## arccos, atan2 from trigonometry

## ElementSets from MeshConnectivity

Faces, OrientedOppositeFaces, VertexOneRing, NeighborVerticesInFace, OrientedVertices from Neighborhoods(M)

$$\begin{aligned} &M: \ \text{TriangleMesh} \\ &x_i \in \mathbb{R}^3 \\ &V, E, F = ElementSets(M) \\ &\theta(i,f) = \arccos\left(\frac{\left(x_j - x_i\right) \cdot (x_k - x_i)}{\left\|x_j - x_i\right\| \left\|x_k - x_i\right\|}\right) \\ &\text{where} \\ &i \in V \\ &f \in F \\ &j, k = Neighbor VerticesInFace(f,i) \\ &K(i) = 2\pi - \sum_{f \in Rese(i)} \theta_{i,f} \text{ where } i \in V \\ &\text{area}(f) = \frac{1}{2}\|(x_j - x_i) \times (x_k - x_i)\| \\ &\text{where} \\ &f \in F \\ &i, j, k = Oriented Vertices(f) \\ &N(f) = \frac{\left(x_j - x_i\right) \times \left(x_k - x_i\right)}{2area(f)} \\ &\text{where} \\ &f \in F \\ &i, j, k = Oriented Vertices(f) \\ &l(i,j) = \|x_j - x_i\| \text{ where } i, j \in V \\ &\phi(i,j) = atan2(e \cdot (n_1 \times n_2), n_1 \cdot n_2) \\ &\text{where} \\ &i, j \in V \\ &e = \frac{x_j - x_i}{\|x_j - x_i\|} \\ &f_1, f_2 = Oriented Opposite Faces(i,j) \\ &n_1 = N(f_1) \\ &n_2 = N(f_2) \\ &H(i) = \frac{1}{4} \sum_{i \in VerticOne Bing(i)} l_{i,j} \phi_{i,j} \text{ where } i \in V \end{aligned}$$