Problem Set 0

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Problem 0-1.

$$\begin{aligned} A &= \left\{ i + \binom{5}{i} \mid i \in Z,\ 0 \le i \le 4 \right\} \\ &= \left\{ 0 + \binom{5}{0},\ 1 + \binom{5}{1},\ 2 + \binom{5}{2},\ 3 + \binom{5}{3},\ 4 + \binom{5}{4} \right\} \\ &= \{1, 6, 12, 13, 8\} \end{aligned} \tag{1}$$

$$\begin{aligned} B &= \{3i \mid i \in \{1, 2, 3, 4, 5\}\} \\ &= \{3, 6, 9, 12, 15\} \end{aligned} \tag{2}$$

(a) $A \cap B = \{6, 12\}$

(b) $|A \cup B| = 7$

(c) $|A - B| = 3$

Problem 0-2.

$$\begin{aligned} X &= \{\text{\# of heads in three coin flips}\} \\ &= \{\text{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}\} \\ &= \{3, 2, 2, 2, 1, 1, 1, 0\} \end{aligned} \tag{3}$$

$$\begin{aligned} Y &= \{\text{products of two six-sided dice}\} \\ &= \{1*1,\ 1*2,\ 2*1,\ \ldots\ 6*5,\ 6*6\} \\ &= \begin{matrix} \{1, & 2, & 3, & 4, & 5, & 6, \\ 2, & 4, & 6, & 8, & 10, & 12, \\ 3, & 6, & 9, & 12, & 15, & 18, \\ 4, & 8, & 12, & 16, & 20, & 24, \\ 5, & 10, & 15, & 20, & 25, & 30, \\ 6, & 12, & 18, & 24, & 30, & 36\}, \end{matrix} \end{aligned} \tag{4}$$

(a) $\mathrm{E}[X] = (3 \cdot 1\ +\ 2 \cdot 3\ +\ 1 \cdot 3)/8 = 12/8 = 1.5$

(b) $\text{E}[Y] = 426/36 = 11.8\overline{33}$

(c) $\text{E}[\{X + Y\}] = 438/44 = 9.9\overline{54}$

Problem 0-3.

$$A = 600/6 = 100 \tag{5}$$

$$B = 60 \bmod 42 = 17 \tag{6}$$

(a) $A \bmod 2 = 0,\ B \bmod 2 = 0,\ \therefore A \equiv B \pmod 2$

(b) $A \bmod 3 = 1,\ B \bmod 3 = 0,\ \therefore A \not\equiv B \pmod 3$

(c) $A \bmod 4 = 0,\ B \bmod 4 = 2,\ \therefore A \not\equiv B \pmod 4$

Problem 0-4. Prove **by induction** that $\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$, for any $n \geq 1$.

Proof. Let $P(n) : \sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$.

Base case, $P(1)$:

$$\sum_{i=1}^{1} i^3 = 1 \tag{7}$$

$$\left[\frac{1(1+1)}{2}\right]^2 = 1 \tag{8}$$

Hence, base case is true. For the induction step, assuming $P(n)$ is true, we get $P(n+1)$:

$$\sum_{i=1}^{n+1} i^3 = (n+1)^3 + \sum_{i=1}^{n} i^3 \tag{9}$$

$$= (n+1)^3 + \left[\frac{n(n+1)}{2}\right]^2 \tag{10}$$

$$= \frac{4(n+1)^3 + n^2(n+1)^2}{4} = \frac{(4(n+1) + n^2)(n+1)^2}{4} \tag{11}$$

$$= \frac{(n^2 + 4n + 1)(n+1)^2}{4} = \frac{(n+2)^2(n+1)^2}{4} \tag{12}$$

$$= \left[\frac{(n+1)((n+1)+1)}{2}\right]^2 \tag{13}$$

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Problem 0-5.

Problem 0-6. Submit your implementation to `alg.mit.edu`.

```
def count_long_subarray(A):
    '''
    Input:  A     | Python Tuple of positive integers
    Output: count | number of longest increasing subarrays of A
    '''
    count = 0
    ##################
    # YOUR CODE HERE #
    ##################
    return count
```