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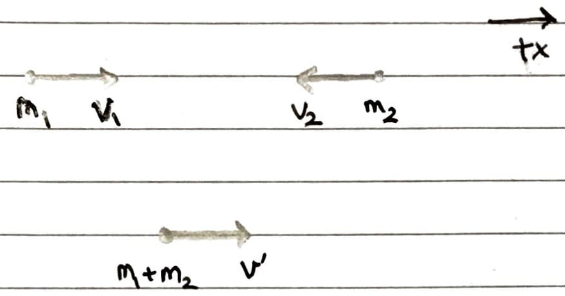
# 8.01 Problem Set - 6

6.1.




a) Conserving momentum

$$P = m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{540 \times 80 + 1400(-80)}{540 + 1400} = -35.46$$

$$\boxed{|v'| = 35.46 \text{ km/h}}$$

b) $$KE_b = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(540)\left(\frac{80 \times 1000}{3600}\right)^2 + \frac{1}{2}(1400)\left(\frac{80 \times 1000}{3600}\right)^2$$

$$\boxed{KE_b = 4.8 \times 10^5 \text{ J}}$$

$$KE_a = \frac{1}{2}(m_1 + m_2)(v')^2 = \frac{1}{2}(\cancel{50}\ 540 + 1400)\left(\frac{35.46 \times 1000}{3600}\right)^2$$

$$\boxed{KE_a = 9.4 \times 10^4 \text{ J}}$$

c) $m_1 = 540$ kg automobile:


$V_0 = +22.2$ m/s
$v' = -9.85$ m/s
$s = 0.60$ m

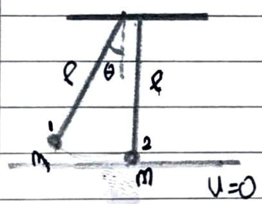
$$a_1 = \frac{(v')^2 - (v_0)^2}{2s} = \underline{\underline{-331 \text{ m/s}^2}}$$

$M_2 = 1400$ kg automobile


$V_0 = +22.2$ m/s
$v' = +9.85$ m/s
$s = 0.60$ m

$$a_2 = \frac{(v')^2 - (v_0)^2}{2s} = \underline{\underline{-331 \text{ m/s}^2}}$$

6. 2 a)




KE of 1 just before collision can be found using the conservation of energy.

$$mgh + 0 = 0 + \frac{1}{2}mv^2$$

$$KE = mgh = mgl(1-\cos\theta)$$

Since the collision is elastic, momentum and KE is conserved.

$p$: $mv_1 + m(0) = mv_1' + mv_2'$

KE: $\frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$

$$mv_1 - mv_1' = mv_2' \Rightarrow v_1 - v_1' = v_2' \longrightarrow ①$$

$$v_1^2 - v_1'^2 = v_2'^2 \Rightarrow (v_1 - v_1')(v_1 + v_1') = (v_2')^2 \longrightarrow ②$$

② / ① : $v_1 + v_1' = v_2' \longrightarrow ③$

① + ③ : $2v_1 = 2v_2' \Rightarrow \boxed{v_2' = v_1}$ and $\boxed{v_1' = 0}$

So, KE is transferred to the other ball. So the ball will reach the same height as the first one started from.

$$mgh + 0 = \frac{1}{2} m v_2'^2 + 0$$

$$mgh = mg\,l(1-\cos\theta) \Rightarrow \boxed{h = l(1-\cos\theta)}$$

b) If the collision is inelastic, only momentum is conserved.

$$mv_1 + 0 = 2mv' \Rightarrow \boxed{v' = \frac{v_1}{2}}$$

$$KE = \frac{1}{2}(2m)(v')^2 = \frac{m v_1^2}{4}$$ (just after collision)

