A Benchmark Test Suite for the Electric Capacitated Vehicle Routing Problem

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***Abstract*—Several logistic companies started utilizing electric vehicles (EVs) in their daily operations to reduce greenhouse gas pollution. However, the limited driving range of EVs may require visits to recharging stations during their operation. These potential visits have to be addressed, avoiding unnecessary long detours. We formulate the electric capacitated vehicle routing problem (E-CVRP), which incorporates the possibility of EVs visiting a recharging station while satisfying the delivery demands of customers. The energy consumption of the EVs is proportional to their cargo load which is an important constraint in real-world logistics applications. A new set of benchmark instances is proposed for the E-CVRP. As solution methods to these new benchmarks, we apply the ant colony optimization metaheuristic method and an exact method. Experimental results on the E-CVRP demonstrate the high complexity of the problem and the efficiency of the applied metaheuristic solution method.**

***Index Terms*—Electric vehicle, capacitated vehicle routing problem, benchmark instance**

I. Introduction

In recent years, there is a growing interest of logistic companies in utilizing electric vehicles (EVs) for their daily operations due to the greenhouse effect. Therefore, a problem of routing a fleet of EVs has emerged, namely the electric vehicle routing problem (EVRP) [1]. The EVRP aims to find the best possible routes for a fleet of EVs so that a set of customers is visited once and only once by an EV, ensuring that the EVs will never run out of energy. Several variations of the EVRP have been introduced that mainly differ on their constraints such as the EVRP with time windows [2], the EVRP with pick up and delivery [3], the EVRP with multiple depots [4], the EVRP with service times [5] and the EVRP with backuals [6]. Refer to the comprehensive survey in [7] for further details.

The common constraint of all EVRP variations, is the consideration of visiting a recharging station due to the limited driving range of EVs. In this work, we formulate the electric capacitated vehicle routing problem (E-CVRP) as a mixed-integer linear program (MILP) where each EV has a limited cargo load to satisfy the delivery demands of the customers and the energy consumption rate of the EV is proportional to that cargo load. Existing EVRP formulations assume that the energy consumption rate is fixed [2], [8], or it is estimated using longitudinal dynamics model [9]. In this work we emphasize on the effect the cargo load of the EV has on its energy consumption.

In addition, a new benchmark set of E-CVRP problem instances is proposed to represent the complexity of the problem. The allocation of the charging stations in the benchmark instances is optimized to ensure energy coverage for EVs from any customer locations. Experiments are conducted in order to provide an initial set of results to this new benchmark set. As E-CVRP extends the well-known capacitated vehicle routing problem (CVRP), the high complexity of the problem makes exact solution methods inefficient for solving realistically sized problem instances [10]. Hence, as an approximation solution method, we apply the ant colony optimization (ACO) metaheuristic [11], which is successfully used to solve world-scale instances of similar problems [12], [13]. The newly generated E-CVRP benchmark set consists of small-sized instances, that they can be solved (within a reasonable time) using the proposed MILP formulation with the Gurobi solver [14] in order to asses the performance of approximation solution methods (i.e., the deviation of the approximation solution from an upper bound solution that could be the optimal), and realistic-sized instances in order to study the efficiency of approximation solution methods.

The rest of the paper is organized as follows. Section II describes the E-CVRP and provides the MILP formulation of the problem. Section III describes the two aforementioned approaches for solving the E-CVRP. Section IV describes a method to generate E-CVRP problem instances from existing CVRP problem instances. Initial results obtained on the newly generated benchmark set are presented in Section V. Section VI gives the conclusions and future work of the paper.

II. The Electric Capacitated Vehicle Routing Problem (E-CVRP)

A. Problem Description

The E-CVRP is a challenging $\mathcal{NP}$-hard combinatorial optimization problem as it is an extension of the conventional

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CVRP problem incorporating additional hard constraints [10]. The problem can be described as follows: given a fleet of EVs with limited battery charge level and limited cargo load capacity, we need to find the best possible route for each EV, starting and ending to the central depot, satisfying the delivery demands of a set of customers.

The E-CVRP can be defined on a complete weighted graph $G = (N, A)$, where $N = \{0\} \cup I \cup F'$ is a set of nodes and $A = \{(i, j) \mid i, j \in N, i \neq j\}$ is a set of arcs connecting these nodes. A non-negative value d_{ij} is associated with each arc which represents the euclidean distance between nodes i and j. Node 0 denotes the central depot. The set $I \subset N$ denotes the set of customers, where each customer $i \in I$ is assigned a positive value δ_i indicating the customer's delivery demand[1]. The set $F' \subset N$ denotes the set of β_i node copies of each charging station $i \in F$ (i.e., $|F'| = \sum_{i \in F} \beta_i$), which are used to permit multiple visits to each charging station $i \in F$ (if required) [15]. The upper bound on the number of node copies for each charging station is equal to $\beta_i = 2|I|$, because in the worst case an EV for each customer is needed and a visit to a charging station before and after serving it is required [16].

