arccos, atan2 from trigonometry

ElementSets from MeshConnectivity

Faces, Vertices, Edges, OppositeFaces, VertexOneRing, OppositeVertices, NeighborVerticesInFace, OrientedVertices, EdgeIndex from Neighborhoods(M)

$$M: \text{TriangleMesh}$$
$$x_i \in \mathbb{R}^3$$

$$V, E, F = \mathit{ElementSets}(M)$$

$$\mathit{VertexNormal}(i) = \left(\sum_{f \in \mathit{Faces}(i)} \frac{(x_j - x_i) \times (x_k - x_i)}{\|x_j - x_i\|^2 \ \|x_k - x_i\|^2} \quad \text{where } j, k = \mathit{NeighborVerticesInFace}(f, i) \right) \text{ where } i \in V$$

$$\theta(i, f) = arccos\left(\frac{(x_j - x_i) \cdot (x_k - x_i)}{\|x_j - x_i\| \ \|x_k - x_i\|} \right)$$

where

$$i \in V$$
$$f \in F$$
$$j, k = \mathit{NeighborVerticesInFace}(f, i)$$

$$\mathit{area}(f) = \frac{1}{2} \ \|(x_j - x_i) \times (x_k - x_i)\|$$

where

$$f \in F$$
$$i, j, k = \mathit{OrientedVertices}(f)$$

$$N(f) = \frac{(x_j - x_i) \times (x_k - x_i)}{2 \ \ \mathit{area}(f)}$$

where

$$f \in F$$
$$i, j, k = \mathit{OrientedVertices}(f)$$

$$l(e) = \|x_j - x_i\|$$

where

$$e \in E$$
$$i, j = \mathit{Vertices}(e)$$

$$\phi(e) = \pi - atan2(\|N_1 \times N_1\|, N_1 \cdot N_2)$$

where

$$e \in E$$
$$f_1, f_2 = \mathit{OppositeFaces}(e)$$
$$N_1 = N(f_1)$$
$$N_2 = N(f_2)$$

$$\mathit{cot}(k, j, i) = \frac{\mathit{cos}}{\mathit{sin}}$$

where

$$i, j, k \in V$$
$$oj, oi = \mathit{OrientedVertices}(k, j, i)$$
$$\mathit{cos} = (x_{oj} - x_k) \cdot (x_{oi} - x_k)$$
$$\mathit{sin} = \|(x_{oj} - x_k) \times (x_{oi} - x_k)\|$$

$$KN(i) = \frac{1}{2} \left(\sum_{j \in \mathit{VertexOneRing}(i)} \frac{\phi_e}{l_e} \ (x_j - x_i) \quad \text{where } e = \mathit{EdgeIndex}(i, j) \right) \text{ where } i \in V$$

$$HN(i) = \frac{1}{2} \left(\sum_{j \in \mathit{VertexOneRing}(i)} (cot(\alpha) + cot(\beta)) \ (x_i - x_j) \quad \text{where } k, p = \mathit{OppositeVertices}(\mathit{EdgeIndex}(i, j)), cot(\alpha) = \mathit{cot}(k, j, i), cot(\beta) = \mathit{cot}(p, i, j) \right) \text{ where } i \in V$$