vec, inversevec, diag, svd from linearalgebra

ElementSets from MeshConnectivity

NeighborVerticesInFace, Faces, Vertices, VertexOneRing, OrientedVertices from TetrahderonNeighborhoods(M)

$$
\begin{aligned}
M &: \text{ TetrahedralMesh} \\
\bar{x}_i &\in \mathbb{R}^3 \text{ rest pos} \\
x_i &\in \mathbb{R}^3 \text{ current pos} \\
bx_j &\in \mathbb{Z}, \text{ index boundary indices} \\
bp_j &\in \mathbb{R}^3 \text{ boundary positions} \\
w &\in \mathbb{R} \text{ penalty} \\
\varepsilon &\in \mathbb{R} \text{ eps} \\
psd &: \mathbb{R}^{p \times p} \to \mathbb{R}^{p \times p}, \text{ sparse}
\end{aligned}
$$

$$
\begin{aligned}
V, E, F, C &= ElementSets(M) \\
vol_{i,j,k,l} &= \tfrac{1}{6} \left| \begin{bmatrix} \bar{x}_j - \bar{x}_i & \bar{x}_k - \bar{x}_i & \bar{x}_l - \bar{x}_i \end{bmatrix} \right| \text{ where } i, j, k, l \in V \\
m_r(s) &= \begin{bmatrix} \bar{x}_b - \bar{x}_a & \bar{x}_c - \bar{x}_a & \bar{x}_d - \bar{x}_a \end{bmatrix} \\
&\quad \text{where} \\
&\qquad s \in C \\
&\quad a, b, c, d = OrientedVertices(s) \\
S(s, x) &= \begin{cases} \infty & \text{if } |m| \leq 0 \\ vol_{a,b,c,d} \left(\|\mathcal{J}\|^2 + \|\mathcal{J}^{-1}\|^2 \right) & \text{otherwise} \end{cases} \\
&\quad \text{where} \\
&\qquad s \in C \\
&\qquad x_i \in \mathbb{R}^3 \\
&\quad a, b, c, d = OrientedVertices(s) \\
&\qquad m = \begin{bmatrix} x_b - x_a & x_c - x_a & x_d - x_a \end{bmatrix} \\
&\qquad \mathcal{J} = m \; m_r(s)^{-1} \\
EXPS(s, x) &= \begin{cases} \infty & \text{if } |m| \leq 0 \\ vol_{a,b,c,d} \; e^{\|\mathcal{J}\|^2 + \|\mathcal{J}^{-1}\|^2} & \text{otherwise} \end{cases} \\
&\quad \text{where} \\
&\qquad s \in C \\
&\qquad x_i \in \mathbb{R}^3 \\
&\quad a, b, c, d = OrientedVertices(s) \\
&\qquad m = \begin{bmatrix} x_b - x_a & x_c - x_a & x_d - x_a \end{bmatrix} \\
&\qquad \mathcal{J} = m \; m_r(s)^{-1} \\
AMIPS(s, x) &= \begin{cases} \infty & \text{if } |m| \leq 0 \\ vol_{a,b,c,d} \; e^{\frac{1}{2} \left(\frac{\|\mathcal{J}\|^2}{|\mathcal{J}|} + \frac{1}{2} \left(|\mathcal{J}| + |\mathcal{J}^{-1}| \right) \right)} & \text{otherwise} \end{cases} \\
&\quad \text{where} \\
&\qquad s \in C \\
&\qquad x_i \in \mathbb{R}^3 \\
&\quad a, b, c, d = OrientedVertices(s) \\
&\qquad m = \begin{bmatrix} x_b - x_a & x_c - x_a & x_d - x_a \end{bmatrix} \\
&\qquad \mathcal{J} = m \; m_r(s)^{-1} \\
CAMIPS(s, x) &= \begin{cases} \infty & \text{if } |m| \leq 0 \\ vol_{a,b,c,d} \left(\frac{\|\mathcal{J}\|^2}{|\mathcal{J}|^{\frac{2}{3}}} \right) & \text{otherwise} \end{cases} \\
&\quad \text{where} \\
&\qquad s \in C \\
&\qquad x_i \in \mathbb{R}^3 \\
&\quad a, b, c, d = OrientedVertices(s) \\
&\qquad m = \begin{bmatrix} x_b - x_a & x_c - x_a & x_d - x_a \end{bmatrix} \\
&\qquad \mathcal{J} = m \; m_r(s)^{-1} \\
E2 &= w \sum_j \left\| bp_j - x_{bx_j} \right\|^2 \\
e &= \sum_{i \in C} S(i, x) + E2 \\
G &= \frac{\partial e}{\partial x} \\
H &= \sum_{i \in C} psd\left(\frac{\partial^2 S(i, x)}{\partial x^2} \right) + psd\left(\frac{\partial^2 E2}{\partial x^2} \right)
\end{aligned}
$$