8.01 Problem Set - II

2.1 $v_1 = 15 - 10t$ $\qquad \cancel{V_2 =}$

$x_1 = 15t - 5t^2$

The stone reaches 11m, at time t,

$$11 = 15t - 5t^2 \Rightarrow 5t^2 - 15t + 11 = 0$$

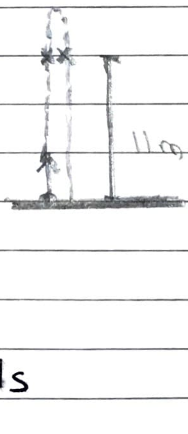
$$t = \frac{15 \pm \sqrt{15^2 - 220}}{10} = \frac{15 \pm \sqrt{5}}{10} \text{ s}$$

The second stone must reach $x = 11$ m at $(t-1)$ sec

$v_2 = v_0 - 10t$ $\qquad$ (where $t > 1$

$x_2 = v_0 t - 5t^2$

$$11 = v_0\left(\frac{5 \pm \sqrt{5}}{10}\right) - 5\left(\frac{5 \pm \sqrt{5}}{10}\right)^2$$



I $\quad v_0 = \left(\frac{5(5 \pm \sqrt{5})^2}{100} + 11\right)\frac{10}{(5+\sqrt{5})}$

$$v_0 = \left(\frac{5(5+\sqrt{5})}{10} + \frac{110}{(5+\sqrt{5})}\right) = \underline{\underline{18.825}} \text{ m/s}$$

II $\quad v_0 = \left(\frac{5(5-\sqrt{5})}{10} + \frac{110}{5-\sqrt{5}}\right) = \underline{\underline{41.18}} \text{ m/s}$

Both these velocities are plausible as there are 2 times ~~in~~ when the first stone crosses 11m. (while going up and down).

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(b) If thrown after 1.30 s.

The second stone must reach 11m in

$$\frac{15 \pm \sqrt{5}}{10} - 1.3 = \frac{2 \pm \sqrt{5}}{10}$$

So only the case when the first stone reaches the highest and going down is consided.

$V = V_0 - 10t$

$x = V_0 t - 5t^2$

$$11 = V_0\left(\frac{2 \pm \sqrt{5}}{10}\right) - 5\left(\frac{2+\sqrt{5}}{10}\right)^2$$

$$V_0 = \left(\frac{5(2+\sqrt{5})}{10} + \frac{110}{(2+\sqrt{5})}\right) = \underline{\underline{28.09 \text{ m/s}}}$$

2.2 $h_1 = 3.000 \pm 0.003 \text{ m}$ (0.1%)

$t_1 = 0.781 \pm 0.002 \text{ s}$ (0.26%)

$t_1^2 = 0.610 \pm 0.003 \text{ s}^2$ (0.49%)