Each EV[2] has a cargo load, u_i ($0 \leq u_i \leq C$), where C is the maximal cargo load of an EV and u_i is the remaining cargo load of an EV on arrival at node i. In addition, each EV has a battery charge level y_i ($0 \leq y_i \leq Q$), where Q is the maximal battery charge level of an EV and y_i is the remaining battery charge level of an EV on arrival at node i. Each traveled arc consumes the amount $h_i d_{ij}$ of the remaining battery charge level, where h_i denotes the energy consumption rate of an EV traversing that arc. The energy consumption rate is proportional to the current cargo load of an EV (i.e., the heavier the EV the more energy will be consumed) defined as follows:

$$h_i = r + \frac{u_i}{C}, \tag{1}$$

where r is a constant value of the energy consumption rate, u_i is the remaining cargo load of an EV on arrival at node i, and C is the maximal cargo load of an EV. There are other minor factors that may affect the energy consumption rate [5], but in this work we focus on the weight of the EV which is one of the most important factors [17].

The objective function of the E-CVRP is to find a set of EV routes that minimize the total distance traveled where:

- Every customer is visited once and only once by exactly one EV.
- All EVs start and end at the depot.
- All EVs always start fully charged from the depot, e.g., $y_0 = Q$.
- All EVs always start with fully loaded cargo from the depot, e.g., $u_0 = C$.
- For every EV route the total delivery demand of customers does not exceed the EV's maximal cargo load C.
- The battery charge level on each traveled arc is never negative.
- EVs always leave a charging station fully charged.
- The charging stations can be visited multiple times by any EV.

B. MILP Problem Formulation

To solve the E-CVRP, a binary decision variable is defined, i.e., x_{ij}, $\forall i, j \in N$, denoting whether arc (i, j) is traversed by an EV (i.e., $x_{ij} = 1$) or not (i.e., $x_{ij} = 0$). The mathematical model of E-CVRP is formulated as:

$$\min \sum_{i \in N, j \in N, i \neq j} d_{ij} x_{ij}, \tag{2a}$$

$$\text{s.t.} \sum_{j \in N, i \neq j} x_{ij} = 1, \forall i \in I, \tag{2b}$$

$$\sum_{j \in N, i \neq j} x_{ij} \leq 1, \forall i \in F', \tag{2c}$$

$$\sum_{j \in N, i \neq j} x_{ij} - \sum_{j \in N, i \neq j} x_{ji} = 0, \forall i \in N, \tag{2d}$$

$$u_j \leq u_i - \delta_j x_{ij} + C(1 - x_{ij}), \forall i \in N, \forall j \in N, i \neq j, \tag{2e}$$

$$u_j \geq u_i - \delta_j x_{ij} - C(1 - x_{ij}), \forall i \in N, \forall j \in N, i \neq j, \tag{2f}$$

$$0 \leq u_i \leq C, \forall i \in N, \tag{2g}$$

$$u_0 = C, \tag{2h}$$

$$y_j \leq y_i - h_i d_{ij} x_{ij} + Q(1 - x_{ij}), \forall i \in I, \forall j \in N, i \neq j, \tag{2i}$$

$$y_j \leq Q - h_i d_{ij} x_{ij}, \forall i \in F', \forall j \in N, i \neq j, \tag{2j}$$

$$0 \leq y_i \leq Q, \ \forall i \in N, \tag{2k}$$

$$y_0 = Q, \tag{2l}$$

$$x_{ij} \in \{0, 1\}, \ \forall i \in N, \ \forall j \in N, i \neq j, \tag{2m}$$

where Eq. (2a) defines the E-CVRP objective function, Eq. (2b) enforces the connectivity of customer visits, Eq. (2c) handles the connectivity of visits to recharging stations, Eq. (2d) establishes the flow conservation by guaranteeing that at each vertex, the number of incoming arcs is equal to the number of outgoing arcs, Eq. (2e), Eq. (2f) and Eq. (2g) guarantee delivery demand fulfillment at all customers by assuring a non-negative cargo load upon arrival at any vertex, Eq. (2h) ensures that EVs start with full cargo load from the depot, Eq. (2i), Eq. (2j) and Eq. (2k) ensure that the battery charge level never falls below 0, Eq. (2l) ensures that EVs start fully charge from the depot, and Eq. (2m) is the binary decision variable described previously.

III. Solving the E-CVRP

In order to provide an initial set of results (e.g., optimal values, upper bounds, or best known values) for each problem instance of the new benchmark set we use the MILP formulation in Eqs. (2) and the ACO metaheuristic, described in Algorithm 1, as solution methods for the E-CVRP.

