

1 The Problem

$$\max_{\{k_{t+J}, n_t, z_{t+1}\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t U(c_t^h, l_t^h) + E \sum_{t=0}^{\infty} \beta^t U(c_t^f, l_t^f) = E \sum_{t=0}^{\infty} \beta^t \frac{1}{\gamma} \left((c_t^h)^\mu (1 - n_t^h)^{1-\mu} \right)^\gamma + E \sum_{t=0}^{\infty} \beta^t \frac{1}{\gamma} \left((c_t^f)^\mu (1 - n_t^f)^{1-\mu} \right)^\gamma,$$

where

$$\begin{aligned} c_t^h + c_t^f &= \left[\left(\lambda_t^h (k_t^h)^\theta (n_t^h)^{1-\theta} \right)^{-\nu} + \sigma (z_t^h)^{-\nu} \right]^{-1/\nu} + \left[\left(\lambda_t^f (k_t^f)^\theta (n_t^f)^{1-\theta} \right)^{-\nu} + \sigma (z_t^f)^{-\nu} \right]^{-1/\nu} - \sum_{j=1}^J \phi_j s_{jt}^h - \sum_{j=1}^J \phi_j s_{jt}^f - z_{t+1}^h - z_{t+1}^f + z_t^h + z_t^f \\ &= \left[\left(\lambda_t^h (k_t^h)^\theta (n_t^h)^{1-\theta} \right)^{-\nu} + \sigma (z_t^h)^{-\nu} \right]^{-1/\nu} + \left[\left(\lambda_t^f (k_t^f)^\theta (n_t^f)^{1-\theta} \right)^{-\nu} + \sigma (z_t^f)^{-\nu} \right]^{-1/\nu} - \frac{1}{J} \sum_{j=1}^J s_{1,t+j-1}^h - \frac{1}{J} \sum_{j=1}^J s_{1,t+j-1}^f - z_{t+1}^h - z_{t+1}^f + z_t^h + z_t^f \\ &= \left[\left(\lambda_t^h (k_t^h)^\theta (n_t^h)^{1-\theta} \right)^{-\nu} + \sigma (z_t^h)^{-\nu} \right]^{-1/\nu} + \left[\left(\lambda_t^f (k_t^f)^\theta (n_t^f)^{1-\theta} \right)^{-\nu} + \sigma (z_t^f)^{-\nu} \right]^{-1/\nu} \\ &\quad - \frac{1}{J} \sum_{j=1}^J (k_{t+j}^h - (1-\delta) k_{t+j-1}^h) - \frac{1}{J} \sum_{j=1}^J (k_{t+j}^f - (1-\delta) k_{t+j-1}^f) - z_{t+1}^h - z_{t+1}^f + z_t^h + z_t^f \\ &\equiv y_t^h + y_t^f - \frac{1}{J} \sum_{j=1}^J (k_{t+j}^h - (1-\delta) k_{t+j-1}^h) - \frac{1}{J} \sum_{j=1}^J (k_{t+j}^f - (1-\delta) k_{t+j-1}^f) - z_{t+1}^h - z_{t+1}^f + z_t^h + z_t^f \end{aligned} \quad (1)$$

Lagrangian multiplier: ξ_t

FOCs (for h variables; f variables are symmetric):

c_t^h :

$$\beta^t u_{tc}^h - \xi_t = 0 \quad (2)$$

n_t^h :

$$0 = \beta^t u_{tn}^h + \xi_t y_{tn}^h \quad (3)$$

k_{t+J}^h :

$$\begin{aligned} 0 &= \xi_{t+J} y_{t+J,k}^h - \frac{1}{J} \left(\sum_t^{t+J-1} \xi_t - \sum_{t+1}^{t+J} \xi_t (1-\delta) \right) \\ &= \xi_{t+J} y_{t+J,k}^h - \frac{1}{J} (\xi_t + \delta \xi_{t+1} + \dots + \delta \xi_{t+J-1} + (\delta-1) \xi_{t+J}) \end{aligned}$$