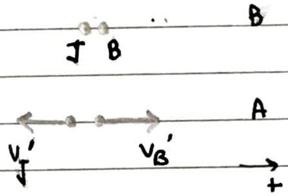
All the KE is converted to potential energy

$$\frac{mv_1^2}{4} = 2mgh \Rightarrow \frac{\cancel{mg}\,l(1-\cos\theta)}{2} = 2\cancel{mg}h$$

$$\boxed{h = \frac{l(1-\cos\theta)}{4}}$$

6.3 a) conserving momentum

$$m_J(0) + m_B(0) = m_J v_J' + m_B v_B'$$




$$100 v_J' + 10 v_B' = 0 \longrightarrow (1)$$

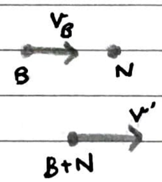
and we know that, $3 = v_B' - v_J' \longrightarrow (2)$

$$v_J' = \frac{-1}{10} v_B'$$

$$3 = v_B' + \frac{1}{10} v_B' \Rightarrow \frac{11 v_B'}{10} = 3 \Rightarrow v_B' = \frac{30}{11} \doteq \underline{\underline{2.73 \text{ m/s}}}$$

$$v_J' = \frac{-3}{11} = \underline{\underline{-0.27 \text{ m/s}}}$$

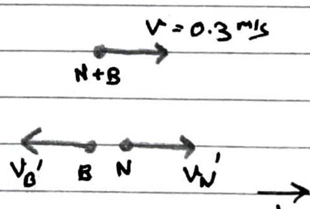
b) conserving momentum,




$$m_B v_B + m_N(0) = (m_B + m_N) v'$$

$$v' = \frac{m_B}{m_B + m_N} v_B = \frac{10}{10+80} \times 2.73 = \underline{\underline{0.3 \text{ m/s}}}$$

c) conserving momentum




$$(m_N + m_B) v = m_B v_B' + m_N v_N' \longrightarrow (1)$$

$$-3 = v_B' - v_N' \longrightarrow (2)$$ (−3 because of choice of direction)

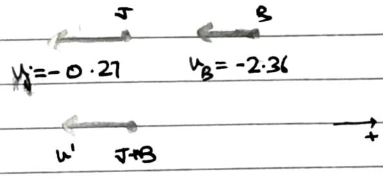
$$27.3 = 10 v_B' + 80 v_N'$$

$$27.3 = 10(v_N' - 3) + 80 v_N'$$

$$90 v_N' - 30 = 27.3 \Rightarrow v_N' = \frac{57.3}{90} = \underline{\underline{0.64 \text{ m/s}}}$$

$$v_B' = v_N' - 3 = \underline{\underline{-2.36 \text{ m/s}}}$$

d) conserving momentum




$$m_J v_J + m_B v_B = (m_J + m_B) u'$$

$$u' = \frac{m_J v_J + m_B v_B}{m_J + m_B}$$

$$= \frac{100(-0.27) + 10(-2.36)}{110} \simeq \underline{\underline{-0.46 \text{ m/s}}}$$

e) $K = \frac{1}{2} m_B (v_B')^2 + \frac{1}{2} m_J (v_J')^2$ (from part (a))

$$= \frac{1}{2}(10)(2.73)^2 + \frac{1}{2}(100)(+0.27)^2 \simeq \underline{\underline{41 \text{ J}}}$$

f) $K = \frac{1}{2} m_J (v_J')^2 + \frac{1}{2}(m_B + m_N)(v')^2$ (from (a) and (b))

$$= \frac{1}{2}(100)(+0.27)^2 + \frac{1}{2}(90)(0.3)^2 \simeq \underline{\underline{7.85 \text{ J}}}$$

g) $K = \frac{1}{2} m_J (v_J')^2 + \frac{1}{2} m_B (v_B')^2 + \frac{1}{2} m_N (v_N')^2$ (from (a), (c))

$$= \frac{1}{2}(100)(+0.27)^2 + \frac{1}{2}(10)(+2.36)^2 + \frac{1}{2}(80)(0.64)^2 = \underline{\underline{47.9 \text{ J}}}$$

h) $K = \frac{1}{2} m_N (v_N')^2 + \frac{1}{2} (m_B + m_J)(v')^2$ (from (c), (d))

$= \frac{1}{2} \times 80 (0.64)^2 + \frac{1}{2} (110)(0.46)^2 \approx \underline{\underline{28\,J}}$

(i) John and Nancy convert the chemical energy stored in their muscles into kinetic energy when they slide the block.

