8.01 Problem Set - 3

3.1 $\vec{F_1} = 2i - 5j + 3k$

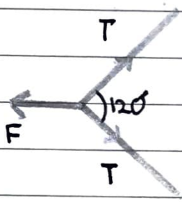
$\vec{F_2} = -4i + 8j + k$

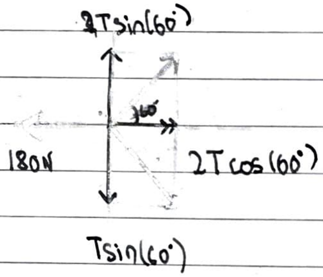
(a) $\vec{F_1} + \vec{F_2} = -2i + 3j + 4k = \vec{F}_{net}$

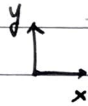
(b) $\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{-2i + 3j + 4k}{6} = \frac{-1}{3}i + \frac{1}{2}j + \frac{2}{3}k$

$|\vec{a}| = \sqrt{\frac{1}{9} + \frac{1}{4} + \frac{4}{9}} = \sqrt{\frac{4+9+16}{36}} = \frac{\sqrt{29}}{6}\ m/s^2$

3.2





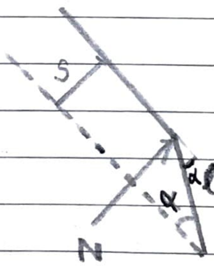


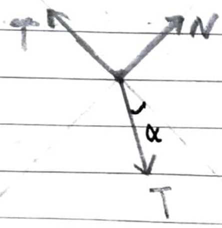
y: $T\sin(60) - T\sin(60) = m(0) \Rightarrow 0 = 0$

x: $2T\cos(60) - 180 = m(0)$

$\boxed{T = 180N}$

3.3





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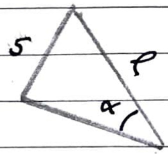
x: $T - T\cos\alpha = m\underset{0}{\cancel{a_x}}$

y: $N - T\sin\alpha = m\underset{0}{\cancel{a_y}}$



for $s \ll l$,
$\cos\alpha \approx 1$

$\sin\alpha = \frac{s}{l}$

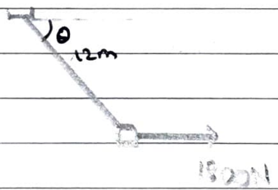
$\Rightarrow N - \frac{Ts}{l} = m(0)$

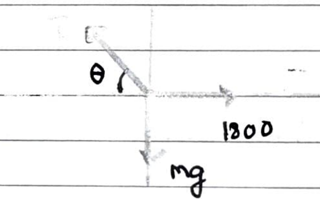
~~Aft~~ $\boxed{T = \frac{Nl}{s}}$

(b) $N = 150N$, $l = 1.5m$, $s = 2.0cm = 0.02$ m

$T = \frac{150 \times 1.5}{0.02} N = 11250 N$

3.4





y: $T\sin\theta - mg = m(0)$

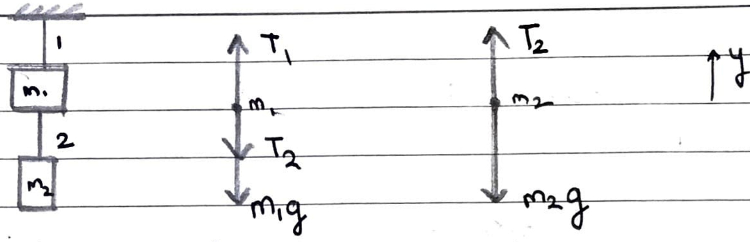
x: $1800 - T\cos\theta = m(0)$

$T\sin\theta = 2000 \times 10$
$T\cos\theta = 1800$

$\boxed{84.86^\circ}$

$\tan\theta = \frac{2000 \times 10}{1800} = \frac{200}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{200}{18}\right) = \underline{\underline{1.48 \text{ rad}}}$

3.5



$\underline{\underline{m_2}}$ $T_2 - m_2g = m_2(0)$

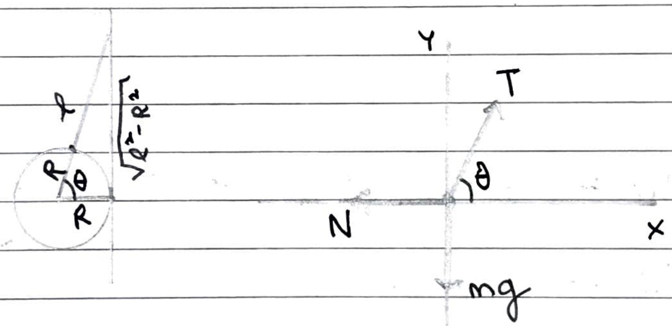
$T_2 = m_2 g = \underline{\underline{30N}} \Rightarrow \boxed{T_2 = 30N}$

