Advanced Monetary Policy

Technical note

The Euler equation and asset pricing - APPENDIX

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Abstract

This note is a follow up to the Euler equation and asset pricing note, to clarify a couple of extra issues that arose when completing the homework and to provide a few additional insights.

Comment 1

For a random gross return from today to tomorrow, we will have the following Euler equation:

$$1 = E_1 \left[\Lambda_{1,2} R_2^{Risk} \right]$$

and recall that a return is just the payoff tomorrow, divided by price today

$$R_2^{Risk} = \frac{\mathcal{P}_2^{Risk}}{P_1^{Risk}}$$

so we can obtain a more obvious asset *pricing* equation by combining these expressions

$$P_1^{Risk} = E_1 \left[\Lambda_{1,2} \mathcal{P}_2^{Risk} \right]$$

This is what you were asked to calculate in the numerical asset pricing example. However, I had not (yet) written the Euler equation in this way so it may not have been obvious you were 'pricing' an asset. The point is that with optimal consumption and savings/investment, the SDF provides the appropriate weighting of the asset payoffs tomorrow, yielding its price today. The value of an asset is its discounted future payoffs and that should align with the price - assuming the agents is able to freely trade it on the margin.

You will have seen that intuition many times before - but here you see it where the discounting is by an SDF. Various 'real world' methods of discounting payoffs (e.g. finding a riskless rate and adding a spread, and then discounting cashflows) are trying to replicate this more theoretically grounded approach. Of course, what exactly $\Lambda_{1,2}$ should be is up for debate. In our simple model it is connected to a particular form of preferences and to consumption growth - i.e. we calculated the SDF and used it conditional on a particular model. That model (like all models) is likely not literally correct.

Comment 2

If the payoff is constant across all contingencies tomorrow we can take it 'out of the expectation'

$$P_1^{Safe} = \mathcal{P}_2^{Safe} E_1 \left[\Lambda_{1,2} \right]$$

which again is the price of the sort of 'riskless' asset we discussed in the main note (think high quality inflation indexed sovereign debt). Clearly (just rearrange), we have

$$\frac{\mathcal{P}_2^{Safe}}{P_1^{Safe}} = \frac{1}{E_1\left[\Lambda_{1,2}\right]}$$

or

$$R_2^{Safe} = \frac{1}{E_1\left[\Lambda_{1,2}\right]}$$

That is, the 'risk free rate' is the inverse of the expected value of the SDF. In our particular 'model' we have

$$\Lambda_{1,2} = \beta\frac{u'(c_2)}{u'(c_1)} = \beta\left(\frac{c_2}{c_1}\right)^{-\gamma}$$

so

$$R_2^{Safe} = \frac{1}{E_1\left[\Lambda_{1,2}\right]} = \frac{\beta^{-1}}{E_1\left[\left(\frac{c_2}{c_1}\right)^{-\gamma}\right]}$$

So economies where β is high ('want to save') and where consumption growth is low, we should see, all else equal, lower risk free rates.

There is a bit of complexity arising from the randomness that means we are still carrying around the E_1 expectations operator.[1] But actually the intuition for what drives riskless rates if we imagine a world without risk in consumption growth:[2]

$$R_1^{Safe} = \beta^{-1}\left(\frac{c_2}{c_1}\right)^{\gamma}$$

Declining riskless real rates (which for a given inflation target implies declining nominal rates - closer to the zero lower bound!) are sometimes connected to theories of slower growth rates in steady state, or to a 'desire to save' (a bit like higher β) among Asian countries, or in countries with a lot of people saving for a long (because of improved life expectancy) retirement.

[1]The inverse of expected SDF is not the same as the expectation of the inverse of the SDF due to Jensen's inequality

[2]Make sure you understand the difference between a world without risk (so consumption growth is known in advance, vs a world with risk, but where we are pricing a riskless bond. Or think of this as considering long horizon or steady state growth rates where we are pretty confident what trend growth is - even if one quarter ahead we are less sure.