6.4


(CM frame)

a) $V_{CM} = 0 = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} \Rightarrow m_1 u_1 + m_2 u_2 = 0 \rightarrow ①$

$\Downarrow$

$v_2 = -\frac{m_1}{m_2} u_1$

elastic collision:

momentum: $m_1 u_1 + m_2 u_2 = m_1 u_1' + m_2 u_2'$

From ①, $m_1 u_1' + m_2 u_2' = 0 \Rightarrow u_2' = -\frac{m_1}{m_2} u_1'$

KE: $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 (u_1')^2 + \frac{1}{2} m_2 (u_2')^2$

$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 \times \frac{m_1^2}{m_2^2} u_1^2 = \frac{1}{2} m_1 (u_1')^2 + \frac{1}{2} m_2 \times \frac{m_1^2}{m_2^2} (u_1')^2$

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$$\left(\cancel{m_1 + \frac{m_1^2}{m_2}}\right) v_1^2 = \left(\cancel{m_1 + \frac{m_1^2}{m_2}}\right) v_1'^2$$

$$v_1^2 = (v_1')^2 \Rightarrow \boxed{|v_1| = |v_1'|}$$

Similarly, $\boxed{|v_2| = |v_2'|}$

b) Consider the 1-D case, we can conclude that

$$v_1' = \pm v_1 \quad \text{and} \quad v_2' = \pm v_2$$

For the velocity of CM ($v_{CM}$) $= 0$, $v_1'$ and $v_2'$ must be in opposide directions just like $v_1$ and $v_2$.

NOTE - $v_1$ and $v_2$ must have opposite direction for $v_{CM} = 0$.

So the possible directions are,

I $v_1' = v_1$, $v_2' = v_2$ and

II $v_1' = -v_1$, $v_2' = -v_2$

- Case I refers to the case in which the pucks pass through each other without colliding.
- Case II refers to the case in which the ~~po~~ pucks collide and reverse direction.

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c) $KE_b = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2} v_1\right)^2$$

$$= \frac{1}{2} m_1 u_1^2 \left(1 + \frac{m_2 m_1}{(m_2)^2}\right) = \underline{\underline{\frac{1}{2} m_1 v_1^2 \left(1 + \frac{m_1}{m_2}\right)}}$$

d) $KE_a = \frac{1}{2} m_1 (v_1')^2 + \frac{1}{2} m_2 (v_2')^2$

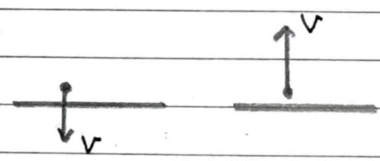
$$= \frac{1}{2} m_1 (v_1')^2 \left(1 + \frac{m_1}{m_2}\right)$$

$KE_b = KE_a$ (elastic collision)

6.5

I Basketball collides with the Ground

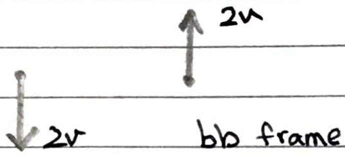
Let $v$ be the velocity just before the bounce, using conservation of energy...




$$mgh = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{2gh}$$

Since, $m_{ground} >> m_{basketball}$, the ball bounces back up with the same velocity $v$.