$$[= \xi_{t+4} y_{t+4,k}^h - \frac{1}{4} (\xi_t + \delta \xi_{t+1} + \delta \xi_{t+2} + \delta \xi_{t+3} + (\delta - 1) \xi_{t+4})] \quad (4)$$

z_{t+1}^h :

$$0 = -\xi_t + \xi_{t+1} (y_{t+1,z}^h + 1) \quad (5)$$

2 First-Order Conditions

From (2),

$$\xi_t = \beta^t u_{tc}^h \quad (6)$$

Substitute (6) back into (4):

$$\begin{aligned} 0 &= \beta^{t+4} u_{t+4,c}^h y_{t+4,k}^h - \frac{1}{4} (\beta^t u_{tc}^h + \delta \beta^{t+1} u_{t+1,c}^h + \delta \beta^{t+2} u_{t+2,c}^h + \delta \beta^{t+3} u_{t+3,c}^h + (\delta - 1) \beta^{t+4} u_{t+4,c}^h) \\ 0 &= \beta^4 u_{t+4,c}^h y_{t+4,k}^h - \frac{1}{4} (u_{tc}^h + \delta \beta u_{t+1,c}^h + \delta \beta^2 u_{t+2,c}^h + \delta \beta^3 u_{t+3,c}^h + (\delta - 1) \beta^4 u_{t+4,c}^h) \end{aligned} \quad (7)$$

Substitute (6) back into (5):

$$\begin{aligned} 0 &= \beta^t u_{tc}^h - \beta^{t+1} u_{t+1,c}^h (y_{t+1,z}^h + 1) \\ 0 &= u_{tc}^h - \beta u_{t+1,c}^h (y_{t+1,z}^h + 1) \end{aligned} \quad (8)$$

Substitute (6) back into (3):

$$0 = u_{tn}^h + u_{tc}^h y_{tn}^h \quad (9)$$

The home- and foreign-country versions of (6) further gives

$$u_{tc}^h = u_{tc}^f \quad (10)$$

3 Steady State

At the steady state, (7) implies

$$\begin{aligned} 0 &= \beta^4 u_c^h y_k^h - \frac{1}{4} (1 + \delta \beta + \delta \beta^2 + \delta \beta^3 + (\delta - 1) \beta^4) u_c^h \\ 0 &= \beta^4 y_k^h (\bar{\lambda}^h, \bar{k}^h, \bar{n}^h, \bar{z}^h) - \frac{1}{4} (1 + \delta \beta + \delta \beta^2 + \delta \beta^3 + (\delta - 1) \beta^4) \end{aligned} \quad (11)$$

(8) implies

$$\begin{aligned} 0 &= u_c^h - \beta u_c^h (y_z^h + 1) \\ 0 &= 1 - \beta (y_z^h (\bar{\lambda}^h, \bar{k}^h, \bar{n}^h, \bar{z}^h) + 1) \end{aligned} \quad (12)$$

(9) implies

$$0 = u_n^h (\bar{c}^h, \bar{n}^h) + u_c^h (\bar{c}^h, \bar{n}^h) y_n^h (\bar{\lambda}^h, \bar{k}^h, \bar{n}^h, \bar{z}^h) \quad (13)$$

and (1) implies

$$\bar{c}^h = \bar{y}^h - \delta \bar{k}^h \quad (14)$$

4 Log-Linearization

Now log-linearize the system of (7), (8), (9), (10) and (1). Let π_{ab} be the elasticity of marginal utility of a with respect to b evaluated at the steady state, τ_{ab} be the elasticity of marginal product of a with respect to b evaluated at the steady state, and ζ_a be the elasticity of output with respect to a evaluated at the steady state.

