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8.01 Problem Set - 5

5.1 $E = E_1$

a) Turning points at $x \approx 0.2$ m (left turning point)

b) The speed is maximum, when U is minimum i.e. at $x \approx 1.0$ m
The speed is minimum ($v=0$), when $U = E_1$ at $x \approx 0.2$ m

c) The orbit is unbound as there is only one turning points.

$E = E_2$

a) Left Turning Point at $x \approx 0.3$ m
Right Turning Point at $x \approx 3$ m

b) Maximum Speed when U is minimum at $x \approx 1.0$ m
Minimum speed when V is maximum at $x \approx 0.3$, and $x \approx 3.0$ m.
The speed also has a local minimum at $x \approx 1.6$ m

c) The orbit is bound.

$E = E_3$

a) Left Turning Point at $x \approx 0.5$ m
Right Turning Point at $x \approx 1.3$ m

b) Maximum speed at $x \approx 1.6\,m$
Minimum speed at $x \approx 0.5$ and $x \approx 1.3\,m$.

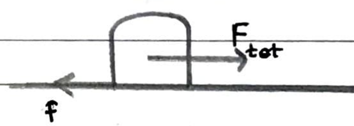
c) The orbit is bound.

5.2 Combustion of 1 gal of gasoline : $1.3 \times 10^{8}\,J$
Yearly Requirement : $8 \times 10^{19}\,J$

Daily requirement $= \frac{8 \times 10^{19}}{365}\,J = 2.2 \times 10^{17}\,J$

No. of gallons needed / day $= \frac{2.2 \times 10^{17}}{1.3 \times 10^{8}} \approx \underline{\underline{1.7 \times 10^{9}\text{ gals}}}$

5.3a) $\mu_k = 0.3$
$F = 360\,N$



$ma = 0$ (constant speed)

$x: \quad F_{tot} = f$

$360 \times 6000 = \mu_k N$

$N = \frac{360 \times 6000}{0.3} = \boxed{7.2 \times 10^{6}\,N = W}$

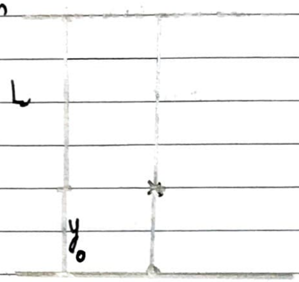
b) $P_{tot} = 6000 \times 0.2\,hp = 1.2 \times 10^{3}\,hp = 1.2 \times 10^{3} \times 746\,W$

$P_{tot} = Fv \Rightarrow v = \frac{P_{tot}}{F}$

$v = \dfrac{1.2 \times 746 \times 10^3}{360 \times 6000} = 0.414 \text{ m/s} \approx \underline{\underline{0.115 \text{ km/hr}}}$

5.4 $k = 100 \text{ N/m}$ $\qquad h = 100 \text{m}$

$m = 50 \text{kg}$



Energy is conserved during the entire motion as only conservative forces act on the body.

E_i (at top) $= mgh$ $\qquad u = 0$

E_f (at botton) $= \frac{1}{2} k y_0^2$

$\frac{1}{2} k y_0^2 = mgh \Rightarrow y_0 = \sqrt{\dfrac{2mgh}{k}} = \sqrt{\dfrac{2 \times 50 \times 10 \times 100}{100}}$

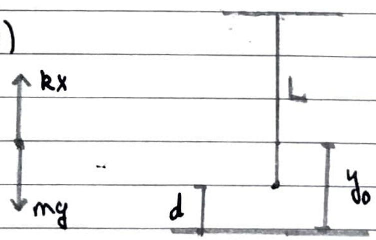
$y_0 = 10\sqrt{10} \approx \underline{31.62 \text{ m}}$

$L_{min} = 100 - y_0 \approx \underline{\underline{68.48 \text{ m}}}$

b) $k(y_0 - d) = mg \quad (\Sigma F = 0)$

$d = y_0 - \dfrac{mg}{k}$

$d \approx \underline{\underline{26.62 \text{ m}}}$



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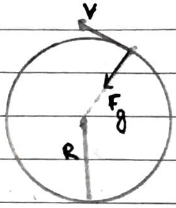
5.5a At the equator, $\frac{mv^2}{R} = F_g$



At any other point, only a component of F_g provides for the centripetal force.

