Lecture 8
Structural parameters and the Lucas critique

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Disclaimer

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Structural parameters

Structural parameters

- Structural parameters are things like β (time preference), σ (intertemporal elasticity of substitution), α (production function curvature), ρ (AR1 shock persistence), ϕ_π (Taylor rule response to inflation)...
- They describe 'deep' and 'fixed' (or very slowly/rarely changing) aspects of the economy that appear in 'primitive' components of models (optimization problems, specifications of constraints, preferences, regulation, policy...)
- When we change one of them, there is no change in any of the others

Structural parameters

- Economists attempt to estimate structural models - and this means finding parameters that ‘match’ the data
  - May seek a **point estimate**
  - Or we may seek a **distribution** expressing our *beliefs* about the parameter, given a model and the observed data (and perhaps prior beliefs about what the parameter might be)
- Once we have them, we can solve the model and make policy predictions
  - If we have a distribution expressing our beliefs, we solve the model using draws from this distribution
  - So our forecasts reflect our uncertainty about underlying parameters
- A frequent use is to solve the model changing a structural parameter describing the policy approach
  - This is distinct from another common exercise, which is shocking the economy with a ‘policy shock’

Structural parameters

As discussed in class and shown in the textbook we have

$$\begin{aligned}\pi_t &= \beta E_t[\pi_{t+1}] - \lambda \hat{\mu}_t + \epsilon_{PC,t} \\ &= \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t + \epsilon_{PC,t}\end{aligned}$$

where

$$\begin{aligned}\lambda &\equiv \theta^{-1}(1-\theta)(1-\beta\theta)\Theta > 0 \\ \Theta &\equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \\ \kappa &\equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\end{aligned}$$

and where

$$\begin{aligned}\hat{\mu}_t &\equiv \mu_t - \mu \\ \mu_t &\equiv p_t - \psi_t \\ \tilde{y}_t &\equiv y_t - y_t^n\end{aligned}$$

Structural parameters

Examining the effects of changing structural parameters can be useful (even before we fully solve the model)

- **Example:** λ is decreasing in the 'Calvo parameter', θ
  - λ influences what value of inflation associated with a given expected path of markup deviations
  - Through κ, λ thus influences how inflation relates to future output gaps, i.e. **the slope of the Phillips curve**

Structural parameters

- Higher θ (more price stickiness) will imply a smaller response of inflation **(all else equal)** to fluctuations in output
  - The 'flattening of the PC' has been a hot policy topic in the last decade
  - To the extent that CBs affect inflation by manipulating the output gap, it suggests larger moves are required to bring π back to target
- Some people think that a 'flattening PC' is why the labor market got very tight (pre-COVID) without substantial inflationary pressure (and why weakness in the great recession didn't drag inflation down as far as expected)
  - *Note:* this argument is often made loosely and frequently takes a 'non-equilibrium' flavor in the sense of ignoring what shocks might have caused the lack of co-movement
  - This is why I constantly say things like 'all else equal'
  - We are looking at an equilibrium condition, not a solved model or a policy function

Structural parameters

Intuition:

- θ being larger naturally suggests more inertia, all else equal, since fewer firms can change prices in a given period
- Note however, that it is not *completely* obvious *a priori*
- One might have wondered if, in equilibrium, the changes made by the firms that *can* reset their prices might be larger to 'compensate' for a larger θ...
- ...possibly to the extent that the net effect would be to make the change in the price level bigger for a given expected path of markups
- However, it turns out that is not the case here

Again - be careful - π, $E_t[\pi_{t+1}]$ and $\hat{\mu}_t$ all depend on the shocks (including $\epsilon_{PC,t}$) in the economy so their comovement/correlation does not only depend on λ

- Contrast this with a regression with uncorrelated shocks

Structural parameters

$$
\begin{aligned}
\mu_t &= p_t - \psi_t \\
&\overset{\psi_t}{=} -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&\overset{HHOLD}{=} -(\sigma c_t + \varphi n_t^S) + (a_t - \alpha n_t^D + \log(1-\alpha)) \\
&= -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha) \qquad (1)
\end{aligned}
$$

Structural parameters

Let us consider the coefficient on y_t in equation (1)

- First note that $-\mu_t$ (the negative of the log markup) is here equal to (log) real marginal cost (right?)
- Things are more intuitive if we explain the association between marginal cost and output. Thus

$$mc_t \equiv \psi_t - p_t = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t - \left(\frac{1 + \varphi}{1 - \alpha}\right) a_t - \log(1 - \alpha)$$

Structural parameters

Recall the production function

$$Y_t = A_t N_t^{1-\alpha}$$

So if $\alpha = 0$ we have constant returns to scale

$$Y_t = A_t N_t$$

No diminishing marginal product of labor (slope is always A_t)

- And marginal cost will be constant

Structural parameters

Let us make the assumption of constant returns to scale in the production technology ($\alpha = 0$).