$\underline{\underline{m_1}}$ $T_1 - T_2 - m_1g = m_1(0)$

$T_1 = T_2 + m_1g = 30 + 100$

$\boxed{T_1 = 130N}$

3.6



$x: \; T\cos\theta - N = m(0)$

$y: \; T\sin\theta - mg = m(0)$

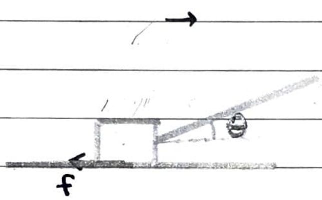
$T\sin\theta = mg \Rightarrow T = \dfrac{mg}{\sin\theta}$

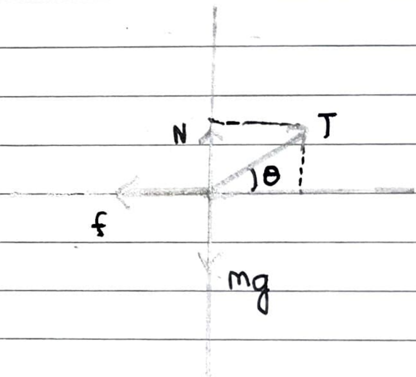
$$\frac{mg}{\sin\theta} \cdot \cos\theta = N$$

$$N = mg\cot\theta = \frac{mgR}{\sqrt{l_1^2 - R^2}} \quad , \; l_1 = (R + l)$$

$$\rightarrow \lim_{l \to \infty} \frac{mgR}{\sqrt{l_1^2 - R^2}} = 0$$

3.7





$$x: \; T\cos\theta = f = \mu_k N \implies N = \frac{T\cos\theta}{\mu_k}$$

$$y: \; T\sin\theta + N - mg = 0$$

$$T\sin\theta + \frac{T\cos\theta}{\mu_k} = mg$$

$$T(\mu_k \sin\theta + \cos\theta) = \mu_k mg$$

$$T = \frac{\mu_k mg}{(\mu_k \sin\theta + \cos\theta)}$$

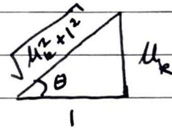
$0 < \theta < \pi/2$

$\frac{dT}{d\theta}$ T is minimum when denominator is the greatest.

$$\frac{d}{d\theta}\left(\mu_k \sin\theta + \cos\theta\right) = 0$$

$$\mu_k \cos\theta - \sin\theta = 0 \Rightarrow \boxed{\tan\theta = \mu_k}$$

$$T_{min} = \frac{\mu_k m g}{(\mu_k \sin\theta + \cos\theta)} = \frac{\tan\theta \, mg}{(\sin^2\theta + \cos^2\theta)} \cdot \cos\theta$$

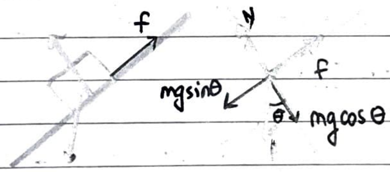


$$T_{min} = mg\sin\theta = \underline{\underline{\frac{\mu_k mg}{\sqrt{\mu_k^2+1}}}}$$ minimum

ends $T(0^+) = \mu_k mg$

$T(\pi/2^-) = mg$

3.8



$N = mg\cos\theta$

$\mu_s N = mg\sin\theta$

$\boxed{\mu_s = \tan\theta}$

θ	μ_s
$20^\circ \pm 2^\circ$	0.36 ± 0.04
$18^\circ \pm 2^\circ$	0.32 ± 0.04

errors:

$\tan(22^\circ) - \tan(20^\circ) \approx 0.04$

$\tan(20^\circ) - \tan(18^\circ) \approx 0.04$

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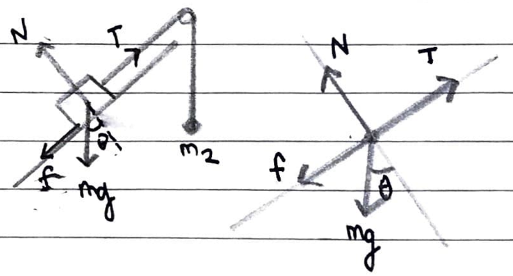


