6.042J Problem Set - I

1.(a) $\exists x \in X, \ S(x) \wedge A(x)$

(b) $\forall x \in X, \ T(x) \wedge S(x) \Rightarrow A(x)$

(c) $\neg(\exists x \in X, \ T(x) \Rightarrow \neg A(x)$

(d) $\exists x,y,z \in X, \ \underbrace{(\neg E(x,y) \wedge \neg E(y,z) \wedge \neg E(x,z))}_{\text{different people}}$

$\wedge T(x) \wedge \neg S(x) \wedge T(y) \wedge \neg S(y) \wedge T(z) \wedge \neg S(z)$

2.(a) $\neg(P \vee (Q \wedge R)) = \neg P \wedge (\neg Q \vee \neg R)$

P	Q	R	$\neg(P \vee (Q \wedge R))$	$\neg P \wedge (\neg Q \vee \neg R)$
T	T	T	F	F
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

The above statements are equivalent.

(b) $\neg(P \wedge (Q \vee R)) = \neg P \vee (\neg Q \vee \neg R)$

P	Q	R	$\neg(P \wedge (Q \vee R))$	$\neg P \vee (\neg Q \vee \neg R)$
T	T	T	F	F
T	T	F	F	T
T	F	T	F	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	P	F	T	T

The above statements are NOT equivalent.

3.(a) (i) $A \wedge B = \neg(A \text{ nand } B)$

(ii) $A \vee B \Rightarrow$ both A and B must NOT be False

$= \neg A \text{ nand } \neg B$

$\left[\neg(\neg A \wedge \neg B) = A \vee B\right]$

(iii) $A \Rightarrow B = \neg A \vee B$

$= A \text{ nand } \neg B$

(b) $\neg A = \neg(A \wedge A) = A \text{ nand } A$

(c) $T = (A \text{ nand } \neg A) = (A \text{ nand } (A \text{ nand } A))$

$\therefore$ one of them will be false

$F = \neg T$

$= (A \text{ nand } (A \text{ nand } A)) \text{ nand } (A \text{ nand } (A \text{ nand } A))$

4. Fake Coin - strategy

1) Divide 12 into 2 groups of 6 and weigh them.
2) The lighter batch has the fake coin.
3) Divide the lighter batch into 2 groups of 3. "
4) The lighter batch has the fake coin.
5) Now, take any 2 coins from the group. There are 2 case:
5.1) If the balance scale doesn't move, the coins on the scale are equal., so the fake coin ~~*~~ is the remaining
5.2) If the scale tilts, the lighter coin is the fake.

5. r is irrational $\Rightarrow$ $r^{1/5}$ is irrational

(by contrapositive)

$r^{1/5}$ is rational $\Rightarrow$ r is rational.

Assume ($r^{1/5}$ is rational) for purpose of proving implication.

$\therefore$ $r^{1/5}$ can be expressed as a fraction

$$r^{1/5} = \frac{a}{b} \Rightarrow r = \frac{a^5}{b^5}$$

$\therefore$ r can be represented as a fraction, so it is rational.

Thus, we have proved if r is irrational, then $r^{1/5}$ is irrational. $\square$

6. $w^2 + x^2 + y^2 = z^2$

To prove: z is even $\Leftrightarrow$ w, x, y are even

Unique cases: No odd, 1 odd, 2 odd, all odd

Case 1: $w, x, y \in$ Even

$$z^2 = (2i)^2 + (2j)^2 + (2k)^2 = 4(i^2+j^2+k^2)$$
$$\Rightarrow z = 2\sqrt{(\ldots)} \Rightarrow z \text{ is even}$$

Assume the root goes away

Case 2: w is odd, x, y are even

$$z^2 = (2i+1)^2 + (2j)^2 + (2k)^2 = \underbrace{4i^2 + 4j^2 + 4k^2 + 4i}_{\text{even}} + \underbrace{1}_{\text{odd}}$$

z^2 = odd $\Rightarrow$ z is odd not even

Case 3: w, x is odd, y is even

$$z^2 = (2i+1)^2 + (2j+1)^2 + (2k)^2$$

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$= 4i^2 + 4j^2 + 4k^2 + 4i + 4j + 2$

z^2 is not a multiple of 4 $\Rightarrow$ z cannot be even.

Case 4: $w, x, y \in$ Odd

$z^2 = (2i+1)^2 + (2j+1)^2 + (2k+1)^2$

$= \underbrace{4i^2 + 4j^2 + 4k^2 + 4(i+j+k)}_{\text{even}} + \underbrace{3}_{\text{odd}} = \text{odd}$

z^2 is odd $\Rightarrow$ z is not even

$\therefore$ By considering all cases, z is even $\Leftrightarrow$ w, x, y are all even.