(7):

$$\begin{aligned} \pi_{cc}\hat{c}_{t+4}^h + \pi_{cn}\hat{n}_{t+4}^h + \tau_{k\lambda}\hat{\lambda}_{t+4}^h + \tau_{kk}\hat{k}_{t+4}^h + \tau_{kn}\hat{n}_{t+4}^h + \tau_{kz}\hat{z}_{t+4}^h &= \frac{1}{1 + \delta\beta + \delta\beta^2 + \delta\beta^3 + (\delta - 1)\beta^4} [(\pi_{cc}\hat{c}_t^h + \pi_{cn}\hat{n}_t^h) + \delta\beta(\pi_{cc}\hat{c}_{t+1}^h + \pi_{cn}\hat{n}_{t+1}^h) \\ &\quad + \delta\beta^2(\pi_{cc}\hat{c}_{t+2}^h + \pi_{cn}\hat{n}_{t+2}^h) + \delta\beta^3(\pi_{cc}\hat{c}_{t+3}^h + \pi_{cn}\hat{n}_{t+3}^h) + (\delta - 1)\beta^4(\pi_{cc}\hat{c}_{t+4}^h + \pi_{cn}\hat{n}_{t+4}^h)] \end{aligned} \quad (15)$$

(8):

$$\pi_{cc}\hat{c}_t^h + \pi_{cn}\hat{n}_t^h = \pi_{cc}\hat{c}_{t+1}^h + \pi_{cn}\hat{n}_{t+1}^h + (1 - \beta) \left(\tau_{z\lambda}\hat{\lambda}_{t+1}^h + \tau_{zk}\hat{k}_{t+1}^h + \tau_{zn}\hat{n}_{t+1}^h + \tau_{zz}\hat{z}_{t+1}^h \right) \quad (16)$$

(The equation above utilizes (12).)

(9):

$$\pi_{nc}\hat{c}_t^h + \pi_{nn}\hat{n}_t^h = \pi_{cc}\hat{c}_t^h + \pi_{cn}\hat{n}_t^h + \tau_{n\lambda}\hat{\lambda}_t^h + \tau_{nk}\hat{k}_t^h + \tau_{nn}\hat{n}_t^h + \tau_{nz}\hat{z}_t^h \quad (17)$$

(10):

$$\pi_{cc}\hat{c}_t^h + \pi_{cn}\hat{n}_t^h = \pi_{cc}\hat{c}_t^f + \pi_{cn}\hat{n}_t^f \quad (18)$$

(1):

$$\bar{c}^h \hat{c}_t^h + \bar{c}^f \hat{c}_t^f = \bar{y}^h \left(\zeta_\lambda \hat{\lambda}_t^h + \zeta_k \hat{k}_t^h + \zeta_n \hat{n}_t^h + \zeta_z \hat{z}_t^h \right) + \bar{y}^f \left(\zeta_\lambda \hat{\lambda}_t^f + \zeta_k \hat{k}_t^f + \zeta_n \hat{n}_t^f + \zeta_z \hat{z}_t^f \right) - \delta \bar{k}^h \sum_{j=1}^4 \frac{1}{4} \left(\frac{1}{\delta} \hat{k}_{t+j}^h - \frac{1-\delta}{\delta} \hat{k}_{t+j-1}^h \right) - \delta \bar{k}^f \sum_{j=1}^4 \frac{1}{4} \left(\frac{1}{\delta} \hat{k}_{t+j}^f - \frac{1-\delta}{\delta} \hat{k}_{t+j-1}^f \right) \quad (19)$$

(The equation above utilizes (14).)