$N = m_1 g\cos\theta$

$T = m_2 g$

$T = f + m_1 g\sin\theta$

$m_2 g = \mu_s m_1 g\cos\theta + m_1 g\sin\theta$

$$\boxed{\mu_s = \frac{m_2 - m_1\sin\theta}{m_1\cos\theta}}$$

m_2	μ_s
270 ± 25 g	0.43 ± 0.10
245 ± 15 g	0.36 ± 0.07

$m_1 = 361 \pm 1 g$

$\theta = 20^\circ \pm 1^\circ$

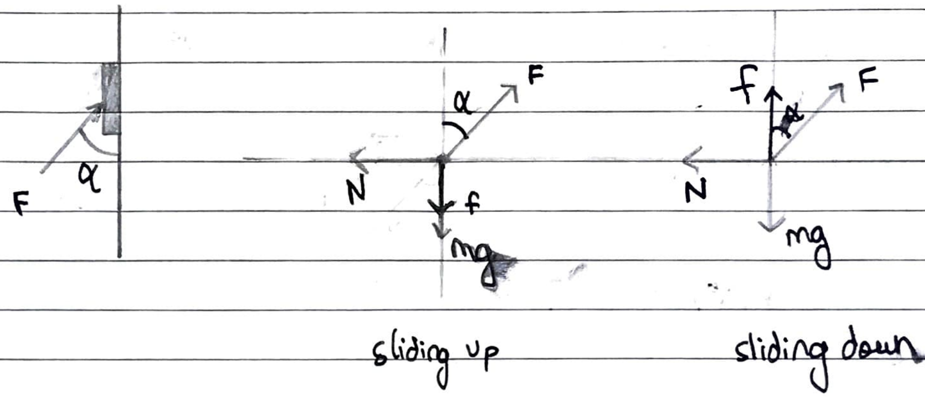
errors:

$$0.10 = \frac{(270+25)-(361-1)\sin(20-1)}{(361-1)\cos(20+1)} - 0.43$$

$$0.07 = \frac{(245+15)-(361-1)\sin(20-1)}{(361-1)\cos(20+1)} - 0.36$$

The values of μ_s are consistent with each other.

3.9 a)



b) sliding up

x: $F\sin\alpha - N = 0 \Rightarrow N = F\sin\alpha$

y: $F\cos\alpha - f - mg = 0$

$F\cos\alpha - \mu F\sin\alpha - mg = 0$

$F(\cos\alpha - \mu\sin\alpha) = mg \Rightarrow \boxed{F = \dfrac{mg}{(\cos\alpha - \mu\sin\alpha)}}$

sliding down

$F\cos\alpha + f = mg$

$F(\cos\alpha + \mu\sin\alpha) = mg \Rightarrow \boxed{F = \dfrac{mg}{\cos\alpha + \mu\sin\alpha}}$

_/_/_

c) friction is zero, when

$$F\cos\alpha - mg = 0 \Rightarrow \boxed{F = \frac{mg}{\cos\alpha}}.$$

$$F(0^{\circ}) = \frac{mg}{\cos(0^{\circ})} = \underline{\underline{mg}}$$

$$F(\pi/2^{-}) = \frac{mg}{\cos(\pi/2^{-})} = \frac{mg}{0^{+}} = \underline{\underline{\infty}}$$

d)

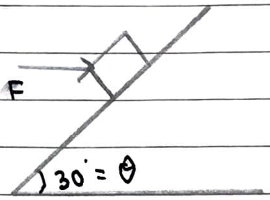
$$F = \frac{mg}{\cos\alpha - \mu\sin\alpha}$$ As μ increase, F increases.

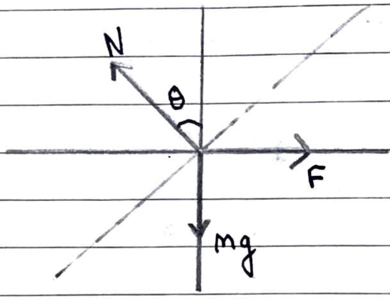
But when $\cos\alpha - \mu\sin\alpha = 0$, f becomes infinite

$$\cos\alpha - \mu\sin\alpha = 0 \Rightarrow \boxed{\mu = \cot\alpha}$$

3.10 Friction force only depends on N and μ. It is independent of the width (surface area) of the tires. Wide tires have larger surface area and so are less likely to melt. Melting decreases the coefficient of friction which causes the car to slip on the road. Wide tire provide better stability.

