arccos, atan2 from trigonometry

ElementSets from MeshConnectivity

Faces, VertexOneRing, OrientedOppositeFaces, OppositeVertices, NeighborVerticesInFace, OrientedVertices from Neighborhoods(M)

$$
\begin{aligned}
& M : \text{TriangleMesh} \\
& x_i \in \mathbb{R}^3
\end{aligned}
$$

$$
\begin{aligned}
V, E, F &= \textit{ElementSets}(M) \\
\textit{VertexNormal}(i) &= \left(\sum_{f \in \textit{Faces}(i)} \frac{(x_j - x_i) \times (x_k - x_i)}{\|x_j - x_i\|^2 \|x_k - x_i\|^2} \qquad \text{where } j, k = \textit{NeighborVerticesInFace}(f, i) \right) \text{ where } i \in V \\
\theta(i, f) &= arccos\left(\frac{(x_j - x_i) \cdot (x_k - x_i)}{\|x_j - x_i\| \|x_k - x_i\|} \right) \\
& \quad \text{where} \\
& \qquad i \in V \\
& \qquad f \in F \\
& \qquad j, k = \textit{NeighborVerticesInFace}(f, i) \\
\textit{area}(f) &= \tfrac{1}{2} \left\| (x_j - x_i) \times (x_k - x_i) \right\| \\
& \quad \text{where} \\
& \qquad f \in F \\
& \qquad i, j, k = \textit{OrientedVertices}(f) \\
N(f) &= \frac{(x_j - x_i) \times (x_k - x_i)}{2\textit{area}(f)} \\
& \quad \text{where} \\
& \qquad f \in F \\
& \qquad i, j, k = \textit{OrientedVertices}(f) \\
l(i, j) &= \|x_j - x_i\| \text{ where } i, j \in V \\
\phi(i, j) &= atan2(e \cdot (n_1 \times n_2), n_1 \cdot n_2) \\
& \quad \text{where} \\
& \qquad i, j \in V \\
& \qquad e = \frac{x_j - x_i}{\|x_j - x_i\|} \\
& \qquad f_1, f_2 = \textit{OrientedOppositeFaces}(i, j) \\
& \qquad n_1 = N(f_1) \\
& \qquad n_2 = N(f_2) \\
\textit{cot}(k, j, i) &= \frac{\textit{cos}}{\textit{sin}} \\
& \quad \text{where} \\
& \qquad i, j, k \in V \\
& \qquad \textit{oj}, \textit{oi} = \textit{OrientedVertices}(k, j, i) \\
& \qquad \textit{cos} = (x_{\textit{oj}} - x_k) \cdot (x_{\textit{oi}} - x_k) \\
& \qquad \textit{sin} = \left\| (x_{\textit{oj}} - x_k) \times (x_{\textit{oi}} - x_k) \right\| \\
\textit{KN}(i) &= \frac{1}{2} \left(\sum_{j \in \textit{VertexOneRing}(i)} \frac{\phi_{i,j}}{l_{i,j}} (x_j - x_i) \right) \text{ where } i \in V \\
\textit{HN}(i) &= \frac{1}{2} \left(\sum_{j \in \textit{VertexOneRing}(i)} (cot(\alpha) + cot(\beta))(x_i - x_j) \qquad \text{where } k, p = \textit{OppositeVertices}(i, j), cot(\alpha) = \textit{cot}(k, j, i), cot(\beta) = \textit{cot}(p, i, j) \right) \text{ where } i \in V
\end{aligned}
$$