- Then the coefficient simply reflects the household's intratemporal optimality condition **and** we have substituted y_t for c_t
- These two elements reflect two features of equilibrium - optimality and market clearing
- The higher is φ, the more an additional hour detracts from utility
- Thus to obtain the greater labor supply associated with greater output, the wage must be higher
- This raises marginal cost, all else equal

Structural parameters

What about the role of σ?

- If σ increases, the marginal utility of consumption is, all else equal, reduced
- Thus, the value of an additional hour to the household is reduced on the margin (consumption purchased with the additional wages yields less utility)
- So, again, all else equal, the wage - and thus the marginal cost - must increase more within any increase in output

Note that we aren't making causal statements - these are simply statements about associations between variables that must hold in equilibrium.

Structural parameters

Interestingly, in equilibrium, there is a positive association between marginal cost and the scale of output (for a given a_t) **even if** there are constant returns to scale ($\alpha = 0$)

- Under a CRS assumption, *from the perspective of an individual wage-taking firm*, MC is independent of their scale of production
  - They treat the wage as given and marginal product of labor is simply A_t
  - Recall, marginal cost is wage over marginal product of labor
- The positive association arises in equilibrium however because wages must adjust
  - In order to make the changed level of output consistent with optimal labor supply decisions by the household
- If there are *decreasing* returns to scale ($\alpha > 0$), marginal cost is positively related to scale, even from the firm's perspective - it
  - This feeds through in the aggregate also. Hence α appearing in the coefficient on y_t.

Structural parameters

If you remember from the Tony 3-equation pre-record (week 6), the coefficients defining the solution of the economy were

$$(c_{i,PC}, c_{i,IS}, c_{y,PC}, c_{y,IS}, c_{\pi,PC}, c_{\pi,IS})'$$

and they were obtained by

$$A^{-1}b$$

where. . .

Structural parameters

$$b = (0, 0, 0, -1, -1, 0)'$$

and

$$A = \begin{pmatrix} -1 & 0 & \alpha_y & 0 & \alpha_\pi & 0 \\ 0 & -1 & 0 & \alpha_y & 0 & \alpha_\pi \\ -\sigma & 0 & \rho_{PC} - 1 & 0 & \rho_{PC} & 0 \\ 0 & -\sigma & 0 & \rho_{IS} - 1 & 0 & \rho_{IS} \\ 0 & 0 & \kappa & 0 & \beta\rho_{PC} - 1 & 0 \\ 0 & 0 & 0 & \kappa & 0 & \beta\rho_{IS} - 1 \end{pmatrix}$$

Structural parameters

What is the solution?

$$
\begin{pmatrix}
\dfrac{\alpha\pi + \alpha y\ \rho PC - \alpha\pi\ \rho PC}{1 + \beta\ \rho PC^2 + \alpha y\ \sigma + \alpha\pi\ \kappa\ \sigma - \rho PC\ (1 + \beta + \kappa + \alpha y\ \beta\ \sigma)} \\
\dfrac{\alpha y + \alpha\pi\ \kappa - \alpha y\ \beta\ \rho IS}{1 + \beta\ \rho IS^2 + \alpha y\ \sigma + \alpha\pi\ \kappa\ \sigma - \rho IS\ (1 + \beta + \kappa + \alpha y\ \beta\ \sigma)} \\
\dfrac{\rho PC - \alpha\pi\ \sigma}{1 + \beta\ \rho PC^2 + \alpha y\ \sigma + \alpha\pi\ \kappa\ \sigma - \rho PC\ (1 + \beta + \kappa + \alpha y\ \beta\ \sigma)} \\
\dfrac{1 - \beta\ \rho IS}{1 + \beta\ \rho IS^2 + \alpha y\ \sigma + \alpha\pi\ \kappa\ \sigma - \rho IS\ (1 + \beta + \kappa + \alpha y\ \beta\ \sigma)} \\
\dfrac{1 - \rho PC + \alpha y\ \sigma}{1 + \beta\ \rho PC^2 + \alpha y\ \sigma + \alpha\pi\ \kappa\ \sigma - \rho PC\ (1 + \beta + \kappa + \alpha y\ \beta\ \sigma)} \\
\dfrac{\kappa}{1 + \beta\ \rho IS^2 + \alpha y\ \sigma + \alpha\pi\ \kappa\ \sigma - \rho IS\ (1 + \beta + \kappa + \alpha y\ \beta\ \sigma)}
\end{pmatrix}
$$