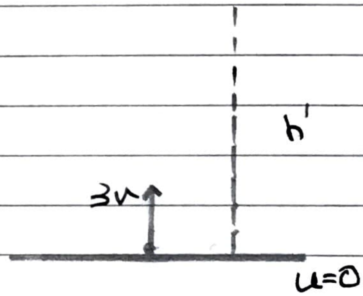
$\text{II}$ Tennisball and Basketball Collision




In the frame of reference of the basketball, the tennis ball approaches it at a speed of $2u$.
Since $m_{bb} >> m_{tb}$, the tennis ball bounces back with the same speed $2u$.
In the ground frame, its speed is $2u + \underbrace{v}_{\text{of basketball}} = 3v$

Using energy conservation

$$\frac{1}{2} \cancel{m_{tb}} (3v)^2 = \cancel{m_{tb}} g h'$$




$$h' = \frac{9v^2}{2g} = \frac{9 \times \cancel{2}gh}{\cancel{2g}} = \underline{\underline{9h}}$$

If $h = 3m$, $\boxed{h' = 27m}$

6.6

$\xrightarrow{}$ 6kg 350 m/s $\qquad$ $\xrightarrow{}$ 2kg 250 m/s $\xrightarrow{}$ 4kg 400 m/s

a) $P_i = 6 \times 350 = \underline{\underline{2100}}$ kg m/s

$P_f = 2 \times 250 + 4 \times 400 = \underline{\underline{2100}}$ kg m/s

Yes, momentum is conserved.

b) $KE_b = \frac{1}{2} \times 6 \times (350)^2 \rightarrow 3.68 \times 10^5$

$KE_a = \frac{1}{2} \times 2 \times (250)^2 + \frac{1}{2} \times 4 \times (400)^2 \approx 3.83 \times 10^5 \, J$

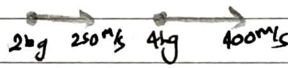
Since $KE_b \neq KE_a$, mechanical energy is not conserved.

(c) Before:

$v_{CM} = 350 \, m/s$

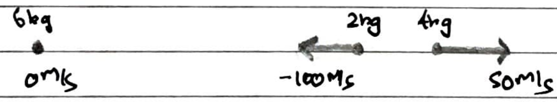

After:

$v_{CM} = \frac{2 \times 250 + 4 \times 400}{6} = 350 \, m/s$




Since the explosive forces are internal, $v_{CM}$ does not change

d)




In the CM frame,

$v_{2kg} = 250 - 350 = -100 \, m/s$

$v_{4kg} = 400 - 350 = 50 \, m/s$

$p = (2)(-100) + 4(50) = 6(0) = \underline{0} \, kg \, m/s$

Makes sense b/c, CM is at rest w.r.t itself

$KE_b = \frac{1}{2}(6)(0)^2 = 0\,J$

$KE_a = \frac{1}{2}(2)(100)^2 + \frac{1}{2}(4)(50)^2 = 15000\,J$

Mechanical energy is not conserved.
None of the previous conclusions changed b/c CM frame is an inertial frame.

6.7a) $\boxed{E = U_0}$ no KE of masses

b) Let $U$ be the potential energy of the spring, since energy is conserved,

$$K_A + K_B + U = U_0 \Rightarrow \boxed{U = U_0 - (K_A + K_B)}$$

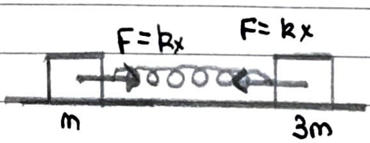
c) There are no external forces on the system so momentum is conserved. Initially, the system is at rest

$$\vec{P} = 0$$

$$m\vec{v}_A + 3m\vec{v}_B = \vec{0} \Rightarrow \boxed{\vec{v}_B = -\frac{1}{3}\vec{v}_A}$$

d) B will move in the opposite direction i.e $-x$ direction

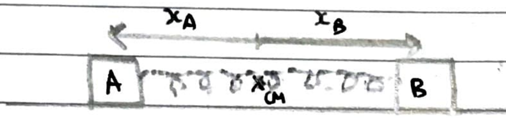
e)



Let $l_0$ be the natural ~~elongatio~~ length of the spring.

