Pre-recording:
Simulating an economy

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Disclaimer

The views expressed in this presentation, and all errors and omissions, should be regarded as those solely of the author, and are not necessarily those of the Bank of England or Qatar Central Bank.

Terminology note

We have discussed how the solution of the model yields **functions** that relate endogenous variables to the state of the economy

- It is common (if slightly awkward/misleading) to refer to these functions as **policy functions**
- This doesn't mean that we're talking only about variables controlled by 'Policymakers' (like central banks or fiscal authorities) or that we are referring to a Taylor Rule, or anything like that
- *It doesn't even necessarily mean* it's a variable chosen by an agent (e.g. the riskless bond price will have a 'policy function')

I will use the term 'policy functions' going forward

Simulating the economy - Only exogenous states

In the models we consider (the Tony 3 equation model and the Galì Ch. 3 model) we have a very simple structure to the state:

- Our state comprises 3 shocks
  - I added a monetary policy shock to Tony's model at the end of the week 6 lecture
  - See also the amended mod file uploaded to Keats
- The shocks are exogenous
  - They are all independent AR(1) processes
  - Their evolution is unaffected by agents' decisions

Simulating the economy - Only exogenous states

To simulate the economy, we can first draw all the shocks and then simply apply the 'policy functions' to those shocks **in one matrix multiplication**

- We can get a n_s-by-T matrix, S, from simulating the n_s separate AR(1)s over a sample period of length T
  - Each row corresponds to a particular shock (in our case $n_s = 3$)
  - Each column corresponds to a time period

- Since our solution is a bunch of linear functions, captured in a matrix of coefficients, C, the associated simulated endogenous variables are:

$$\mathcal{E} \equiv C * S$$

  - $\mathcal{E}$ is a n_{end}-by-T matrix, where n_{end} is the number of endogenous variables considered
  - Each row corresponds to a particular endogenous variable
  - Each column corresponds to a time period

Simulating the economy - Endogenous states

In most macro models there will be at least one 'endogenous' state:

- In time t it is not enough to know only the exogenous shocks, we may need to know other stuff
  - So the functions (from the solution of the model) connecting endogenous variables to the state, will load not only on shocks, but on other variables too
  - Those other variables are themselves **endogenous**, but **predetermined** from earlier periods (before t)

- The classic example would be the economy's **'capital stock'**
  - The amount of 'machines' sitting around to be used today will reflect **historical** shocks and decisions
  - An economy inheriting a lot of (little) predetermined capital, all else equal, will produce more (less)
  - Capital **tomorrow** will depend on our decisions taken **today**
  - Indeed, k_{t+1} will take a value given by a function of **today's state** that is part of the model solution ($k_{t+1} = f_k(s_t)$)

Simulating the economy - Endogenous states

What are other examples of endogenous states?

- Lagged consumption (in a model with habits)
- Lagged investment (in a model with investment adjustment costs)
- Lagged nominal interest rate (in a model with a policymaker hoping to smooth the path of policy changes)
- Banks or firms' net worth (in a model where financial frictions require them to have 'skin in the game' to obtain funding - see week 5)

In practice this means that obtaining the simulated endogenous variables is not as simple as first drawing all the exogenous variables and hitting them with a single matrix multiplication

Simulating the economy - Endogenous states

We still can simulate the exogenous states separately (obviously)

- But then we need to use a loop to calculate the evolution of the state
- Within that loop we need to use the policy functions for any variable that is an endogenous state variable to get the value of the state in the next period
  - In $t = 1$: What value should be assumed for the endogenous state (what happened in $t = 0$?)
  - Depends on application - but often start at 'steady state' or draw from unconditional distribution implied by equilibrium (will clarify)
- Can then stack the endogenous states with the exogenous states and calculate any further endogenous variables with a single matrix multiplication
  - But, in practice, typically will calculate *all* endogenous variables (not just those that are states) in the loop unless there is a computational issue (will clarify)

This is easier to show than to explain verbally - see the uploaded examples

Using the simulations

Once we have the simulation we can do various things with it:

- Common to calculate means, variances, correlations, autocorrelations
  - Matlab has routines to obtain these
- If the model is linear we can do much of this **without** simulation
  - Linearity - combined with Normal innovations - implies there are clean mathematical expressions we can appeal to
- But we will simulate as it's a good skill to have **since it is necessary for richer models**