vec, inversevec, diag, svd from linearalgebra

ElementSets from MeshConnectivity

NeighborVerticesInFace, Faces, VertexOneRing, OrientedVertices from Neighborhoods(M)

M : FaceMesh
$\bar{x}_i \in \mathbb{R}^3$ rest pos in 3D
$x_i \in \mathbb{R}^2$ current pos in 2D
$\varepsilon \in \mathbb{R}$ eps
$psd : \mathbb{R}^{p \times p} \to \mathbb{R}^{p \times p}$, sparse

$V, E, F = ElementSets(M)$

$m_r(f) = [br - ar \quad cr - ar]$
$\quad$ where
$\qquad f \in F$
$\qquad a, b, c = OrientedVertices(f)$
$\qquad n = (\bar{x}_b - \bar{x}_a) \times (\bar{x}_c - \bar{x}_a)$
$\qquad b1 = \dfrac{\bar{x}_b - \bar{x}_a}{\|\bar{x}_b - \bar{x}_a\|}$
$\qquad b2 = \dfrac{n \times b1}{\|n \times b1\|}$
$\qquad ar = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\qquad br = \begin{pmatrix} (\bar{x}_b - \bar{x}_a) \cdot b1 \\ 0 \end{pmatrix}$
$\qquad cr = \begin{pmatrix} (\bar{x}_c - \bar{x}_a) \cdot b1 \\ (\bar{x}_c - \bar{x}_a) \cdot b2 \end{pmatrix}$

$S(f, x) = \begin{cases} \infty & \text{if } |m| \le 0 \\ A\left(\|J\|^2 + \|J^{-1}\|^2\right) & \text{otherwise} \end{cases}$
$\quad$ where
$\qquad f \in F$
$\qquad x_i \in \mathbb{R}^2$
$\qquad a, b, c = OrientedVertices(f)$
$\qquad m = [x_b - x_a \quad x_c - x_a]$
$\qquad A = \frac{1}{2}|m_r(f)|$
$\qquad J = m m_r(f)^{-1}$

$e = \displaystyle\sum_{i \in F} S_i(x)$

$H = \displaystyle\sum_{i \in F} psd\left(\dfrac{\partial^2 S_i(x)}{\partial x^2}\right)$

$G = \dfrac{\partial e}{\partial x}$