5 Derivatives and Elasticities

$$\begin{aligned} u_{tc}^h &= \mu (c_t^h)^{\gamma\mu-1} (1 - n_t^h)^{\gamma(1-\mu)} \\ u_{tn}^h &= (\mu - 1) (c_t^h)^{\gamma\mu} (1 - n_t^h)^{\gamma(1-\mu)-1} \\ y_{tk}^h &= \theta \left[\left(\lambda_t^h (k_t^h)^\theta (n_t^h)^{1-\theta} \right)^{-\nu} + \sigma (z_t^h)^{-\nu} \right]^{-\frac{1+\nu}{\nu}} \left(\lambda_t^h (n_t^h)^{1-\theta} \right)^{-\nu} (k_t^h)^{-\nu\theta-1} \\ &= \theta (y_t^h)^{\nu+1} \left(\lambda_t^h (n_t^h)^{1-\theta} \right)^{-\nu} (k_t^h)^{-\nu\theta-1} \\ &= \theta (y_t^h)^{\nu+1} \left[(y_t^h)^{-\nu} - \sigma (z_t^h)^{-\nu} \right] (k_t^h)^{-1} \\ y_{tn}^h &= (1 - \theta) \left[\left(\lambda_t^h (k_t^h)^\theta (n_t^h)^{1-\theta} \right)^{-\nu} + \sigma (z_t^h)^{-\nu} \right]^{-\frac{1+\nu}{\nu}} \left(\lambda_t^h (k_t^h)^\theta \right)^{-\nu} (n_t^h)^{\nu(\theta-1)-1} \\ &= (1 - \theta) (y_t^h)^{\nu+1} \left(\lambda_t^h (k_t^h)^\theta \right)^{-\nu} (n_t^h)^{\nu(\theta-1)-1} \\ &= (1 - \theta) (y_t^h)^{\nu+1} \left[(y_t^h)^{-\nu} - \sigma (z_t^h)^{-\nu} \right] (n_t^h)^{-1} \\ y_{tz}^h &= \sigma \left[\left(\lambda_t^h (k_t^h)^\theta (n_t^h)^{1-\theta} \right)^{-\nu} + \sigma (z_t^h)^{-\nu} \right]^{-\frac{1+\nu}{\nu}} (z_t^h)^{-\nu-1} \\ &= \sigma (y_t^h)^{\nu+1} (z_t^h)^{-\nu-1} \\ \pi_{cc} &= \gamma\mu - 1 \\ \pi_{cn} &= \gamma(1 - \mu) \frac{\bar{n}^h}{\bar{n}^h - 1} \\ \pi_{nc} &= \gamma\mu \\ \pi_{nn} &= (\gamma(1 - \mu) - 1) \frac{\bar{n}^h}{\bar{n}^h - 1} \\ \zeta_\lambda &= \left[\left(\bar{\lambda}^h (\bar{k}^h)^\theta (\bar{n}^h)^{1-\theta} \right)^{-\nu} + \sigma (\bar{z}^h)^{-\nu} \right]^{-1} \left((\bar{k}^h)^\theta (\bar{n}^h)^{1-\theta} \right)^{-\nu} (\bar{\lambda}^h)^{-\nu} \end{aligned}$$

$$\begin{aligned}
&= (\bar{y}^h)^\nu \left((\bar{k}^h)^\theta (\bar{n}^h)^{1-\theta} \right)^{-\nu} (\bar{\lambda}^h)^{-\nu} \\
&= 1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \\
\zeta_k &= \theta \left[\left(\bar{\lambda}^h (\bar{k}^h)^\theta (\bar{n}^h)^{1-\theta} \right)^{-\nu} + \sigma (\bar{z}^h)^{-\nu} \right]^{-1} \left(\bar{\lambda}^h (\bar{n}^h)^{1-\theta} \right)^{-\nu} (\bar{k}^h)^{-\nu\theta} \\
&= \theta (\bar{y}^h)^\nu \left(\bar{\lambda}^h (\bar{n}^h)^{1-\theta} \right)^{-\nu} (\bar{k}^h)^{-\nu\theta} \\
&= \theta \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] \\
\zeta_n &= (1 - \theta) \left[\left(\bar{\lambda}^h (\bar{k}^h)^\theta (\bar{n}^h)^{1-\theta} \right)^{-\nu} + \sigma (\bar{z}^h)^{-\nu} \right]^{-1} \left(\bar{\lambda}^h (\bar{k}^h)^\theta \right)^{-\nu} (\bar{n}^h)^{\nu(\theta-1)} \\
&= (1 - \theta) (\bar{y}^h)^\nu \left(\bar{\lambda}^h (\bar{k}^h)^\theta \right)^{-\nu} (\bar{n}^h)^{\nu(\theta-1)} \\
&= (1 - \theta) \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] \\
\zeta_z &= \sigma \left[\left(\bar{\lambda}^h (\bar{k}^h)^\theta (\bar{n}^h)^{1-\theta} \right)^{-\nu} + \sigma (\bar{z}^h)^{-\nu} \right]^{-1} (\bar{z}^h)^{-\nu} \\
&= \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \\
\tau_{k\lambda} &= \tau_{n\lambda} = (\nu + 1) \zeta_\lambda - \nu \\
&= (\nu + 1) \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] - \nu \\
\tau_{kk} &= (\nu + 1) \zeta_k - \nu\theta - 1 \\
&= (\nu + 1) \theta \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] - \nu\theta - 1 \\
\tau_{kn} &= (\nu + 1) \zeta_n + \nu(\theta - 1) \\
&= (\nu + 1) (1 - \theta) \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] + \nu(\theta - 1) \\
\tau_{kz} &= \tau_{nz} = (\nu + 1) \zeta_z \\
&= (\nu + 1) \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \\
\tau_{nk} &= (\nu + 1) \zeta_k - \nu\theta \\
&= (\nu + 1) \theta \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] - \nu\theta \\
\tau_{nn} &= (\nu + 1) \zeta_n + \nu(\theta - 1) - 1 \\
&= (\nu + 1) (1 - \theta) \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] + \nu(\theta - 1) - 1 \\
\tau_{z\lambda} &= (\nu + 1) \zeta_\lambda
\end{aligned}$$