If $\frac{mv^2}{R} > F_g$, the gravity cannot hold the material

and particles will launch off the surface.

a) $\frac{mv^2}{R} = F_g = \frac{GMm}{R^2}$

$M = \rho \times \frac{4}{3} \pi R^3$

$v^2 = \frac{G\rho \times 4 \pi R^{\cancel{3}2}}{\cancel{R}\ 3} = 4 \frac{G\rho}{3} \pi R^2$

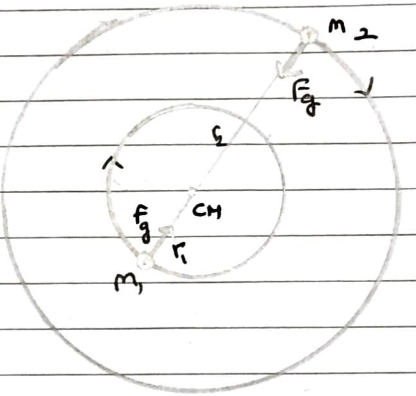
$v = 2R \left(\frac{G\rho\pi}{3} \right)^{1/2}$

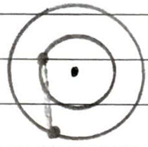
$T = \frac{2\pi R}{v} = \frac{2\pi \cancel{R}}{2\cancel{R} \left(\frac{G\rho\pi}{3} \right)^{1/2}} = \underline{\underline{\left(\frac{3\pi}{G\rho} \right)^{1/2}}}$ sec

b) $T = \left(\frac{3 \times \pi}{6.67 \times 10^{-11} \times \rho} \right)^{1/2}$; $\rho = 3\ g/cm^3 = 3 \times 10^3\ kg/m^3$

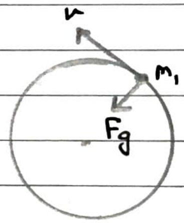
$T = \left(\frac{3\pi}{6.67 \times 10^{-11} \times 3 \times 10^3} \right)^{1/2} = 6.9 \times 10^3$ sec $\approx \underline{\underline{1.9\ hr}}$

5.6 Since there are no external forces on the system, the CM does not move.
Thus both stars must have the same time period (move together).





One star cannot move faster than the other star because the center of mass will change.



$$\frac{\cancel{m_1} v^2}{r_1} = \frac{G m_2 \cancel{m_1}}{(r_1 + r_2)^2}$$

$$v^2 = \frac{G m_2 r_1}{(r_1+r_2)^2}$$

$$T^2 = \frac{(2\pi r_1)^2}{v^2} = \frac{4\pi^2 r_1^{\cancel{2}} (r_1+r_2)^2}{G m_2 \cancel{r_1}} = \frac{4\pi^2 r_1 (r_1+r_2)^2}{G m_2}$$

we know that, $\frac{r_1}{r_2} = \frac{m_2}{m_1} \longrightarrow$ ①

Adding 1 to ①, $\frac{r_1+r_2}{r_2} = \frac{m_1+m_2}{m_1}$

$$\frac{r_2}{m_1} = \frac{r_1}{m_2} = \frac{r_1+r_2}{m_1+m_2} \longrightarrow ②$$

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Substituting (2), we get

$$\boxed{T^2 = \frac{4\pi^2}{G(m_1+m_2)}(r_1+r_2)^3}$$

5.7 $T = 5.6$ days $= 5.6 \times 24 \times 60 \times 60$ sec

$M_s = 1.989 \times 10^{30}$ kg

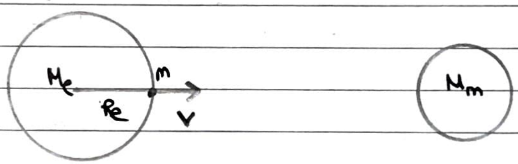
$m_1 = 25 M_s$

$m_2 = 10 M_s$

$$T^2 = \frac{4\pi^2}{G(m_1+m_2)}(r_1+r_2)^3$$

$$(r_1+r_2) = \left(\frac{G(m_1+m_2)T^2}{4\pi^2}\right)^{1/3} = \underline{\underline{3.02 \times 10^{10} \text{ m}}}$$

5.8



(a) Only conservative forces are acting on the projectile after firing, So mechanical energy is conserved

