8.01 Problem Set - 4

4.1 In Regime I, $v_{term} \ll v_{crit}$

$$\frac{mg}{c_1 r} \ll \frac{c_1}{c_2 r}$$

$$m = \frac{4}{3}\pi r^3 \cdot \rho \;; \quad \frac{4}{3}\pi r^3 \rho g \ll \frac{c_1^2}{c_2}$$

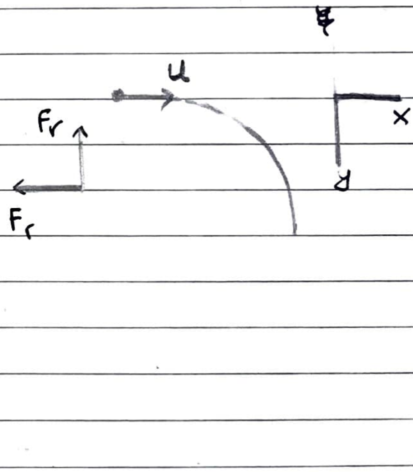
$$r^3 \ll \frac{3c_1^2}{4\pi c_2 \rho g} \approx 4\times 10^{-12}$$

$$\boxed{r \ll 1.6\times 10^{-4}\ \text{m}}$$

4.2a)

x// $ma = -c_1 r v$

$$\boxed{m\ddot{x} = -c_1 r \dot{x}}$$



y// $ma = +mg \mp c_1 r v$

$$\boxed{m\ddot{y} = -c_1 r\dot{y} + mg}$$

b) $m\frac{dv}{dt} = -c_1 r v$

$$\frac{dv}{v} = -\frac{c_1 r}{m}\, dt$$

$$\int_u^v \frac{dv}{v} = -\frac{c_1 r}{m} \int_0^t dt$$

$$\ln v - \ln u = -\frac{c_1 r}{m} t \Rightarrow \ln(v/u) = -\frac{c_1 r}{m} t$$

$$\boxed{v_x = u e^{-\frac{c_1 r}{m} t}}$$

c) $$m \frac{dv}{dt} = mg - c_1 r v$$

$$\frac{m\,dv}{mg - c_1 r v} = dt$$

$$\int_0^v \frac{m\,dv}{mg - c_1 r v} = \int_0^t dt \Rightarrow \left[\frac{-m}{c_1 r} \ln|mg - c_1 r v|\right]_0^v = t$$

$$\ln(mg - c_1 r v) - \ln(mg) = -\frac{c_1 r}{m} t$$

$$\ln\left[\frac{mg - c_1 r v}{mg}\right] = -\frac{c_1 r}{m} t$$

$$\frac{mg - c_1 r v}{mg} = e^{-\frac{c_1 r}{m} t}$$

$$c_1 r v = mg - mg e^{-\frac{c_1 r}{m} t}$$

$$\boxed{v_y = \frac{mg}{c_1 r}\left(1 - e^{-(c_1 r/m)t}\right)}$$

d) $v_y = 0.99 \frac{mg}{c_1 r}$

$$e^{-\frac{c_1 r}{m} t} = 0.01$$

$$-\frac{c_1 r}{m} t = \ln(0.01) \Rightarrow t = \frac{-m \ln(0.01)}{c_1 r}$$

$$\boxed{t = \frac{m \ln(100)}{c_1 r} \text{ sec}}$$

$c_1 = 1.6 \times 10^{2}$ kg/ms

$r = 0.00635$ m $= 6.35 \times 10^{-3}$ m

$\rho_{steel} = 7.8 \times 10^{3}$ kg/m^3

$m = \frac{4}{3}\pi r^3 \rho_s = 8.37 \times 10^{-3}$ kg

$$\boxed{t = 0.038 \text{ sec}}$$

e) As $t \to \infty$

$v_x \to 0$ //

$v_y \to \frac{mg}{c_1 r} = V_{term}$ //

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4.3 $x = A\cos(\omega t + \delta)$