$$\begin{aligned}
&= (\nu + 1) \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] \\
\tau_{zk} &= (\nu + 1) \zeta_k \\
&= (\nu + 1) \theta \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] \\
\tau_{zn} &= (\nu + 1) \zeta_n \\
&= (\nu + 1) (1 - \theta) \left[1 - \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} \right] \\
\tau_{zz} &= (\nu + 1) \zeta_z - \nu - 1 \\
&= (\nu + 1) \sigma (\bar{y}^h)^\nu (\bar{z}^h)^{-\nu} - \nu - 1
\end{aligned}$$

6 Steady State, Cont'd

Substitute the derivatives back to the steady-state equations (11), (12), (13) and (14):

$$0 = \beta^4 \theta (\bar{y}^h)^{\nu+1} \left((\bar{y}^h)^{-\nu} - \sigma (\bar{z}^h)^{-\nu} \right) (\bar{k}^h)^{-1} - \frac{1}{4} (1 + \delta\beta + \delta\beta^2 + \delta\beta^3 + (\delta - 1)\beta^4) \quad (20)$$

$$0 = 1 - \beta \left(\sigma (\bar{y}^h)^{\nu+1} (\bar{z}^h)^{-\nu-1} + 1 \right) \quad (21)$$

$$0 = (1 - \mu) \bar{c}^h + \mu (1 - \bar{n}^h) (1 - \theta) (\bar{y}^h)^{\nu+1} \left[(\bar{y}^h)^{-\nu} - \sigma (\bar{z}^h)^{-\nu} \right] (\bar{n}^h)^{-1} \quad (22)$$

$$\bar{c}^h = \bar{y}^h - \delta \bar{k}^h \quad (23)$$

From (21),

$$\bar{z}^h = \left(\sigma \frac{\beta}{1 - \beta} \right)^{\frac{1}{\nu+1}} \bar{y}^h$$

From (20):

$$\bar{k}^h = \frac{4\beta^4 \theta}{1 + \delta\beta + \delta\beta^2 + \delta\beta^3 + (\delta - 1)\beta^4} (\bar{y}^h)^{\nu+1} \left((\bar{y}^h)^{-\nu} - \sigma (\bar{z}^h)^{-\nu} \right)$$

Given (23), from (22):

$$\bar{n}^h = \left(\left[(1 - \theta) (\bar{y}^h)^{\nu+1} \left((\bar{y}^h)^{-\nu} - \sigma (\bar{z}^h)^{-\nu} \right) \right]^{-1} \left(\frac{1}{\mu} - 1 \right) (\bar{y}^h - \delta \bar{k}^h) + 1 \right)^{-1}$$

7 Log-Linearization, Cont'd

From (17), we can write \hat{n}_t^h as

$$\hat{n}_t^h = \frac{1}{\pi_{nn} - \pi_{cn} - \tau_{nn}} \left[(\pi_{cc} - \pi_{nc}) \hat{c}_t^h + \tau_{n\lambda} \hat{\lambda}_t^h + \tau_{nk} \hat{k}_t^h + \tau_{nz} \hat{z}_t^h \right]$$

Substitute this back to (15), (16), (18) and (19).