The CM doesn't move since no external force acts on the system. In the frame of reference of CM, $(r_{cm} = 0)$

$$m_A x_A + m_B x_B = 0 \Rightarrow \boxed{x_B = -\frac{1}{3} x_A}$$




$$F = m\ddot{x}_A = -k\underbrace{(|x_A| + |x_B| - l_0)}_{\text{elongation of spring}} = -k\left(x_A + \frac{1}{3}x_A - l_0\right)$$

$$m\ddot{x}_A = -k\left(\frac{4}{3}x_A - l_0\right) = -\frac{4k}{3}\left(x_A - \frac{3}{4}l_0\right)$$

$$\ddot{x}_A = \frac{-4k}{3m}\left(x_A - \frac{3}{4}l_0\right)$$

This is of the form $\ddot{x} = -\omega^2(x - x_0)$ which is the eqn. for SHM.

$$\omega^2 = \frac{4k}{3m} \Rightarrow \omega = \sqrt{\frac{4k}{3m}}$$

$$T_A = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{3m}{4k}} = T_A}$$

Since $x_B = -1/3\, x_A$, $T_B = T = 2\pi\sqrt{\frac{3m}{4k}}$

6.9 $m\frac{dv}{dt} = -u\frac{dm}{dt} \Rightarrow m\,dv = -u\,dm$ (− b/c dm is negative)

$dv = -u\frac{dm}{m} \Rightarrow \int dv = -u\int\frac{dm}{m}$

$v_f - v_i = -u\ln\frac{m_f}{m_i} + C$

When $v_f = v_i$ (not moved), no mass is released, $m_f = m_i$

$v_f - v_f = -u\ln 1 + c \Rightarrow c = 0$

$v_f - v_i = -u\ln\frac{m_f}{m_i}$

$u = 2.5$ km/s
$v_f = 8.5$ km/s
$v_i = 8.0$ km/s
$m_i = 10^5$ kg

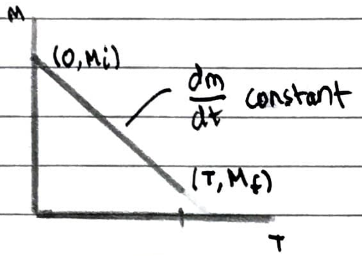
$0.5 = -2.5\ln\left(\frac{m_f}{10^5}\right) \Rightarrow m_f = 10^5 e^{-1/5}$ kg

Fuel Used $= m_f - m_i = 10^5(1 - e^{-1/5}) = \underline{\underline{1.8\times10^4 \text{ kg}}}$

6.10 a) $F_{thr} = u \dfrac{dm}{dt} \Rightarrow u = \dfrac{F_{thr}}{dm/dt}$

$u = \dfrac{34 \times 10^6}{13.8 \times 10^3} \simeq \underline{\underline{2.46 \times 10^3 \ m/s}}$

b)




$\dfrac{M_f - M_i}{T - 0} = -13.8 \times 10^3$

$\dfrac{0.77 \times 10^6 - 2.85 \times 10^6}{T} = -13.8 \times 10^3$

$T = \left( \dfrac{-13.8 \times 10^3}{-2.08 \times 10^6} \right)^{-1} = \dfrac{2.08 \times 10^6}{13.8 \times 10^3} \simeq \underline{\underline{151 \text{ sec}}}$

c) $ma = F_{thr} - mg$

At $t = 0$,

$M_i a = u \cdot \dfrac{dm}{dt} - M_i g = (13.8 \times 10^3)(2.46 \times 10^3) - M_i g$

$a = \dfrac{F_{thr}}{M_i} - g = \underline{\underline{1.9 \ m/s^2}}$

d) At $t = T$, $m = M_f$

$a = \dfrac{F_{thr}}{M_f} - g = \dfrac{34 \times 10^6}{0.77 \times 10^6} - 10 \simeq \underline{\underline{34 \ m/s^2}}$

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e) $v_f - v_i = -u \ln\left(\frac{m_f}{m_i}\right) - gt$

$u = 2.5 \times 10^3$ m/s $\qquad t = T$

$m_f = 0.77 \times 10^6$ kg

$m_i = 2.85 \times 10^6$ kg

$v_i = 0$

$$v_f = -2.5\times10^3 \ln\left(\frac{0.77}{2.85}\right) - 10\times(151) \approx \underline{\underline{1.8 \times 10^3 \text{ m/s}}}$$