3.11 a)





b) $mg = 60g = \underline{\underline{588\ N}}$

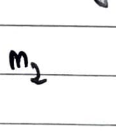
$y: \; N\cos\theta - mg = 0 \;\Rightarrow\; N = \dfrac{mg}{\cos\theta}$

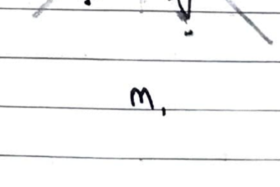
$N = \underline{\underline{679\ N}}$

$x: \; F - N\sin\theta = 0 \;\Rightarrow\; F = N\sin\theta$

$F = \underline{\underline{339\ N}}$

3.12





m_2 m_1

$m_1 = 1.5kg$
$m_2 = 3kg$
$\theta = 35°$
$\mu_k = 0.40$

$\underline{m_2}$: $m_2 g - T = m_2 a \longrightarrow (1)$

$\underline{m_{1y}}$: $N - m_2 g\cos\theta = m_1(0)$

$N = m_1 g\cos\theta$

$\underline{m_{1x}}$: $T - \mu_k N - m_1 g\sin\theta = m_1 a \longrightarrow (2)$

(1) + (2)

$$m_2 g - \mu_k m_1 g\cos\theta - m_1 g\sin\theta = (m_1+m_2)a$$

$$\cancel{a = g(m_2 - \mu_k m_1 - m_1)}$$

$$a = \frac{g(m_2 - \mu_k m_1\cos\theta - m_1\sin\theta)}{(m_1+m_2)}$$

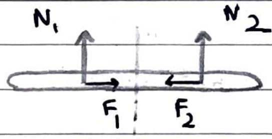
$$a = \frac{9.8}{4.5}(3 - 0.4\times1.5\times\cos(35^\circ) - 1.5\sin(35^\circ))$$

$$\boxed{a = 3.58 \ m/s^2}$$

This confirms our assumption that ~~a~~ m_1 is accelerating uphill. ~~i.e.~~ (or)

$$m_2 g > \mu_k m_1 g\cos\theta + m_1 g\sin\theta$$

3.13



$F_1 \rightarrow$ friction force ON the rod by 1.

※ The fraction of weight supported by each finger can be different. The figer closest to the center will bear a larger fraction of weight, so will have greater normal force.

$$N_1 + N_2 = mg.$$

Consider the case when fingers are same distance from the center ($N_1 = N_2$), and finger 1 just starts sliding. Since $\mu_s > \mu_k$,

$$F_1 = \mu_k N_1 < \mu_s N_2 = F_2.$$

As ~~the~~ finger 1 moves, $F_1 = \mu_k N_1$ increases (N_1 increases) and $F_2 = \mu_s N_2$ starts to decrease (N_2 decreases). Finger 1 will move until,

$$F_1 = \mu_k N_1 = \mu_s N_2 = F_2$$

After this, finger 2 will no longer be able to sustain F_1 and will start to move and finger 1 will stop. The whole process begins again.

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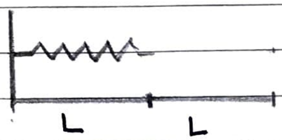
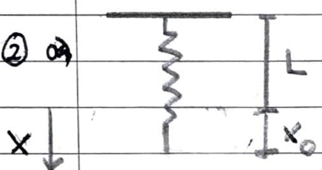
3.14. 

$L = 0.15\ m$
$k = 150\ N/m$

① a) $F = kx = kL = 22.5N \quad (x = L)$

b) $F = -\frac{kL}{2} = -11.25N \quad (x = -\frac{L}{2})$

② a) 

Because of gravity the natural length of the spring is more.

$$kx_0 = mg$$

$$m\ddot{x} = -kx + mg$$
$$= -k\cancel{g}x + kx_0$$
$$m\ddot{x} = -k(x - x_0)$$

$$m\ddot{x} = -k(x - x_0) = -kx'_0 \rightarrow \text{measured from } x_0.$$

$$x' = x - x_0 \Rightarrow \ddot{x}' = \ddot{x}$$

$$\boxed{\ddot{x}' = -\frac{k}{m}x'} \qquad \omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

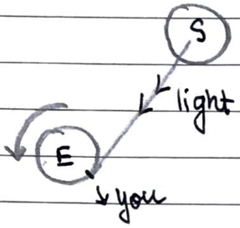
a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{150}{3}} = \sqrt{50} = \underline{\underline{5\sqrt{2}\ rad/s}}$

b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{5\sqrt{2}} = \underline{\underline{0.89\ sec}}$

c) $f = T^{-1} = \underline{\underline{1.13\ Hz}}$

3. 15 Remember: The earth is rotating NOT the sun.

Aiming directly at the sun will hit the sun dead center.



Ignoring atmospheric refraction, light from the Sun travels in a straight path and reaches you.

Although you are viewing the light which was emitted 8 minutes ago by the sun, it doesn't matter because at that moment there is a direct path b/w you and the sun.