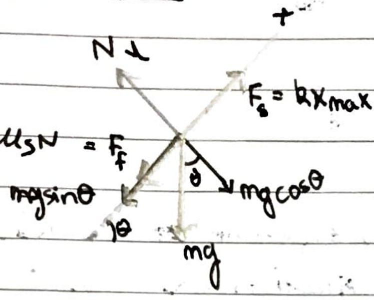
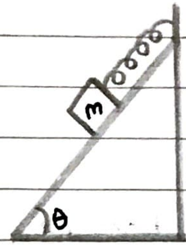
8.01 Exam - 2

1. a,b)





y: $N - mg\cos\theta = m(0) \Rightarrow N = mg\cos\theta$

x: $kx_{max} - \mu_s N - mg\sin\theta = 0$

$kx_{max} = mg(\mu_s\cos\theta + \sin\theta) \Rightarrow \boxed{x_{max} = \frac{mg}{k}(\mu_s\cos\theta + \sin\theta)}$

b) Forces acting on the block:

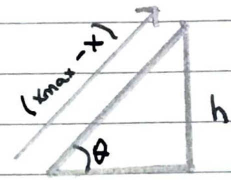
Spring Force (F_s) = kx_{max}
Normal Force (N) = $N = mg\cos\theta$
Gravity (mg) = mg
Friction (F_f) = $\mu_s mg\cos\theta$

c) After the block is gently touched, stat friction switches to kinetic and their is a net force towards the right (towards the spring).

The block keeps accelerating until $F_{net} > 0$ after which it starts to decellerate.

Hence, the block reaches maximum speed when $F_{net} = 0$

$$kx = \mu_k N + mg\sin\theta \Rightarrow \boxed{x = \frac{mg}{k}(\mu_k \cos\theta + \sin\theta)}$$



d) i) $W_g : -mg(x_{max} - x)\sin\theta$

$x_{max} > x$

(ii) $W_s : -\Delta U = -\frac{1}{2}kx^2 + \frac{1}{2}kx_{max}^2$

$$= \frac{1}{2}kx_{max}^2 - \frac{1}{2}kx^2$$

(iii) $W_f : -\mu_k mg\cos\theta\,(x_{max} - x)$

e) Work-Energy Theorem

$$W_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Substitute in d) for $x = 0$. For the block to reach this position, the net work done by all the forces must be ≥ 0

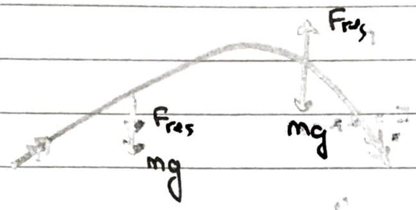
$$W_g + W_s + W_f \geq 0$$

$$-mg\,x_{max}\sin\theta + \frac{1}{2}kx_{max}^2 - \mu_k mg\cos\theta\, x_{max} \geq 0$$

$$\frac{1}{2}kx_{max}^2 \geq mg\,x_{max}(\mu_k\cos\theta + \sin\theta)$$

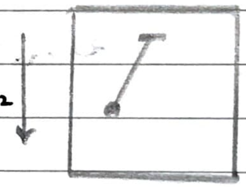
$$\boxed{x_{max} \geq \frac{2mg}{k}(\mu_k\cos\theta + \sin\theta)}$$

2. a)



While going up, the y component of the resistive force is in the downward direction (as mg).
When the object is going down, the direction is opposite. Hence the net force acting is smaller while going down and so it will take longer than 2 sec to come down.

b) In normal gravity, $(a=0)$



$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{10}}$$

When $a = \cdot 5\, m/s$, perceive weight $= \cancel{mg}\ m(g-a)$

$$T' = 2\pi\sqrt{\frac{l}{g-a}} = 2\pi\sqrt{\frac{l}{5}} = \sqrt{2} \times 2\pi\sqrt{\frac{l}{10}}$$

$$\boxed{T' = \sqrt{2}\, T}$$

c) The viscous term will dominate when $v_{term} \ll v_{crit}$

$$mg = c_1 r v_{term} \Rightarrow v_{term} = \frac{mg}{c_1 r}$$

$$c_1 r v_t \gg c_2 r^2 v_t^2 \Rightarrow v_t \ll \underbrace{\frac{c_1}{c_2 r}}_{v_{crit}}$$

$$\frac{mg}{c_1 r} \ll \frac{c_1}{c_2 r}$$

$$m = \rho . \frac{4}{3} \pi r^3$$

$$\frac{4\pi r^3 \rho g}{3c_1} \ll \frac{c_1}{c_2} \Rightarrow r^3 \ll \frac{3c_1^2}{4\pi \rho g c_2}$$