Initial: $E = \frac{1}{2}mv^2 - \frac{GM_e m}{R_e}$

Distance b/w Earth and Moon $\approx 384400 \times 10^3$ m $= r$

Final: $E = -\frac{GM_e m}{r}$

$$\frac{1}{2}mv^2 - \frac{GM_e m}{R_e} = -\frac{GM_e m}{r}$$

$$v^2 = 2GM_e\left(\frac{1}{R_e} - \frac{1}{r}\right)$$

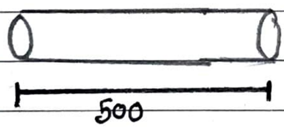
$$v = \underline{\underline{11083\ m/s}} \simeq 11.08\ km/s$$

(b) $m = 2000\ kg$

$$E_{req} = \frac{1}{2}mv^2 \approx 1.23 \times 10^{11}\ J$$

$$E = 1.23\times10^{11}\ J = \frac{1.23\times10^{11}}{4.2\times10^{9}}\ ton = \underline{\underline{29.25}}\ \text{tons of TNT}$$

(c)



$V_0 = 0$, $x - x_0 = 500m$

$v = 11083\ m/s$

$$x - x_0 = \cancel{v_0}^{\,0}t + \frac{a}{2}t^2 \Rightarrow \frac{500\times2}{a} = t^2$$ (1D constant acceleration)

$$a = \frac{v-0}{t} \Rightarrow a^2 = \frac{v^2}{t^2} \Rightarrow a^2 = \frac{v^2\times \cancel{a}}{1000}$$

$$a = \frac{v^2}{1000} \approx \underline{\underline{1.23\times10^{5}\ m/s^2}}$$

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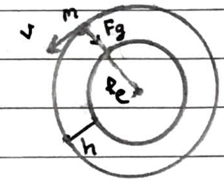
5.9 $m = 1300$ kg

$R_e = 6371$ km

$h = 100$ km

$M_e = 5.972 \times 10^{24}$ kg



(a) $E_i = \underbrace{\frac{1}{2}mv^2}_{K} - \underbrace{\frac{GMm}{(R_e+h)}}_{U}$

$$\frac{mv^2}{(R_e+h)} = \frac{GM_em}{(R_e+h)^2} \Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2(R_e+h)}$$

$$E_i = \frac{-GM_em}{2(R_e+h)} = \underline{\underline{-4 \times 10^{10}\ J}}$$

$$E_f = \frac{-GM_em}{R_e} = \underline{\underline{-8.13 \times 10^{10}\ J}}$$

$$\Delta E = E_f - E_i = \underline{\underline{-4.13 \times 10^{10}\ J}}$$

(b) Total Energy Absorbed $= 4.13 \times 10^{10}$ J

heat of fusion for Aluminium $= 95.3$ kcal/kg $\simeq 3.99 \times 10^2$ J/kg

Energy required to melt the entire satellite.

$$E_{melt} = 3.99 \times 10^2 \times 1300 \simeq 5.2 \times 10^5\ J$$

heat of vaporization for Al $= 2520$ kcal/kg $\simeq 1.06 \times 10^4$ J/kg

Energy required to vaporize the satellite.

$$E_{vap} = 1.06 \times 10^4 \times 1300 \simeq 1.38 \times 10^7\ J$$

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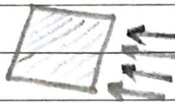
Total energy to melt and then vaporize = $E_{melt} + E_{vap}$

$$= \underline{\underline{1.43 \times 10^7 \, J}} < E_{ab}$$

The energy absorbed is enough to both melt and then vaporize the satellite.

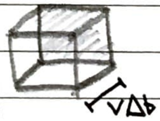
5.11 $m = 1.7 \times 10^{-27}$ kg

 $A = 1 \, m^2$

$u = 4.0 \times 10^5$ m/s

$\rho = 1.0 \times 10^7$ ions / m^3 ← density

Consider a time interval Δt. The volume (of ions) that will stick to the surface is:



$$V = \underbrace{A}_{\text{base}} \underbrace{u \Delta t}_{\text{height}}$$

The ~~charge~~ number of ions occupied in this volume.