[1] For the central depot and charging stations the demand is set to zero, i.e., $\delta_0 = 0 \wedge \delta_i = 0, \forall i \notin I$.

[2] A homogeneous fleet of EVs is considered.

A. Using the MILP Formulation

Although the formulation defined in Eqs. (2) fully describes the E-CVRP problem, it cannot be handled by standard mathematical optimization solvers, such as the Gurobi solver [14]. This is because that formulation does not adhere to any MILP standard form, because of the constraints in Eq. (2i) and Eq. (2j) as they involve a product of a continuous and a binary variable, that is, $u_i x_{ij}$. Recall that the current cargo load u_i is used to calculate the variable energy consumption rate h_i in Eq (1). To cope with this issue, the continuous variable m_{ij} is introduced to replaced the product, $m_{ij} = u_i x_{ij}$, so that it will be equivalently transformed to a set of MILP inequalities according to the big "M" notation [28] as follows:

$$m_{ij} \leq u_i, \tag{3a}$$

$$m_{ij} \leq M x_{ij}, \tag{3b}$$

$$m_{ij} \geq 0, \tag{3c}$$

$$m_{ij} \geq u_i - M(1 - x_{ij}), \tag{3d}$$

with $M \geq C$.

To summarize, the problem presented in Eqs. (2) can be reformulated into a MILP problem by replacing the product of variables (i.e., $u_i x_{ij}$) in Eq. (2i) and Eq. (2j) with m_{ij} and adding Eqs. (3). As a result, we get a transformed mathematical program that involves only linear constraints.

B. Using the ACO Metaheuristic

The $\mathcal{MAX}$-$\mathcal{MIN}$ Ant System ($\mathcal{MM}$AS) [19] variant is used which is one of the most-studied ACO variants and has already proven its good performance on relevant EVRPs [5], [20]. In this section we describe the application of $\mathcal{MM}$AS to the E-CVRP, including the proposed method to construct feasible solutions.

1) Initialization: A colony of ω ants is initially positioned at the central depot, i.e., component 0. All the solution components of the problem are associated with a pheromone trail value which is uniformly initialized at the start of the execution as follows:

$$\tau_{ij} \leftarrow \tau_0, \forall (i,j) \in A, \tag{4}$$

where τ_{ij} is the pheromone trail value associated with arc (i, j) connecting solution components i and j, and τ_0 is the initial pheromone trail value. A good value for τ_0 was found to be $1/\rho C^{nn}$, where ρ is the evaporation rate (more details are provided later on) and C^{nn} is the solution quality of the solution generated by the nearest-neighbor heuristic [21].

2) Solution Construction: Each ant k represents a complete E-CVRP solution T^k (i.e., the routes of all EVs) and makes selections biased by the existing pheromone trails and some heuristic information associated with the solution components of the problem, until all the components (i.e., customers) are selected.

The probability distribution with which ant k selects the next customer component j from solution component i is defined as follows:

$$p_{ij}^k = \begin{cases} \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in \mathcal{N}_i^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}, \text{if } j \in \mathcal{N}_i^k, \\ 0, \qquad\qquad\qquad \text{otherwise}, \end{cases} \tag{5}$$

where τ_{ij} and $\eta_{ij} = 1/d_{ij}$ are, respectively, the existing pheromone trail and the heuristic information available a priori between components i and j. α and β are the two parameters which determine the relative influence of τ_{ij} and η_{ij}, respectively. $\mathcal{N}_i^k$ is the set of unselected customers components for the k–th ant adjacent to component i. Note that the depot and charging station components are not included in the $\mathcal{N}_i^k$ set (the selection of these components is described later in Section III-B3).

The selected customer component j must satisfy the EV's capacity constraints defined in Eqs. (2e) and (2g), and the energy constraints defined in Eqs. (2i), (2j), and (2k). It is worth mentioning that the construction method proposed in this section can be utilized in other constructive or local search heuristics to construct feasible solutions when addressing similar EVRPs. The following section describes in more detail the handling of the aforementioned constraints.

3) Constraints Handling: To construct a feasible solution the next customer component j to be selected, using Eq. (5), must satisfy all the following criteria:

1) The delivery demand of the customer component j, i.e., δ_j, must not cause the cargo load to fall below zero, that is:

$$i, j \in N \wedge u_i - \delta_j \geq 0. \tag{6}$$

2) The energy consumption to travel from solution component i to customer component j must not cause the energy level, i.e., y_i, to fall below zero, that is:

$$i, j \in N \wedge y_i - h_i d_{ij} \geq 0. \tag{7}$$

3) The customer component j must have at least one charging station or the central depot within its energy range [5], that is:

$$i, j \in N \wedge y_i - h_i d_{ij} - h_j d_{jl} \geq 0, \exists\ l \in F \cup \{0\}. \tag{8}$$

