arccos, atan2 from trigonometry

ElementSets from MeshConnectivity

Faces, Vertices, Edges, OppositeFaces, OrientedOppositeFaces, VertexOneRing, NeighborVerticesInFace, OrientedVertices from Neighborhoods(M)

$$M: \text{TriangleMesh}$$
$$x_i \in \mathbb{R}^3$$

$$V, E, F = \mathit{ElementSets}(M)$$

$$\theta(i, f) = arccos\left(\frac{(x_j - x_i) \cdot (x_k - x_i)}{\|x_j - x_i\| \ \|x_k - x_i\|}\right)$$

where

$$i \in V$$
$$f \in F$$
$$j, k = \mathit{NeighborVerticesInFace}(f, i)$$

$$\mathit{area}(f) = \tfrac{1}{2}\ \|(x_j - x_i) \times (x_k - x_i)\|$$

where

$$f \in F$$
$$i, j, k = \mathit{OrientedVertices}(f)$$

$$A(i) = \tfrac{1}{3} \sum_{f \in \mathit{Faces}(i)} \mathit{area}(f) \text{ where } i \in V$$

$$N(f) = \frac{(x_j - x_i) \times (x_k - x_i)}{2\ \ \mathit{area}(f)}$$

where

$$f \in F$$
$$i, j, k = \mathit{OrientedVertices}(f)$$

$$K(i) = \frac{2\ \ \pi - \sum_{f \in \mathit{Faces}(i)} \theta_{i,f}}{A_i} \text{ where } i \in V$$

$$l(i, j) = \|x_j - x_i\| \text{ where } i, j \in V$$

$$\phi(i, j) = atan2(e \cdot (n_1 \times n_2), n_1 \cdot n_2)$$

where

$$i, j \in V$$
$$e = \frac{x_j - x_i}{\|x_j - x_i\|}$$
$$f_1, f_2 = \mathit{OrientedOppositeFaces}(i, j)$$
$$n_1 = N(f_1)$$
$$n_2 = N(f_2)$$

$$H(i) = \frac{1}{4} \sum_{j \in \mathit{VertexOneRing}(i)} l_{i,j}\ \ \phi_{i,j} \text{ where } i \in V$$