$A = 0.2\,m$

$T = 1.2s$

$0.20 = A\cos(\delta)$

(a) $f = T^{-1} = \underline{\underline{5/6\ Hz}}$

$\omega = \frac{2\pi}{T} = \underline{\underline{\frac{5}{3}\pi\ rad/sec}}$

(b) $A = \underline{\underline{0.2m}}$

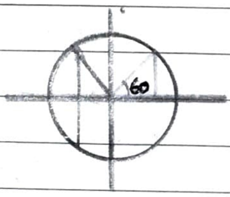
(c) $\cos(\delta) = 1 \Rightarrow \delta = 0$

$x = 0.2 \times \cos(\frac{5}{3}\pi t)$

At $x = 0$, $\cos(\omega t) = 0$

$\omega t = \frac{\pi}{2} + n\pi \Rightarrow t\frac{5}{3}\pi = \frac{\pi}{2} + n\pi$

$\boxed{t = \frac{3}{10} + \frac{3n}{5}\ sec}$ $(n = 0, 1, \dots)$



$-0.1 = 0.2\cos(\omega t) \Rightarrow \cos(\omega t) = -1/2$

$\omega t = \pm\frac{2\pi}{3} + 2n\pi \Rightarrow \frac{5}{3}\pi t = \pm\frac{2\pi}{3} + 2n\pi$

$$t = \pm\frac{2}{5} + \frac{6n}{5}$$

$$t = \frac{2}{5} + \frac{6n}{5}, \quad n = 0, 1, \ldots$$

and,

$$t = \frac{-2}{5} + \frac{6n}{5}, \quad n = 1, 2, \ldots \quad (t \geq 0).$$

(d) $x = A\cos(\omega t)$

$v = -\omega A\sin(\omega t)$

At $x = 0$, $v = -\omega A$

$$\|v\| = \frac{5}{3}\pi \times 0.2 \ \text{m/s} = \underline{1.05 \ \text{m/s}}$$

At $x = -0.10\,\text{m}$, $\cos(\omega t) = -1/2$

$$v = -\omega A\sin(\omega t) = \pm\frac{\omega A\sqrt{3}}{2}$$

$$\|v\| = \frac{5}{3}\pi \times \frac{1}{2} \times 0.2 \times \sqrt{3} = \underline{0.907 \ \text{m/s}}$$

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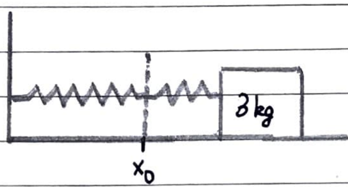
4.5



$x = A\cos(\omega t + \phi)$
$m = 3kg$
$T = 0.4$

(a) $v = -\omega A \sin(\omega t + \phi)$

$0.1 = A\cos(\phi) \longrightarrow 1$
$-3 = -\omega A \sin(\phi) \Rightarrow 3 = \omega A \sin\phi \longrightarrow 2$

$\omega = 2\pi/T = 5\pi$ rad/sec

$1/2 \Rightarrow 30 = \omega \tan\phi$

$\tan\phi = \frac{30}{5\pi} = \frac{6}{\pi} \Rightarrow \cancel{\phi}\ \phi = 1.089$ rad

$A = \frac{0.1}{\cos(\phi)} = 0.216$ m

$$\boxed{x = 0.216\cos(5\pi t + 1.089)}$$

(b) $x = 0$, $\cos(\omega t + \phi) = 0$

$\omega t + \phi = \frac{\pi}{2} + n\pi$

For the first time,

$5\pi t + 1.089 = \pi/2 \Rightarrow t = \underline{\underline{0.031 \text{ sec}}}$