When the next customer violates the delivery demand criteria in Eq. (6), then the depot component is added to the E-CVRP solution to close the route of the EV (i.e., denotes the return of the EV to the central depot). In case this return to the depot violates the energy criteria in Eq. (7), then the EV visits a recharging station prior to its return to the depot. When the next customer violates any of the energy criteria in Eq. (7) or the range criteria in Eq. (8), then the closest energy recharging station s from the current solution component i to the (potentially) next customer j is selected as follows:

$$s = \arg\min_{l \in F} \{d_{il} + d_{lj}\}, \tag{9}$$

and is added to the E-CVRP solution (i.e., denotes the visit to an energy recharging station). Then, the next customer to

Algorithm 1 ACO for E-CVRP

```
$C^{bs} \leftarrow \infty$
for each arc $(i, j)$ do
    $\tau_{ij} \leftarrow$ InitializePheromoneTrails($\tau_0$) {Eq. (4)}
end for
while maximum number of iterations not exceeded do
    $C^{ib} \leftarrow \infty$
    for $k = 1$ to $\omega$ do
        $T^k \leftarrow \emptyset$
        $T^k \leftarrow$ InsertDepotComponent()
        while ant $k$ has not selected all customers do
            $i \leftarrow$ GetLastInsertedComponentFrom($T^k$)
            $\mathcal{N}_i^k \leftarrow$ GenerateUnselectedCustomerSetFrom($i$)
            $j \leftarrow$ SelectNextCustomerComponentUsing($\mathcal{N}_i^k$) {Eq. (5)}
            if $j$ satisfies all the criteria then
                $T^k \leftarrow$ InsertCustomerComponent($j$)
            else if $j$ violates the capacity constraint in Eq. (6) then
                $T^k \leftarrow$ InsertDepotComponent(0)
            else if $j$ violates the energy constraint in Eq. (7) or in Eq. (8) then
                $s \leftarrow$ FindClosestStationFromTo($i, j$) {Eq. (9)}
                $T^k \leftarrow$ InsertStationComponent($s$)
            end if
        end while
        $T^k \leftarrow$ InsertDepotComponent(0)
        $C^k \leftarrow$ Evaluate($T^k$) {e.g., Eq. (2a)}
        if $C^k$ is better than $C^{ib}$ then
            $T^{ib} \leftarrow T^k$
            $C^{ib} \leftarrow C^k$
        end if
    end for
    if $C^{ib}$ is better than $C^{bs}$ then
        $T^{bs} \leftarrow T^{ib}$
        $C^{bs} \leftarrow C^{ib}$
        $\tau_{max} \leftarrow$ CalculateMaximumTrailLimit()
        $\tau_{min} \leftarrow$ CalculateMinimumTrailLimit()
    end if
    $T^{best} \leftarrow T^{ib}$ {occasionally set $T^{best} \leftarrow T^{bs}$}
    for each arc $(i, j) \in A$ do
        $\tau_{ij} \leftarrow$ PheromoneEvaporation($\rho$) {Eq. (10)}
        if arc $(i, j) \in T^{best}$ then
            $\tau_{ij} \leftarrow$ PheromoneDeposit($T^{best}$) {Eq. (11)}
        end if
        $\tau_{ij} \leftarrow$ PheromoneTrailLimits($\tau_{min}$,$\tau_{max}$) {Eq. (12)}
    end for
    if stagnation behavior is detected then
        for each arc $(i, j)$ do
            $\tau_{ij} \leftarrow$ InitializePheromoneTrails($\tau_{max}$)
        end for
    end if
end while
OUTPUT: $T^{bs}$
```

be visited will be most probably j if all three criteria defined in Eq. (6), Eq. (7) and Eq. (8) are now satisfied, otherwise another customer satisfying all the aforementioned criteria will be selected. The possibility of not having enough energy to travel to any energy recharging station or back to the central depot is eliminated because the customer component must always satisfy the range criteria in Eq. (8).

4) Pheromone Update: In the $\mathcal{MM}$AS variant [19], [21], [22], the pheromone trails are updated by first decreasing the pheromone trails on all arcs (using pheromone evaporation), and then increasing the pheromone trails on the arcs of the solution constructed by the best ant (using pheromone deposit). The pheromone evaporation is applied as follows:

$$\tau_{ij} \leftarrow (1 - \rho)\tau_{ij}, \forall (i, j) \in A, \tag{10}$$

where ρ ($\rho \in (0, 1]$) is the evaporation rate.