(15): Let

$$\begin{aligned} m &= 1 + \delta\beta + \delta\beta^2 + \delta\beta^3 \\ s &= \pi_{nn} - \pi_{cn} - \tau_{nn} \\ p_c &= \pi_{cc} + \pi_{cn} (\pi_{cc} - \pi_{nc}) \\ q_c &= \pi_{cc} (\pi_{nn} - \tau_{nn}) - \pi_{cn} \pi_{nc} \\ p_\lambda &= \pi_{cn} \tau_{n\lambda} \\ p_k &= \pi_{cn} \tau_{nk} \\ p_z &= \pi_{cn} \tau_{nz}; \end{aligned}$$

then

$$\begin{aligned} & \left[m q_c + (m + (\delta - 1) \beta^4) \tau_{kn} (\pi_{cc} - \pi_{nc}) \right] \hat{c}_{t+4}^h \\ & + \left[m \pi_{cn} \tau_{n\lambda} + (m + (\delta - 1) \beta^4) (\tau_{kn} \tau_{n\lambda} + s \tau_{k\lambda}) \right] \hat{\lambda}_{t+4}^h \\ & + \left[m \pi_{cn} \tau_{nk} + (m + (\delta - 1) \beta^4) (\tau_{kn} \tau_{nk} + s \tau_{kk}) \right] \hat{k}_{t+4}^h \\ & + \left[m \pi_{cn} \tau_{nz} + (m + (\delta - 1) \beta^4) (\tau_{kn} \tau_{nz} + s \tau_{kz}) \right] \hat{z}_{t+4}^h \\ = & p_c \hat{c}_t^h + p_\lambda \hat{\lambda}_t^h + p_k \hat{k}_t^h + p_z \hat{z}_t^h \\ & + \delta\beta p_c \hat{c}_{t+1}^h + \delta\beta p_\lambda \hat{\lambda}_{t+1}^h + \delta\beta p_k \hat{k}_{t+1}^h + \delta\beta p_z \hat{z}_{t+1}^h \\ & + \delta\beta^2 p_c \hat{c}_{t+2}^h + \delta\beta^2 p_\lambda \hat{\lambda}_{t+2}^h + \delta\beta^2 p_k \hat{k}_{t+2}^h + \delta\beta^2 p_z \hat{z}_{t+2}^h \\ & + \delta\beta^3 p_c \hat{c}_{t+3}^h + \delta\beta^3 p_\lambda \hat{\lambda}_{t+3}^h + \delta\beta^3 p_k \hat{k}_{t+3}^h + \delta\beta^3 p_z \hat{z}_{t+3}^h \end{aligned} \tag{24}$$

(16):

$$\begin{aligned} & [q_c + (1 - \beta) \tau_{zn} (\pi_{cc} - \pi_{nc})] \hat{c}_{t+1}^h \\ & + [\pi_{cn} \tau_{n\lambda} + (1 - \beta) (\tau_{zn} \tau_{n\lambda} + s \tau_{z\lambda})] \hat{\lambda}_{t+1}^h \\ & + [\pi_{cn} \tau_{nk} + (1 - \beta) (\tau_{zn} \tau_{nk} + s \tau_{zk})] \hat{k}_{t+1}^h \end{aligned} \tag{25}$$

$$\begin{aligned}
& + [\pi_{cn}\tau_{nz} + (1 - \beta) (\tau_{zn}\tau_{nz} + s\tau_{zz})] \hat{z}_{t+1}^h \\
= & q_c \hat{c}_t^h + p_\lambda \hat{\lambda}_t^h + p_k \hat{k}_t^h + p_z \hat{z}_t^h
\end{aligned}$$