$\|v\| = \omega A = \underline{\underline{3.39}}$ m/s

$a = -\omega^2 x = \underline{\underline{0 \text{ m/s}^2}}$

$K = \frac{1}{2} m v^2 = \frac{1}{2} \times 3 \times (3.39)^2 = 17.27$ J

$U = \frac{1}{2} k x^2 = 0$ J

(c) At the turning point $v = 0$

$0 = -\omega A \sin(\omega t + \phi)$

$\omega t + \phi = n\pi \Rightarrow t = \frac{n\pi - \phi}{5\pi}$

($t > 0$, so $n = 1$) first time

$t = \frac{\pi - 1.089}{5\pi} = \underline{\underline{0.131 \text{ sec}}}$

$\|v\| = 0$

$x = A\cos(\pi) = -A = \underline{\underline{-0.216 \text{ m}}}$

$a = -\omega^2 x = \underline{\underline{53.3 \text{ m/s}^2}}$

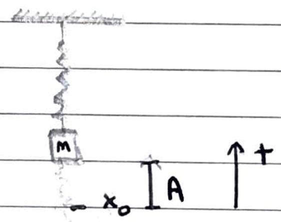
$K = \frac{1}{2} m v^2 = \frac{1}{2} m (0)^2 = \underline{\underline{0 \text{ J}}}$

$U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2$

$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = \omega^2 m$

$U = \frac{1}{2}\omega^2 m A = \underline{\underline{17.27 \text{ J}}}$

4.6 $m = 2.5$ kg
$k = 90$ N/m



A vertical spring performs SHM about the point x_0 (equilibrium pt.)

NOTE $\Rightarrow$ At $x = x_0$, $\sum F = 0$ ie $mg = kx_0$

$x = A\cos(\omega t + \phi)$ where x is measured from x_0

At $t = 0$, $x = A \Rightarrow \cos\phi = 1 \Rightarrow \phi = 0$

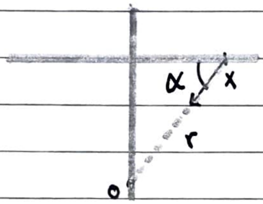
$x = A\cos(\omega t)$ where $\omega = \sqrt{k/m}$

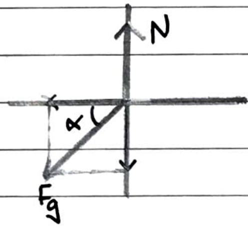
$\cancel{xa}\ A = x_0 = \dfrac{mg}{k} = 0.278\ m$

$\omega = \sqrt{k/m} = \sqrt{36} = 6\ rad/s$

$\boxed{x = 0.278 \cos(6t)}$ measured from x_0

47 a)





$y:\ N - F_g \sin\alpha = 0$

$x:\ -F_g \cos\alpha = m g_x$

$-\dfrac{GM}{R^3} \cdot \underbrace{r\cos\alpha}_{x} \cdot \cancel{m} = \cancel{m} g_x$

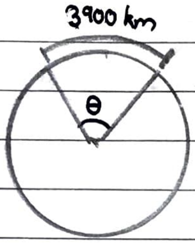
$$\boxed{g_x = -\frac{GM}{R^3} x}$$

b) $\ddot{x} = -\dfrac{GM}{R^3} x \Rightarrow \ddot{x} = -\omega^2 x \rightarrow$ eqn for SHM.

$\omega^2 = \dfrac{GM}{R^3}$, $\omega = \sqrt{\dfrac{GM}{R^3}}$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM}}}$$

(c)

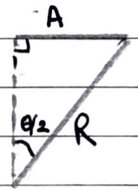


$f \times 2\pi R = 3900\ km$

$f = \dfrac{3900}{2\pi R} = 0.097$

$\theta = 2\pi f = 0.612 \approx 35^\circ$

$\sin\left(\frac{\theta}{2}\right) = \frac{A}{R} \Rightarrow A = R\sin\left(\frac{\theta}{2}\right)$

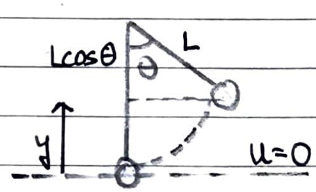


$A = \underline{\underline{1919.7}}\ km$

Time Taken $= \frac{T}{2} = 2550\ sec \approx \underline{\underline{42.5\ mins}}$