After pheromone evaporation, the best ant deposits pheromone on the arcs of its solution components as follows:

$$\tau_{ij} \leftarrow \tau_{ij} + \Delta\tau_{ij}^{best}, \forall (i, j) \in T^{best}, \tag{11}$$

where $\Delta\tau_{ij}^{best} = 1/C^{best}$ is the amount of pheromone that the best ant deposits and C^{best} is the quality of the best solution T^{best}. The "best" ant that is allowed to deposit pheromone may be either the best-so-far ant[3], in which $C^{best} = C^{bs}$, or the iteration-best ant, in which case $C^{best} = C^{ib}$. These two ants are allowed to deposit pheromone in an alternate way. More precisely, the iteration-best ant deposits pheromone at each iteration and the best-so-far ant deposits pheromone occasionally (i.e., every 25 iterations in this work, more details are provided in [22]). Pheromone evaporation enables the algorithm to gradually decrease the pheromone trails on the arcs that belong to low-quality solutions so that the pheromone trails on the arcs that belong to high-quality solutions consist of higher values (e.g., they will be increased regularly because of pheromone deposit).

Since only the best ant is allowed to deposit pheromone the search is focused on a specific area of the search space. Hence, there is consequent danger of getting trapped in search stagnation[4] from the early stages of the optimization process. To address this issue, the lower and upper limits of the pheromone trail values are explicitly imposed, preventing in this way the excessive growth of pheromone trails on the arcs of the best ant, as follows:

$$\tau_{ij} \leftarrow \begin{cases} \tau_{max}, & \text{if } \tau_{ij} > \tau_{max}, \\ \tau_{min}, & \text{if } \tau_{ij} < \tau_{min}, \quad \forall (i, j) \in A, \\ \tau_{ij}, & \text{otherwise}, \end{cases} \tag{12}$$

where τ_{min} and τ_{max} are, respectively, the minimum and maximum pheromone trails. The τ_{max} value is bounded

[3] A special ant that represents the best solution from all iterations so far and, hence, may not necessarily belong in the constructing colony at the current iteration.

[4] A behavior in which all ants follow the same path and construct the same solution over and over again.

Algorithm 2 Convert CVRP to E-CVRP

```
1: INPUT: Classical CVRP benchmark instance
2: Read initial benchmark problem of size $n$
3: Define maximal battery charge level (i.e., $Q$)
4: Add charging stations
5: OUTPUT: Electric CVRP benchmark instance
```

by $1/\rho C^{bs}$, where C^{bs} is the solution quality of the best-so-far ant[5]. The τ_{min} value is set to $\tau_{min} = \tau_{max}(1 - \sqrt[n]{0.05})/((avg - 1)\sqrt[n]{0.05})$ where avg is the average number of different choices available to an ant at each solution construction step and n is the size of the problem instance. Note that the τ_{max} and τ_{min} values are updated whenever whenever a new best-so-far ant is discovered.

In addition, all the pheromone trails are uniformly re-initialized to the τ_{max} value whenever the stagnation behavior occurs or when no improved solution is found for a given number of iterations. The stagnation behavior is detected using the λ-branching factor that measures the distribution of the pheromone trail values [23]. The λ-branching factor is given by the number of arcs incident to node i satisfying the following condition: $\tau_{ij} \geq \tau^i_{min} + \lambda(\tau^i_{max} - \tau^i_{min})$, where τ^i_{max} and τ^i_{min} are, respectively, the maximum and minimum pheromone values on arcs incident to node i, and $\lambda \in [0, 1]$ is a constant. The average λ-branching factor from all nodes' λ-branching factors indicates whether the search has entered stagnation behavior or not.

IV. E-CVRP Benchmark Test Suite

A. Converting CVRP to E-CVRP

In this section, we provide the algorithmic steps followed to convert a CVRP problem instance to an E-CVRP problem instance with the basic procedures outlined in Algorithm 2. More specifically, at the first step we import the classical CVRP problem instance where each customer node $i \in I$ represents a point in the Cartesian plane (e.g., lines 1-2). In the sequel, the maximum battery charge level Q is defined as follows:

$$Q = \gamma \bar{d}, \tag{13}$$

where $\bar{d}$ is the maximum euclidean distance between the depot and all other customer nodes, i.e., $\bar{d} = \arg\max_{i \in I}\{d_{0i}\}$, and γ is a user defined constant.

The goal of stations placement (Algorithm 2, line 4) is to find the minimum number of charging stations together with their positions so that all customer nodes are covered by at least one charging station. This procedure ensures that problem instances with remote customer nodes stay feasible. In particular, it is assumed that each charging station covers an area of a circle with a specific radius $R = \phi Q$, where ϕ is a user selected constant. Increasing the value of ϕ results in a larger coverage area of charging stations. Each customer $i \in I$ can be covered from at least one charging station placed within the area of the circle that has radius R with customer i being the center of the circle. All potential positioning points are in the convex-hull area of all customer points that can be computed by the *Quickhull* method [18]. The set P denotes any potential discrete point in the above convex area in which a charging station can be potentially placed (excluding all the points of customer nodes). Furthermore, the set $A_i, \forall i \in I$, determines all the potential charging stations points in set P that cover each customer $i \in I$ (i.e., the area of the circle).