$x - X_0 = + h_1$ m

$V_0 = 0$

$$x = x_0 + \underbrace{V_0 t_1}_{0} + \frac{1}{2} a t_1^2$$

$$\Rightarrow a = \frac{2(x - x_0)}{t_1^2}$$

$$\boxed{a = 9.836 \pm 0.058 \ m/s^2} \qquad (0.59\%)$$

$h_2 = 1.500 \pm 0.003 \qquad (0.2\%)$

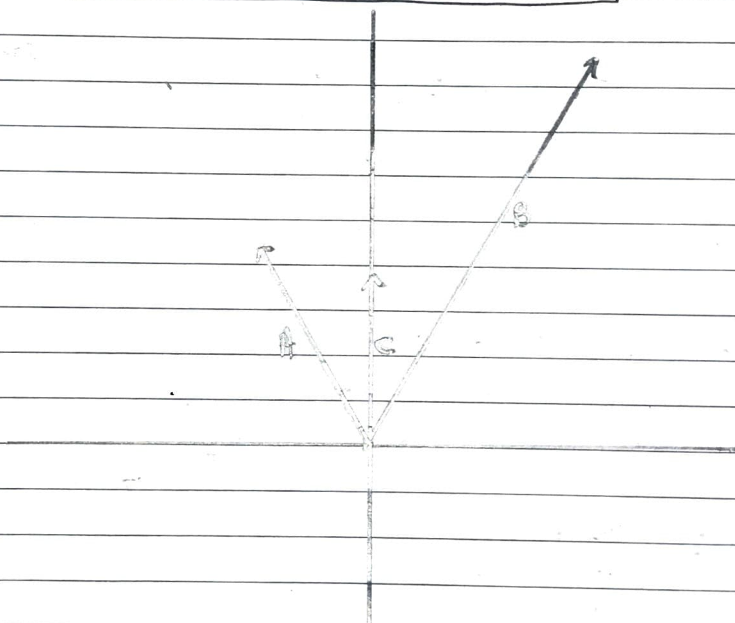
$t_2 = 0.551 \pm 0.002 \qquad (0.36\%)$

$t_2^2 = 0.304 \pm 0.002 \qquad (0.66\%)$

$$a = \frac{2h_2}{t_2^2}$$

$$\boxed{a = 9.868 \pm 0.085 \ m/s^2} \qquad (0.86\%)$$

2.3



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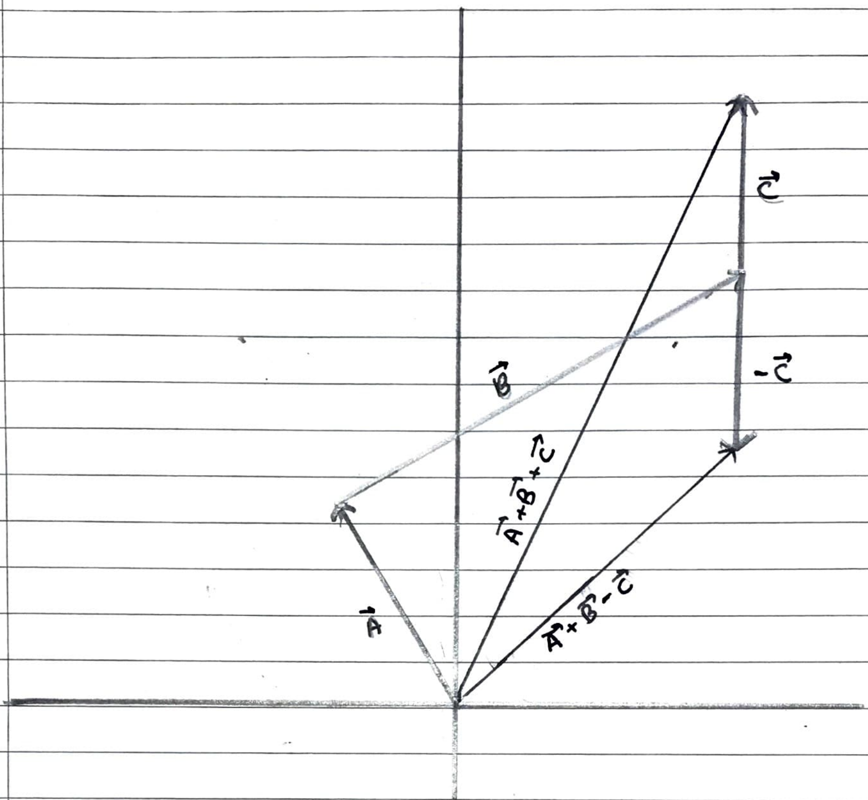
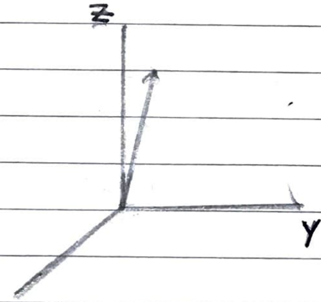


2.4 $\vec{A} = 5.0i - 3.0j + 1.0k$

$|A| = \sqrt{(5)^2 + (-3)^2 + (1)^2} = \underline{\underline{\sqrt{35}}}$



x-axis $\cos\theta_x = \frac{A_x}{|A|} = \frac{5}{\sqrt{35}}$

$\theta_x = \underline{\underline{32.31^\circ}}$

y-axis $\cos\theta_y = \frac{A_y}{|A|} = \frac{-3}{\sqrt{35}}$

$\theta_y = \underline{\underline{120.47^\circ}}$

Z-axis $\cos\theta_z = \dfrac{A_z}{|A|} = \dfrac{1}{\sqrt{35}}$

$$\theta_z = \underline{\underline{80.27^\circ}}$$

2.5 $A = 3i - 6j + 2k$

$|A| = \sqrt{9+36+4} = 7$

$B = \left(\dfrac{3i - 6j + 2k}{7}\right) \times 2 = \underline{\underline{\dfrac{6}{7}i - \dfrac{12}{7}j + \dfrac{4}{7}k}}$