$|v_{max}| = \omega A \approx \underline{\underline{2.3}}\ km/s$

4.8 The energy of the system is conserved, as there are only conservative forces acting on the body ($W_T = 0$)



$ME_i = E_i = mgy$

$= mg(L - L\cos\theta) = mgL(1 - \cos(30^\circ))$

$E_i = \underline{\underline{8.04}}\ J$

$$E_f = mg(0) + \frac{1}{2}mv^2$$

$$\therefore \frac{1}{2}mv^2 = mgy \Rightarrow v^2 = 2gy \Rightarrow \boxed{v = 2.31\ m/s}$$

b) At $\theta = 10^\circ$,

$$E = \frac{1}{2}mv^2 + mgL(1-\cos(10^\circ)) = mgL(1-\cos(30^\circ))$$

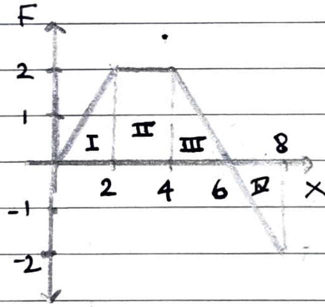
$$K + U = E$$

$$K = E - U = mgL(\cos(10^\circ) - \cos(30^\circ)) = 7.13\ J$$

$$\boxed{KE = 7.13J}$$

c) The velocity at the bottom is independent of the mass (m's cancel)
The KE of the bob doubles if the mass is doubled.

4.9



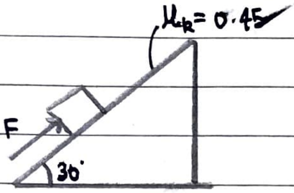
W = area of the F-X curve

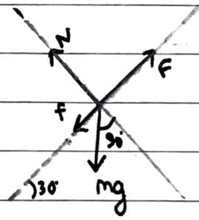
= I + II + III + IV

$$= \frac{1}{2}\times 2\times 2 + 2\times 2 + \cancel{\frac{1}{2}\times 2\times 2} - \cancel{\frac{1}{2}\times 2\times 2}$$

$$= \frac{3\times 4}{2} = \underline{\underline{6\ Nm}}$$

4.10





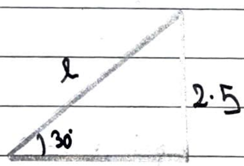
y: $N - mg\cos(30°) = 0$

$N = mg\cos(30°)$

x: $F - f - mg\sin(30°) = 0$

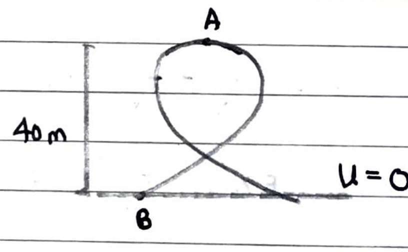
$F = \mu_k N - mg\sin 30° = mg\,(\mu_k \cos(30°) - \sin(30°))$

$\underline{\underline{F = 523.2\ N}}$



$W = Fl = \dfrac{F \times 2.5}{\sin(30°)} = \underline{\underline{2615.8\ Nm}}$

4.11 For the car to not fall off,



$\dfrac{V_t^2}{R} \geqslant g$

$V_t \geqslant \sqrt{10g} \Rightarrow \underline{\underline{V_t \geqslant 10\ m/s}}$

Only conservative forces do work on the car, energy is conserved.