To optimally solve the described charging station placement problem we introduce a MILP formulation that aims to select the minimum number (together with their position) to cover each customer node. Let the variable $\psi_j \in \{0, 1\},\ j \in P$, denote whether the potential point $j \in P$ is selected to place a charging station ($\psi_j = 1$) or not ($\psi_j = 0$). The related mathematical formulation is as follows:

$$\min \sum_{i \in P} \psi_i, \tag{14a}$$

$$\text{s.t.} \sum_{j \in A_i} \psi_j \geq 1, \forall i \in I, \tag{14b}$$

$$\psi_i \in \{0, 1\},\ \forall i \in P, \tag{14c}$$

where Eq. (14a) is the objective function as described previously, Eq. (14b) is responsible to ensure that each customer $i \in I$ is covered from at least one charging station and Eq. (14c) defines the binary decision variable. Finally, in line 5 of Algorithm 2 the defined maximum battery charge level value and the positioning of the charging stations are used to generate the E-CVRP benchmark instance.

B. Description of E-CVRP Problem Instances

Four different sets of CVRP benchmark instances are used to generate the new E-CVRP benchmark instances in the experiments[6], including the popular instances of Christofides and Eilon [24], Christofides *et al.* [25], and Fisher [26], and the recent instances of Uchua *et al.* [27]. The details for converting a CVRP instance to an E-CVRP instance are given in Section IV. The converted E-CVRP benchmark problem instances are described in Tables I, II, III, IV, where the columns (starting from the left) denote the name[7] of the problem instance, the number of customers (#customers), the number of charging stations (#stations), the minimum number of EV routes a solution can have[8] (#routes), the maximum cargo load of an EV (C), and the maximum battery charge level of an EV (Q). Note that all E-CVRP benchmark instances consist of a single central depot. To generate reasonable (i.e., enforcing EVs to visit charging stations) E-CVRP instances the γ and ϕ parameters are set to $\gamma = 2$ and $\phi = 0.125$. Increasing the

[5] C^{bs} is initialized with the solution quality of the estimated optimal tour C^{nn}.

[6] The generated instances of the E-CVRP benchmark set can be downloaded from https://github.com/KIOS-Research/e-cvrp_benchmark_instances

[7] The numbers after n, k, and s letters in the instance name identifies the size of the problem, the minimum number of routes and the number of charging stations, respectively.

[8] A lower bound based on the total customer delivery demands and the maximum cargo load of the EV.

TABLE I
DETAILS OF THE E-CVRP INSTANCES GENERATED FROM THE CVRP BENCHMARK SET OF [24].

instance name	#customers	#stations	#routes	C	Q
`E-n29-k4-s7`	21	7	4	6000	99
`E-n30-k3-s7`	22	7	3	4500	162
`E-n35-k3-s5`	29	5	3	4500	138
`E-n37-k4-s4`	32	4	4	8000	238
`E-n60-k5-s9`	50	9	5	160	88
`E-n89-k7-s13`	75	13	7	220	87
`E-n112-k8-s11`	100	11	8	200	100

TABLE II
DETAILS OF THE E-CVRP INSTANCES GENERATED FROM THE CVRP BENCHMARK SET OF [25].

instance name	#customers	#stations	#routes	C	Q
`M-n110-k10-s9`	100	9	10	200	118
`M-n126-k7-s5`	120	5	7	200	199
`M-n163-k12-s12`	150	12	12	200	100
`M-n212-k16-s12`	199	12	16	200	100

TABLE III
DETAILS OF THE E-CVRP INSTANCES GENERATED FROM THE CVRP BENCHMARK SET OF [26].

instance name	#customers	#stations	#routes	C	Q
`F-n49-k4-s4`	44	4	4	2010	260
`F-n80-k4-s8`	71	8	4	30000	53
`F-n140-k5-s5`	134	5	5	2210	307

TABLE IV
DETAILS OF THE E-CVRP INSTANCES GENERATED FROM THE CVRP BENCHMARK SET OF [27].

instance name	#customers	#stations	#routes	C	Q
`X-n147-k7-s4`	142	4	7	1190	2762
`X-n221-k11-s9`	213	7	11	944	1204
`X-n360-k40-s9`	351	9	40	436	1236
`X-n469-k26-s10`	458	10	26	1106	1230
`X-n577-k30-s4`	577	30	4	210	2191
`X-n698-k75-s13`	698	13	75	408	1336
`X-n759-k98-s10`	748	10	98	396	1367
`X-n830-k171-s11`	818	11	171	358	1385
`X-n920-k207-s4`	915	4	207	33	2773
`X-n1006-k43-s5`	1000	5	43	131	2536

value of γ and decreasing the value of ϕ results in a higher number of charging stations.