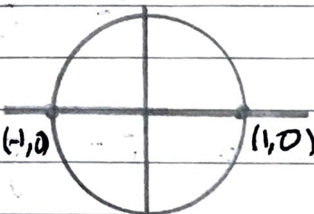
$$\boxed{r \ll \left(\frac{3c_1^2}{4\pi\rho g c_2}\right)^{1/3}}$$

d) $x = -0.3 \sin(2t + \pi/4)$

$\omega = 2$, $T = \frac{2\pi}{\omega} = \pi$ sec

$f = 1/T = 1/\pi$ Hz

e) $\frac{dx}{dt} = -0.6 \cos(2t + \pi/4)$



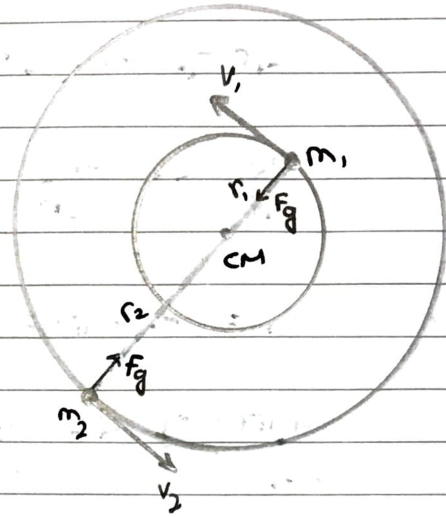
The speed is maximum when $\cos(2t + \pi/4) = \pm 1$

$2t + \pi/4 = n\pi$ (where $n = 0, \pm 1, \pm 2, \ldots$)

$2t = \pi\left(n - \frac{1}{4}\right)$

$$\boxed{t = \frac{\pi}{2}\left(n - \frac{1}{4}\right)}$$ where $n = 1, 2 \ldots$ $(t \geq 0)$

3. a) Since there are no external forces, the center of mass remains still.
So both planets should move together i.e have the same angular velocity.



b) $$F_g = \frac{G m_1 m_2}{(r_1+r_2)^2}$$

c) $$a_{m_1} = \frac{F_g}{m_1} = \frac{G m_2}{(r_1+r_2)^2}$$

$$a_{m_2} = \frac{F_g}{m_2} = \frac{G m_1}{(r_1+r_2)^2}$$

d) $$T = \frac{2\pi r_1}{v_1} \quad (\text{for } m_1)$$

$$\frac{v_1^2}{r_1} = a_{m_1} = \frac{G m_2}{(r_1+r_2)^2} \Rightarrow v_1^2 = \frac{G m_2 r_1}{(r_1+r_2)^2}$$

$$T^2 = \frac{(2\pi r_1)^2}{v_1^2} = \frac{4\pi r_1^2}{v_1^2}$$

$$T^2 = \frac{4\pi r_1^2}{G m_2 r_1}(r_1+r_2)^2 = \frac{4\pi r_1}{G m_2}(r_1+r_2)^2$$

$m_1 r_1 = m_2 r_2$ (CM is fixed at origin)

$$\frac{r_2}{r_1} = \frac{m_1}{m_2} \Rightarrow \frac{r_2}{r_1} + 1 = \frac{m_1}{m_2} + 1$$

$$\frac{r_1 + r_2}{r_1} = \frac{m_1 + m_2}{m_2} \rightarrow \frac{r_1}{m_2} = \frac{r_1 + r_2}{m_1 + m_2}$$

$$T^2 = \frac{4\pi (r_1+r_2)^3}{G(m_1+m_2)} \rightarrow \boxed{T = \left(\frac{4\pi (r_1+r_2)^3}{G(m_1+m_2)}\right)^{1/2}}$$