(18):

$$q_c \hat{c}_t^h + p_\lambda \hat{\lambda}_t^h + p_k \hat{k}_t^h + p_z \hat{z}_t^h = q_c \hat{c}_t^f + p_\lambda \hat{\lambda}_t^f + p_k \hat{k}_t^f + p_z \hat{z}_t^f \quad (26)$$

(19):

$$\begin{aligned}
& \delta s \bar{k}^h \hat{k}_{t+1}^h + \delta s \bar{k}^h \hat{k}_{t+2}^h + \delta s \bar{k}^h \hat{k}_{t+3}^h + s \bar{k}^h \hat{k}_{t+4}^h \\
& + \delta s \bar{k}^f \hat{k}_{t+1}^f + \delta s \bar{k}^f \hat{k}_{t+2}^f + \delta s \bar{k}^f \hat{k}_{t+3}^f + s \bar{k}^f \hat{k}_{t+4}^f \\
= & 4 [\zeta_n (\pi_{cc} - \pi_{nc}) \bar{y}^h - s \bar{c}^h] \hat{c}_t^h \\
& + 4 [\zeta_n (\pi_{cc} - \pi_{nc}) \bar{y}^f - s \bar{c}^f] \hat{c}_t^f \\
& + 4 \bar{y}^h (\zeta_\lambda s + \zeta_n \tau_{n\lambda}) \hat{\lambda}_t^h \\
& + 4 \bar{y}^f (\zeta_\lambda s + \zeta_n \tau_{n\lambda}) \hat{\lambda}_t^f \\
& + [4 \bar{y}^h (\zeta_k s + \zeta_n \tau_{nk}) + (1 - \delta) s \bar{k}^h] \hat{k}_t^h \\
& + [4 \bar{y}^f (\zeta_k s + \zeta_n \tau_{nk}) + (1 - \delta) s \bar{k}^f] \hat{k}_t^f \\
& + 4 \bar{y}^h (\zeta_z s + \zeta_n \tau_{nz}) \hat{z}_t^h \\
& + 4 \bar{y}^f (\zeta_z s + \zeta_n \tau_{nz}) \hat{z}_t^f
\end{aligned} \quad (27)$$

8 A Detour: $J = 1$

Change in steady-state values:

$$\bar{k}^h = \frac{\beta\theta}{1 + (\delta - 1)\beta} (\bar{y}^h)^{\nu+1} \left((\bar{y}^h)^{-\nu} - \sigma (\bar{z}^h)^{-\nu} \right)$$

Change in log-linearized system (pre-substitution):

$$\begin{aligned}
& \pi_{cc} \hat{c}_{t+1}^h + \pi_{cn} \hat{n}_{t+1}^h + \tau_{k\lambda} \hat{\lambda}_{t+1}^h + \tau_{kk} \hat{k}_{t+1}^h + \tau_{kn} \hat{n}_{t+1}^h + \tau_{kz} \hat{z}_{t+1}^h = \frac{1}{1 + (\delta - 1)\beta} [(\pi_{cc} \hat{c}_t^h + \pi_{cn} \hat{n}_t^h) + (\delta - 1)\beta (\pi_{cc} \hat{c}_{t+1}^h + \pi_{cn} \hat{n}_{t+1}^h)] \\
& \bar{c}^h \hat{c}_t^h + \bar{c}^f \hat{c}_t^f = \bar{y}^h \left(\zeta_\lambda \hat{\lambda}_t^h + \zeta_k \hat{k}_t^h + \zeta_n \hat{n}_t^h + \zeta_z \hat{z}_t^h \right) + \bar{y}^f \left(\zeta_\lambda \hat{\lambda}_t^f + \zeta_k \hat{k}_t^f + \zeta_n \hat{n}_t^f + \zeta_z \hat{z}_t^f \right) - \delta \bar{k}^h \left(\frac{1}{\delta} \hat{k}_{t+1}^h - \frac{1 - \delta}{\delta} \hat{k}_t^h \right) - \delta \bar{k}^f \left(\frac{1}{\delta} \hat{k}_{t+1}^f - \frac{1 - \delta}{\delta} \hat{k}_t^f \right)
\end{aligned}$$