V. EXPERIMENTAL RESULTS

A. Experimental Setup

In order to provide an initial set of results to the new benchmark, experiments have been conducted with the $\mathcal{MM}$AS approach described in Section III-B and the MILP approach defined in Section III-A. The experiments were performed under a Linux System with an Intel Core i7-3930K 3.20GHz processor with 12MB cache and 16GB RAM.

The MILP approach was solved using the Gurobi solver [14] and a time limit of approximately 3 weeks was imposed. The $\mathcal{MM}$AS approach performs $25000n$ evaluations[9] for 10 independent executions (with different random seeds), where n is the problem size. The total distance traveled of the solution with the best quality together with the CPU time are recorded. The colony size of $\mathcal{MM}$AS was set to $\omega = n$ ants. The evaporation rate was set to $\rho = 0.02$ and the two parameters of the decision rule were set to typical values as follows: $\alpha = 1$ and $\beta = 2$.

B. Analysis of the Results

The experimental results for small-scale problem instances of $\mathcal{MM}$AS and MILP approaches are given in Table V, whereas the experimental results for large-scale problem instance are given in Table VI. Note that the MILP approach was not able to provide a solution for the large-scale problem instances within the pre-determined time limit, and, thus, only the results of the $\mathcal{MM}$AS approach are reported in Table VI. "UB" is the best upper bound within the time limit found by the MILP approach (in some cases the given upper bound is also the optimal). "best", "worst", "mean" and "stdev" are, respectively, the minimum, maximum, average and standard deviation values found by $\mathcal{MM}$AS from its 10 independent runs. Furthermore, "t_{avg}" is the CPU time required (averaged over 10 runs) for $\mathcal{MM}$AS to find the solution with the best quality. The "UB" and "best" values also include the number of routes that the solution consists of within the brackets.

From Table V it can be observed that the MILP approach can find better solutions than the $\mathcal{MM}$AS[10] approach on most of the smallest problem instances. Nevertheless, the deviation of the solution quality of the $\mathcal{MM}$AS approach from the upper bound solution found by the MILP approach on `E-n29-k4-s7`, `E-n30-k3-s7` and `E-n35-k3-s5` problem instances (which is also the optimal solution for these cases) is very small. However, the $\mathcal{MM}$AS approach requires a few seconds whereas the MILP approach requires weeks. As the problem size increases it can be observed that the $\mathcal{MM}$AS approach can find better solutions than the MILP approach (e.g., on the `E-n60-k5-s9` problem instance) because the pre-determined time appears to be short. This observation was expected considering the high complexity of the E-CVRP.

From Table VI it can be observed that the $\mathcal{MM}$AS approach was able to find a solution for all E-CVRP problem instances, requiring a few minutes for most problem instances (with less than 300 nodes) and couple of hours for the remaining problem instances. This observation demonstrates the efficiency of the $\mathcal{MM}AS$ approach against the MILP

[9] The same termination condition was proposed at the *2020 Congress on Evolutionary Computation* competition for the Electric Vehicle Routing Problem [8], where each evaluation is of complexity $\mathcal{O}(n^2)$.

[10] $\mathcal{MM}$AS is not utilizing any local search improvements in these experiments.

TABLE V
SOLUTION QUALITY AND CPU TIME (IN SECONDS) RESULTS OBTAINED FROM THE MILP AND $\mathcal{MMAS}$ APPROACHES FOR SMALL-SCALE E-CVRP PROBLEM INSTANCES.

E-CVRP instance name	MILP	$\mathcal{MMAS}$			
	UB	best	mean±stdev	worst	t_{avg}
`E-n29-k4-s7`	383(4)†	383(4)	384.5±0.7	385	0.1
`E-n30-k3-s7`	577(3)†	582(3)	582.0±0.0	582	3.1
`E-n35-k3-s5`	527(3)†	530(4)	533.0±1.4	534	2.2
`E-n37-k4-s4`	865(4)	865(4)	868.5±1.6	871	3.4
`E-n60-k5-s9`	585(5)	544(5)	557.0±6.4	562	20.7
`F-n49-k4-s4`	740(4)	769(4)	775.7±6.9	791	8.9

† indicates optimal value.

TABLE VI
SOLUTION QUALITY AND CPU TIME (IN SECONDS) RESULTS OBTAINED FROM THE $\mathcal{MMAS}$ APPROACH FOR LARGE-SCALE E-CVRP PROBLEM INSTANCES.