2.6 Minimum Distance :

$d = 8$ km

Maximum Distance

$d = 70$ km

2.7 $A = 2\hat{x} - 3\hat{y}$

$B = -\hat{x} + a\hat{y} - 5\hat{z}$

$A \cdot B = 0$

$-2 - 3a = 0 \Rightarrow \boxed{a = \dfrac{-2}{3}}$

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2.8 $A = -5i - 3j + k$

$B = 2i + j - 3k$

(a) $A + B = (-5+2)i + (-3+1)j + (1-3)k$

$= -3i - 2j - 2k$ //

(b) $A - B = -5i - 3j + k - 2i - j + 3k$

$= (-5-2)i + (-3-1)j + (1+3)k$

$= -7i - 4j + 4k$ //

(c) $2A - 3B = -10i - 6j + 2k - 6i - 3j + 9k$

$= (-10-6)i + (-6-3)j + (2+9)k$

$= -16i - 9j + 11k$ //

(d) $A \cdot B = (-5) \times 2 + (-3) \times 1 + (1) \times -3$

$= -10 - 3 - 3 = -16$ //

$B \cdot A = 2 \times (-5) + (1)(-3) + (-3) \times 1$

$= -10 - 3 - 3 = -16$ //

(e) $\vec{A} \times B$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -5 & -3 & 1 \\ 2 & 1 & -3 \end{vmatrix}$$

$A \times B = (9-1)\hat{x} + -(15-2)\hat{y} + (-5+6)\hat{z}$

$= 8\hat{x} - 13\hat{y} + \hat{z}$ //

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 1 & -3 \\ -5 & -3 & 1 \end{vmatrix}$$

$B \times A = (1-9)\hat{x} - (2-15)\hat{y} + (-6+5)\hat{z}$

$= -8\hat{x} + 13\hat{y} - \hat{z}$ //

$A \times B = -(B \times A)$

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2.9 $A = 2\hat{x} - 3\hat{y}$

$B = -\hat{x} + 4\hat{y} - 5\hat{z}$

we can find 2 perpendicular vectors by finding $A\times B$ and $B\times A$.

$A\times B$:

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & -3 & 0 \\ -1 & 4 & -5 \end{vmatrix} \qquad A\times B = (15)\hat{x} - (-10)\hat{y} + (8-3)\hat{z}$$

$$= 15\hat{x} + 10\hat{y} + 5\hat{z}$$

$$\cancel{A}B\times A = -(A\times B) = -15\hat{x} - 10\hat{y} - 5\hat{z}$$

$$u_1 = \frac{A\times B}{|A\times B|} = \frac{1}{5\sqrt{14}}(15\hat{x} + 10\hat{y} + 5\hat{z}) = \underline{\frac{3}{\sqrt{14}}\hat{x} + \frac{2}{\sqrt{14}}\hat{y} + \frac{1}{\sqrt{14}}\hat{z}}$$

$$u_2 = -u_1 = \underline{\frac{-3}{\sqrt{14}}\hat{x} - \frac{2}{\sqrt{14}}\hat{y} - \frac{1}{\sqrt{14}}\hat{z}}$$

2.10 $r = (6-2t)\hat{x} + (3+4t-6t^2)\hat{y} - (1+3t-2t^2)\hat{z}$

(a) $\vec{v} = \frac{dr}{dt} = -2\hat{x} + (4-12t)\hat{y} - (3-4t)\hat{z}$ //

$t = 3$,

$$v = -2\hat{x} - 32\hat{y} + 9\hat{z} \quad m/s$$

(b) Speed $= \sqrt{(-2)^2 + (-32)^2 + (9)^2} = \underline{33.302} \ m/s$

(c) $a = \frac{dv}{dt} = -12\hat{y} - 4\hat{z}$

$t = 3$, $|a| = \sqrt{144 + 4^2} = \underline{12.64 \ m/s^2}$

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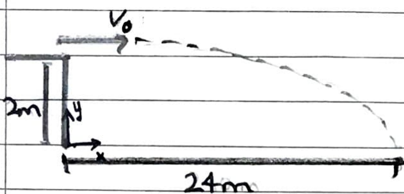


