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18.01 Exam - I

1. tangent line

$(y - y_0) = m(x - x_0)$

a) point $(x_0, y_0) = (1, 1/3)$

b) slope $m = \frac{d}{dx} \frac{1}{3} x^2 \Big|_{x=1} = \frac{1}{3} \cdot 2x = \frac{2}{3}$ //

Tangent Line Equation:

$$\boxed{y - \frac{1}{3} = \frac{2}{3}(x - 1)}$$

2-a. $\frac{d}{dx} \frac{x}{\sqrt{1-x}}$ $\quad (u/v)' = \frac{vu' - uv'}{v^2}$

$$\frac{d}{dx}\frac{x}{\sqrt{1-x}} = \left((\sqrt{1-x}) \cdot 1 - x \cdot \frac{1}{\sqrt{1-x}} \cdot -1 \right) \cdot \frac{1}{1-x}$$

$$= \frac{1 - \cancel{x} + \cancel{x}}{(\sqrt{1-x})(1-x)} = \underline{\frac{1}{(1-x)^{3/2}}}$$

b. $\frac{d}{dx} \frac{\cos(2x)}{x} = \frac{x \cdot -\sin(2x) \cdot 2 - \cos(2x) \cdot 1}{x^2} = \underline{\frac{-2x\sin 2x - \cos 2x}{x^2}}$

c. $e^{2f(x)} = g(x)$

differentiating on both sides

$(e^{2f(x)})' = g'(x)$

$\boxed{e^{2f(x)} \cdot 2f'(x) = g'(x)}$ chain rule

d. $\frac{d}{dx} \ln(\sin x) = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \underline{\underline{\cot x}}$

3. $y^4 + xy = 4$. Find y' at $x=3$, $y=1$

implicit differentiation.

$4y^3 \cdot y' + xy' + y = 0$

$(4y^3 + x)y' = -y \Rightarrow y' = \frac{-y}{4y^3+x}$

$y' \Big|_{\substack{x=3 \\ y=1}} = \frac{-1}{-4+3} = \underline{\underline{1}}$

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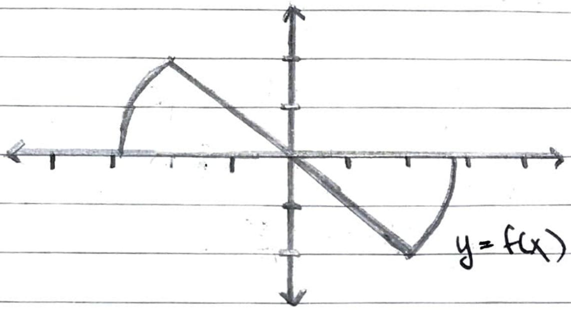
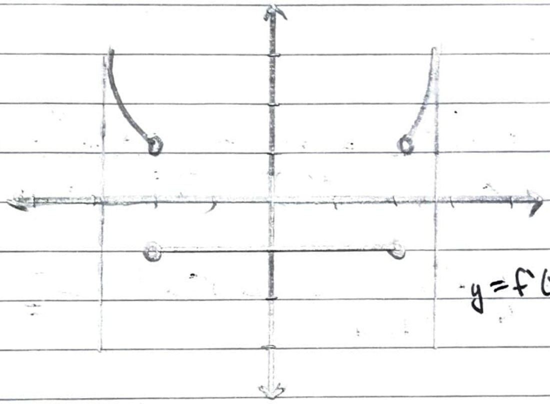
4.



odd



even

5. $f(x) = \begin{cases} ax+b & , x<1 \\ x^4+x+1 & , x \geq 1 \end{cases}$

(i) slope of tangents from both sides must be equal.

$$\frac{d}{dx}(ax+b)\Big|_{x=1} = \frac{d}{dx}(x^4+x+1)\Big|_{x=1}$$

$$a = 4x^3+1\Big|_{x=1} = 5$$

$$\boxed{a=5}$$

(ii) diff $\Rightarrow$ cont.

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^4 + x + 1 = 3$$

$$\parallel$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (ax+b) = 5 + b$$

$$\boxed{b = -2}$$

6. a) $$\lim_{x \to 0} \frac{(1+2x)^{10} - 1}{x} = \lim_{2x \to 0} 2\left(\frac{(1+2x)^{10} - 1^{10}}{2x}\right)$$

$$= 2 \cdot \frac{d}{dx}(x^{10})\Big|_{x=1} = 2 \cdot 10 x^9 = \underline{\underline{20}}$$

b) $$\lim_{x \to 0} \frac{\sqrt{\cos x} - 1}{x} = \lim_{x \to 0} \frac{\sqrt{\cos(0+x)} - \sqrt{\cos(0)}}{x} = \frac{d}{dx}\sqrt{\cos x}\Big|_{x=0}$$

$$\frac{d}{dx}\sqrt{\cos x} = \frac{1}{2\sqrt{\cos x}} \cdot -\sin x \Big|_{x=0} = \underline{\underline{0}}$$

7.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Using defn to find $\frac{d}{dx} a^x$.

$$\frac{d}{dx} a^x = \lim_{\Delta x \to 0} \frac{a^{x+\Delta x} - a^x}{\Delta x} = a^x \lim_{\Delta x \to 0} \frac{a^{\Delta x} - 1}{\Delta x}$$

a^x is a constant as Δx is changing.

$$M(a) = \lim_{\Delta x \to 0} \frac{a^{\Delta x} - 1}{\Delta x}$$

$$\boxed{\frac{d}{dx} a^x = a^x \cdot M(a)} \quad \text{where } \boxed{M(a) = \lim_{\Delta x \to 0} \frac{a^{\Delta x} - 1}{\Delta x}}$$