\begin{aligned}
V, E, F, C &= \mathit{ElementSets}(M) \\
\mathit{vol}_{i,j,k,l} &= \tfrac{1}{6}\left|\begin{bmatrix}\bar{x}_j - \bar{x}_i & \bar{x}_k - \bar{x}_i & \bar{x}_l - \bar{x}_i\end{bmatrix}\right| \text{ where } i,j,k,l \in V \\
m_r(s) &= \begin{bmatrix}\bar{x}_b - \bar{x}_a & \bar{x}_c - \bar{x}_a & \bar{x}_d - \bar{x}_a\end{bmatrix} \\
&\quad \text{where} \\
&\quad\quad s \in C \\
&\quad a,b,c,d = \mathit{OrientedVertices}(s) \\
S(s,x) &= \begin{cases}\infty & \text{if } |m| \le 0 \\ \mathit{vol}_{a,b,c,d}\left(\left\|J\right\|^2 + \left\|J^{-1}\right\|^2\right) & \text{otherwise}\end{cases} \\
&\quad \text{where} \\
&\quad\quad s \in C \\
&\quad\quad x_i \in \mathbb{R}^3 \\
&\quad a,b,c,d = \mathit{OrientedVertices}(s) \\
&\quad\quad m = \begin{bmatrix}x_b - x_a & x_c - x_a & x_d - x_a\end{bmatrix} \\
&\quad\quad J = m m_r(s)^{-1} \\
\mathit{EXPS}(s,x) &= \begin{cases}\infty & \text{if } |m| \le 0 \\ \mathit{vol}_{a,b,c,d}e^{\left\|J\right\|^2 + \left\|J^{-1}\right\|^2} & \text{otherwise}\end{cases} \\
&\quad \text{where} \\
&\quad\quad s \in C \\
&\quad\quad x_i \in \mathbb{R}^3 \\
&\quad a,b,c,d = \mathit{OrientedVertices}(s) \\
&\quad\quad m = \begin{bmatrix}x_b - x_a & x_c - x_a & x_d - x_a\end{bmatrix} \\
&\quad\quad J = m m_r(s)^{-1} \\
\mathit{AMIPS}(s,x) &= \begin{cases}\infty & \text{if } |m| \le 0 \\ \mathit{vol}_{a,b,c,d}e^{\frac{1}{2}\left(\frac{\|J\|^2}{|J|} + \frac{1}{2}\left(|J| + |J^{-1}|\right)\right)} & \text{otherwise}\end{cases} \\
&\quad \text{where} \\
&\quad\quad s \in C \\
&\quad\quad x_i \in \mathbb{R}^3 \\
&\quad a,b,c,d = \mathit{OrientedVertices}(s) \\
&\quad\quad m = \begin{bmatrix}x_b - x_a & x_c - x_a & x_d - x_a\end{bmatrix} \\
&\quad\quad J = m m_r(s)^{-1} \\
\mathit{CAMIPS}(s,x) &= \begin{cases}\infty & \text{if } |m| \le 0 \\ \mathit{vol}_{a,b,c,d}\left(\frac{\left\|J\right\|^2}{|J|^{\frac{2}{3}}}\right) & \text{otherwise}\end{cases} \\
&\quad \text{where} \\
&\quad\quad s \in C \\
&\quad\quad x_i \in \mathbb{R}^3 \\
&\quad a,b,c,d = \mathit{OrientedVertices}(s) \\
&\quad\quad m = \begin{bmatrix}x_b - x_a & x_c - x_a & x_d - x_a\end{bmatrix} \\
&\quad\quad J = m m_r(s)^{-1} \\
\mathit{E2} &= w\sum_j \left\|\mathit{bp}_j - x_{\mathit{bx}_j}\right\|^2 \\
e &= \sum_{i \in C} S_i(x) + \mathit{E2} \\
G &= \frac{\partial e}{\partial x} \\
H &= \sum_{i \in C} \mathit{psd}\left(\frac{\partial^2 S_i(x)}{\partial x^2}\right) + \mathit{psd}\left(\frac{\partial^2 \mathit{E2}}{\partial x^2}\right)
\end{aligned}
$$