$g = 10\ m/s^2$

$x = V_0 t$

$V_x = V_0$

$v_y = -gt$

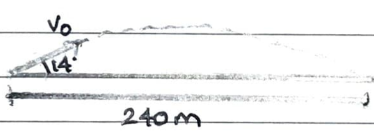
$y = 2 - 5t^2$

$24 = V_0 t_f \Rightarrow t_f = \frac{24}{V_0}$

At $t = t_f$, $y = 0$

$0 = 2 - 5(t_f)^2 \Rightarrow \left(\frac{24}{V_0}\right)^2 = \frac{2}{5} \Rightarrow V_0 = 12\sqrt{10}\ m/s$

2.12a)



$t_{flight} = \frac{2 V_0 \sin\theta}{g}$

$x = V_0 \cos\theta \left(\frac{2 V_0 \sin\theta}{g}\right) = \frac{2 V_0^2 \cos\theta \sin\theta}{g}$

$V_0 = \sqrt{\frac{240 \cdot g}{2 \cos\theta \sin\theta}} = \sqrt{\frac{2400}{2\cos(14°)\sin(14°)}} = 71.5\ m/s$

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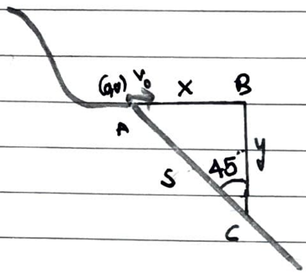
b) $x = \dfrac{2(72.1)^2 \sin(14°)\cos(14°)}{10} = \underline{\underline{244.045\ m}}$

$\Delta x = 4.045\ m$

c) $x = \dfrac{2(71.5)^2 \sin(14.5)\cos(14.5)}{g} = \underline{\underline{247.85\ m}}$

$\Delta x = 7.85\ m$

2.13 a)



$v_0 = 110\ km/h = 30.56\ m/s$

$x = v_0 t$

$y = -\frac{1}{2} g t^2$

From ΔABC,

$x = y \Rightarrow v_0 t = \frac{1}{2} g t^2$

$t = \dfrac{2v_0}{g} = 6.11\ s$

$x = 30.56 \times 6.11 = \underline{\underline{186.73}}\ m$

$s = \sqrt{x^2 + y^2} = \underline{\underline{264.07\ m}}$

b) Air ~~redu~~ resistance reduces the velocity of the skier thus a decreased range.

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2.14 $R = 1.50 \times 10^{11}$ m

$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi R}{T} = \frac{2\pi \times 1.50 \times 10^{11} \text{ m}}{365 \times 24 \times 60 \times 60 \text{ s}}$

$v = 29900$ m/s

$a_c = \frac{v^2}{R} = \frac{(29900)^2}{R} = \underline{\underline{5.96 \times 10^{-3} \text{ m/s}^2}}$

2.15

Planets	R (10^6 km)	T (yr)	a_c (10^9 km/yr²)
Mercury	57.9	0.24	39.4
Venus	108.2	0.62	11.24
Earth	149.6	1.00	5.92

$a_c \propto \frac{1}{R^2} \Rightarrow a_c \cdot R^2 = \text{constant}$

M $a_c R^2 = 132084$
V $a_c R^2 = 131589$ (similar)
E $a_c R^2 = 132490$

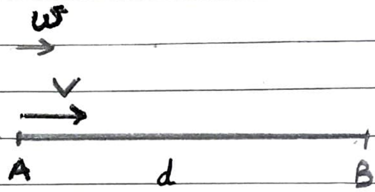
2.16 Guess: same time, with/without wind. (wrong guess)

speed of plane w.r.t air $= v$
speed of air w.r.t ground $= w$

speed of plane w.r.t ground $= v + w$ (same direction)
$= v - w$ (opposite direction)

I Without wind

$$\boxed{T = \frac{2d}{v}}$$



II With wind

$$T = t_{AB} + t_{BA} = \frac{d}{v+w} + \frac{d}{v-w} = d\left(\frac{1}{v+w} + \frac{1}{v-w}\right)$$

$$= \frac{2d}{v}\left(\frac{v^2}{v^2-w^2}\right) = \frac{2d}{v}\left(\frac{1}{1-w^2/v^2}\right)$$

$$\boxed{T = \frac{2d}{v}\left(\frac{1}{1-w^2/v^2}\right)}$$

When $|w| < v$, $\underline{\underline{T_{wind} > T_{without}}}$

- As $w \to v$, $T \to \infty$. i.e. the plane cannot travel against the direction of wind. ($t_{BA} \to \infty$)

- Changing the direction of the wind will not change the total time $T = t_{AB} + t_{BA}$.