$$n = \rho V = \rho A u \Delta t.$$

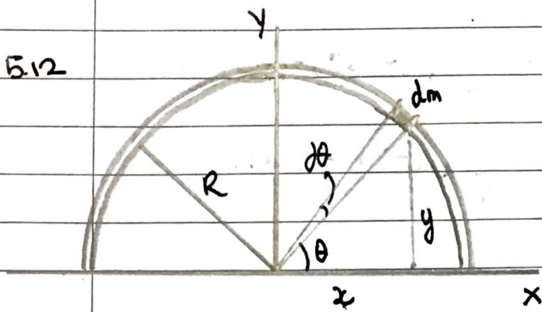
The charge in momentum of the surface is -

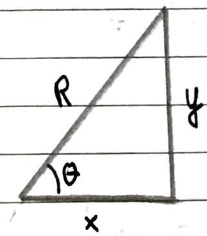
$$\Delta p = nmu - \underbrace{0}_{\text{initially at rest}}$$

$$\Delta p = \rho A m u^2 \Delta t$$

$$F \simeq \frac{\Delta p}{\Delta t} = \frac{\rho A m u^2 \cancel{\Delta t}}{\cancel{\Delta t}} = \rho A m u^2 = \underline{\underline{2.72 \times 10^{-9} \, N}}$$

5.12





Consider a small mass element dm part of the rod.
Since the rod has uniform density

$$\rho = \frac{M}{\pi R} = \frac{dm}{R\,d\theta} \qquad (\theta \text{ measured in radians})$$

length of rod

$$\rightarrow \quad dm = \frac{M}{\pi}\,d\theta$$

x_{CM}: By symmetry, we can conclude that the COM must lie on the y-axis

$$\boxed{x_{CM} = 0}$$

y_{CM}: $$M y_{cm} = \int_0^R y\,dm$$

From the Δ, $y = R\sin\theta$

$$y_{CM} = \frac{1}{\cancel{M}}\int_0^{\pi} R\sin\theta \cdot \frac{\cancel{M}}{\pi}\,d\theta = \frac{R}{\pi}\int_0^{\pi} \sin\theta\,d\theta$$

$$y_{CM} = \frac{R}{\pi}(-\cos\theta)\Big|_0^{\pi} = \frac{R}{\pi}\Big((-\cos\pi)-(-\cos 0)\Big)$$

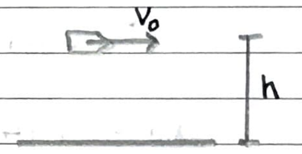
$$= \frac{R}{\pi}\Big((-(-1))-(-1)\Big)$$

$$\boxed{y_{CM} = \frac{2R}{\pi}}$$

$$\therefore (x_{CM}, y_{CM}) = \underline{\underline{\left(0, \frac{2R}{\pi}\right)}}$$

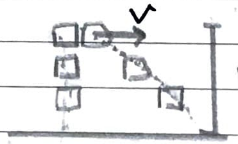
5.13 $h = 2.5 \times 10^4$ m

$V_0 = 5.0 \times 10^3$ m/s



$V_{CM} = V_0 = 5.0 \times 10^3$ m/s

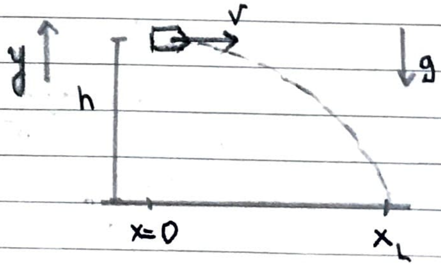
This velocity doesn't change b/c no external forces are acting on the system.



$$V_{CM} = \frac{m(0) + m(v)}{2m} \quad \text{(just after explosion)}$$

$$V_{CM} = \frac{v}{2} \Rightarrow v = 2V_{CM} = 10^4 \text{ m/s}$$

Projectile Motion ($a = -g$)



x: $x = vt$

To find x_L:

y: $y - y_0 = \overset{0}{\cancel{v_y}}t - \frac{1}{2}gt^2$

$0 + 2.5\times10^4 = +\frac{1}{2}\times10\times t^2 \Rightarrow t^2 = 5\times10^3$

$t = \underline{\underline{70.71 \text{ sec}}}$

$x_L = vt = 10^4 \times 70.71 = \underline{\underline{7.07\times10^5 \text{ m}}}$

The second piece is 7.07×10^5 m = 707 km from the point of explosion.