$ME = [\text{constant}]$

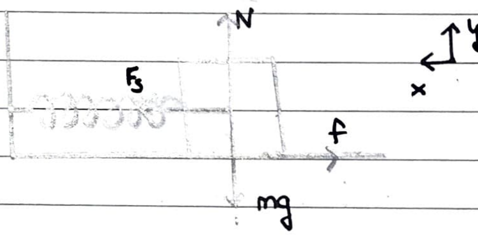
Energy at A = Energy at B

$\frac{1}{2}\cancel{m} v_t^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$\frac{1}{\cancel{2}} \times \overset{5}{\cancel{10}} \times 10 \times 10 \times 40 = \frac{1}{2} \times v^2 \Rightarrow \quad 450 = \frac{v2}{2}$

$v^2 = 900 \Rightarrow \boxed{v = 30\ m/s}$

4.12 $m = 3\,kg$
$\mu_s = 0.30$
$\mu_k = 0.20$
$k = 80\,N/m$



a) $y: \; N = mg = 30\,N$

Static friction force balances the spring force

$x: \quad F_s - f = m(0)$

$kx_{max} = \mu_s N \Rightarrow 80\,x_{max} = 0.3 \times 30$

$x_{max} = \frac{9}{80} = \underline{\underline{0.1125\,m}}$

b) The gentle push makes the friction kinetic, from static.
The block starts moving towards the equilibrium point.

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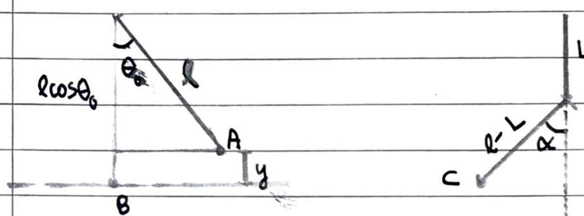
The block will keep accelerating until $F_S = f_k$ after which the velocity starts to decrease

$F_S = \cancel{k} f_k$

$kx = \mu_k N \Rightarrow x = \frac{0.2 \times 30}{80} = \underline{\underline{0.075\ m}}$

4.13



a) Energy conserved between A and B

$E_A = E_B$

$mgy = \frac{1}{2}mv^2 \Rightarrow mg(l(1-\cos\theta_0)) = \frac{1}{2}mv^2$

Since the string is massless, none of the KE is absorbed when it hits the pin.

$E_A = E_B = E_C$

$mgl(1-\cos\theta_0) = mg(l-L)(1-\cos\alpha)$

$\cos\alpha = 1 - \frac{l}{l-L}(1-\cos\theta_0) = \frac{l - L - l + l\cos\theta_0}{l-L}$

$$\boxed{\cos\alpha = \frac{l\cos\theta_0 - L}{l-L}}$$

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b) Initial KE at A

$$E_A = mg\ell(1-\cos\theta_0) + \frac{1}{2}mv_0^2$$

$$E_C = mg(\ell-L)(1-\cos\alpha)$$

$$E_A = E_C$$

$$\cancel{m}g\ell(1-\cos\theta_0) + \frac{1}{2}\cancel{m}v_0^2 = \cancel{m}g(\ell-L)(1-\cos\alpha)$$

$$g(\ell-L)(1-\cos\alpha) = g\ell(1-\cos\theta_0) + \frac{v_0^2}{2}$$

$$1-\cos\alpha = \frac{\ell}{\ell-L}(1-\cos\theta_0) + \frac{v_0^2}{2g(\ell-L)}$$

$$\boxed{\cos\alpha = \frac{\ell\cos\theta_0 - L}{\ell - L} - \frac{v_0^2}{2g(\ell-L)}}$$

The direction of tangential velocity doesn't matter as both directions provide the same amount of KE.

4.14 a) There is relative motion b/w the surfaces, so the friction is Kinetic.

b) Friction allways opposes the relative motion. The work, however ~~This motion~~ depends on the frame of reference.

From the frame of reference of the ground, the net displacement is to the right. Also, the friction is to the right. So, the work done by friction is positive.

In the frame of reference of the conveyor belt, the displacement is to the left, so the work done by friction is negative.

c) After the suitcase reaches the speed of the belt, there is no relative motion between the two. So there is no friction. The net horizontal force is thus zero.