E-CVRP instance name	$\mathcal{MMAS}$			
	best	mean±stdev	worst	t_{avg}
`E-n89-k7-s13`	724(7)	739.4±2.9	748	31.8
`E-n112-k8-s11`	860(8)	878.3±14.6	910	71.6
`M-n110-k10-s9`	914(10)	923.7±9.7	937	57.6
`M-n126-k7-s5`	1099(7)	1103.6±5.7	1119	63.7
`M-n163-k12-s12`	1109(12)	1129.2±8.9	1139	158.4
`M-n212-k16-s12`	1398(17)	1419.2±11.8	1436	266.1
`F-n80-k4-s8`	240(4)	243.9±2.8	249	23.7
`F-n140-k7-s5`	1229(7)	1233.8±2.9	1239	92.5
`X-n147-k7-s4`	17704(5)	17792.3±89.7	17884	104.5
`X-n221-k11-s9`	12235(12)	12487.9±109.4	12586	161.0
`X-n360-k40-s9`	27701(41)	28034.4±168.6	28252	1119.4
`X-n469-k26-s10`	26881(26)	27391.7±257.0	27675	1905.2
`X-n577-k30-s4`	55266(30)	55627.6±209.4	55856	3182.0
`X-n698-k75-s13`	75048(77)	75646.6±323.7	76051	5511.7
`X-n759-k98-s10`	84996(101)	85683.7±558.8	86349	7258.8
`X-n830-k171-s11`	167575(181)	168465±376.1	168927	6612.6
`X-n920-k207-s4`	345214(216)	346619.4±901.3	347946	6774.9
`X-n1006-k43-s5`	80765(43)	81549.3±568.8	82442	8380.6

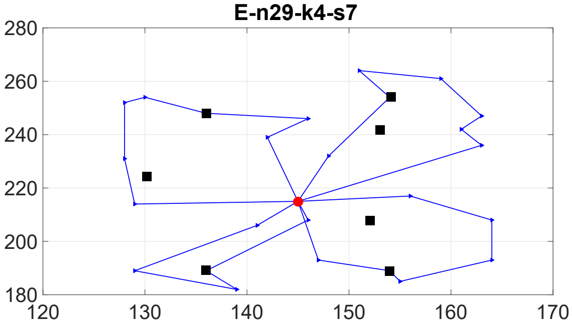

Fig. 1. Illustration of the solution generated by the $\mathcal{MMAS}$ approach on the `E-n29-k4-s7` problem instance with solution quality 385. ●, ▶, and ■ represent the depot, customer and station components, respectively.

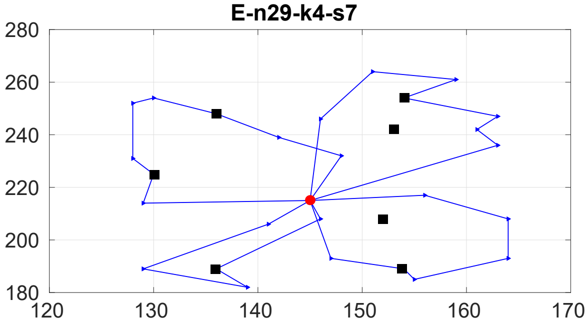

Fig. 2. Illustration of the optimal solution generated by the MILP approach on the `E-n29-k4-s7` problem instance with solution quality 383. ●, ▶, and ■ represent the depot, customer and station components, respectively.

approach which requires several days for even smaller problem instances, e.g., the `E-n37-k4-s4` problem instance in Table V. In addition, it verifies that ACO algorithms are effective solvers for these type of problems.

Finally, the median solution (from the solutions of the 10 runs) generated by the $\mathcal{MM}$AS approach is plotted in Fig. 1 for the `E-n29-k4-s7` problem instance and the optimal solution generated by the MILP approach is plotted in Fig. 2 for the same problem instance. From Fig.1 it can be observed that the solution generated by the $\mathcal{MM}$AS approach consists of four routes, in which all of them contain a single visit to a recharging station. On the contrary, from Fig. 2 the optimal solution consists of four routes in which one route contains two visits to recharging stations and one visit for each of the remaining three routes. Note that the solutions generated by both approaches can be operated by four EVs simultaneously or by a single EV sequentially. Although the median solution generated by the $\mathcal{MM}$AS approach requires less visits to recharging stations than the optimal solution, its solution quality is inferior. This observation shows the challenge when addressing the E-CVRP because fewer detours to recharging stations does not necessarily lead to better solution quality.

VI. Conclusions

In this work, we present a new variation of the vehicle routing problem for determining minimum distance routes for EVs. The E-CVRP considers limited battery charge level and limited cargo load capacity for the EVs. The energy consumption of the EVs is based on the cargo load of the EVs, that is, the heavier the EV the more energy will be consumed. The problem is formulated as a MILP and it is solved using an exact solution method and an approximation solution method. The experimental results on a set of newly generated benchmark E-CVRP instances represent the complexity of the problem because the applied approaches experience challenges either in terms of efficiency or solution quality.

For future work, it will be interesting to design heuristic methods that are problem specific to this E-CVRP in order to further improve the solution quality. In fact, there is a lot of room for improvements on these benchmark problem instances considering the initial experimental results presented in this work.

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