New final system of equations (only the first and fourth equations change):

$$\begin{aligned}
& [q_c + (1 + (\delta - 1)\beta) \tau_{kn} (\pi_{cc} - \pi_{nc})] \hat{c}_{t+1}^h \\
& + [\pi_{cn} \tau_{n\lambda} + (1 + (\delta - 1)\beta) (\tau_{kn} \tau_{n\lambda} + s\tau_{k\lambda})] \hat{\lambda}_{t+1}^h \\
& + [\pi_{cn} \tau_{nk} + (1 + (\delta - 1)\beta) (\tau_{kn} \tau_{nk} + s\tau_{kk})] \hat{k}_{t+1}^h \\
& + [\pi_{cn} \tau_{nz} + (1 + (\delta - 1)\beta) (\tau_{kn} \tau_{nz} + s\tau_{kz})] \hat{z}_{t+1}^h \\
= & p_c \hat{c}_t^h + p_\lambda \hat{\lambda}_t^h + p_k \hat{k}_t^h + p_z \hat{z}_t^h
\end{aligned}$$

$$\begin{aligned}
& [q_c + (1 - \beta) \tau_{zn} (\pi_{cc} - \pi_{nc})] \hat{c}_{t+1}^h \\
& + [\pi_{cn} \tau_{n\lambda} + (1 - \beta) (\tau_{zn} \tau_{n\lambda} + s\tau_{z\lambda})] \hat{\lambda}_{t+1}^h \\
& + [\pi_{cn} \tau_{nk} + (1 - \beta) (\tau_{zn} \tau_{nk} + s\tau_{zk})] \hat{k}_{t+1}^h \\
& + [\pi_{cn} \tau_{nz} + (1 - \beta) (\tau_{zn} \tau_{nz} + s\tau_{zz})] \hat{z}_{t+1}^h \\
= & q_c \hat{c}_t^h + p_\lambda \hat{\lambda}_t^h + p_k \hat{k}_t^h + p_z \hat{z}_t^h
\end{aligned}$$

$$\begin{aligned}
& q_c \hat{c}_t^h + p_\lambda \hat{\lambda}_t^h + p_k \hat{k}_t^h + p_z \hat{z}_t^h \\
= & q_c \hat{c}_t^f + p_\lambda \hat{\lambda}_t^f + p_k \hat{k}_t^f + p_z \hat{z}_t^f
\end{aligned}$$

$$\begin{aligned}
s\bar{k}^h \hat{k}_{t+1}^h + s\bar{k}^f \hat{k}_{t+1}^f = & [\zeta_n (\pi_{cc} - \pi_{nc}) \bar{y}^h - s\bar{c}^h] \hat{c}_t^h \\
& + [\zeta_n (\pi_{cc} - \pi_{nc}) \bar{y}^f - s\bar{c}^f] \hat{c}_t^f \\
& + \bar{y}^h (\zeta_\lambda s + \zeta_n \tau_{n\lambda}) \hat{\lambda}_t^h \\
& + \bar{y}^f (\zeta_\lambda s + \zeta_n \tau_{n\lambda}) \hat{\lambda}_t^f \\
& + [\bar{y}^h (\zeta_k s + \zeta_n \tau_{nk}) + (1 - \delta) s\bar{k}^h] \hat{k}_t^h \\
& + [\bar{y}^f (\zeta_k s + \zeta_n \tau_{nk}) + (1 - \delta) s\bar{k}^f] \hat{k}_t^f \\
& + \bar{y}^h (\zeta_z s + \zeta_n \tau_{nz}) \hat{z}_t^h \\
& + \bar{y}^f (\zeta_z s + \zeta_n \tau_{nz}) \hat{z}_t^f
\end{aligned}$$