Structural parameters

Note how each parameter pervades all the coefficients - through nonlinear transformations

- This leads to ‘cross equation’ restrictions
- You can’t (shouldn’t) model the dependence of one variable on another separately from how you model another variable’s dependence
- Both will be functions of (typically) the same - small set of - parameters
- You have far fewer degrees of freedom than you (or early post war modelers) think!

Structural parameters

Recall our dynare ‘example1.mod’ RBC model

- In particular, remember that k_{t+1} was a function of k_t and the shocks
- The state in that model was k_t and the shocks, ϵ_t
- The shocks can be modeled as a VAR

$$\epsilon_t = A_\epsilon \epsilon_{t-1} + u_t$$

Structural parameters

We can actually model a new vector, $v_t \equiv (k_{t+1}, \epsilon_{t+1})'$ as a VAR

$$v_t = A_v v_{t-1} + \tilde{u}_t$$

where A_v is essentially A_ϵ but augmented with a row containing the policy function coefficients for the choice of capital tomorrow, and where u_t is essentially u_t but with a zero on top

- Clearly, we can obtaining other endogenous variables by $C \times v_t$ where C contains the other policy function coefficients
- But note that all the dynamics/correlations, autocorrelations will be driven by the coefficients in C and the coefficients in A_v in a highly nonlinear way
- If we estimated an *unrestricted* VAR on observed data, we would get the estimates of these combinations of deep structural parameters

If we change a structural parameter, all the aspects of the VAR, in general, will change

Lucas critique

Lucas critique

The ‘policy functions’ functions that relate the endogenous variables to the state, will generally depend on *all* structural parameters in the economy

- This dependence will typically be highly nonlinear

A parameter from one part of the economy (say, a Taylor rule coefficient) can influence the shape of *any of these functions*

- This is an example of the ‘cross equation restrictions that are the hallmark of rational expectations’ (a phrase typically attributed to my PhD adviser, Tom Sargent)

Lucas critique

Why does this matter?

- Suppose you have estimated a VAR (summarizing covariances between variables and across time) on data from a period where a parameter takes a particular value
- Imagine the parameter being the coefficient on inflation in the Taylor rule
- Suppose a policymaker asks you what the effect of changing the policy parameter will be, you cannot/should not assume *only* the comovement of i_t and π_t changes

Lucas critique

We refer to 'parameters' of non-structural models (such as a correlation coefficient, or coefficients of a VAR) as being 'reduced form'

- They encode lots of structural parameters (relating to preferences, technologies, shock processes, policy. . .)

In the post-war period large VARs (and other systems of equations) were estimated, yielding lots of reduced form coefficients

- The coefficients (including some that were 'arbitrarily' set to zero) were often treated as invariant to policy changes
- But while structural parameters are supposed to be invariant to policy changes, reduced form parameters are not

Lucas critique

- Most famous example of Lucas Critique is perhaps the assumption that the (apparent) relationship between inflation and output, noted by Phillips (the original Phillips curve) was something that could be exploited by policymakers
- The relationship emerged in a period where the ‘policy parameters’ were such that policymakers were not trying to engineer lower unemployment by increasing inflation
- Once policymakers did begin to attempt this (i.e. when they changed their policy parameters) the correlation between inflation and output weakened/vanished, so people ended up with higher inflation and no lower unemployment

Lucas critique

The 'Lucas Critique' (after Bob Lucas) is the formalization of this intuition:

> *Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models.*

- Lucas (1976)

Lucas critique

The Lucas Critique is one of the most important cautionary insights for economic policymakers to be aware of

- Absolutely key (ideally with a model or less formally by extended introspection/thought) that one considers the effects of proposed policy changes from the perspective of 'structural' modelling
- If you have a model (and you trust it) you should resolve the model under the new parameters and see what the policy change implies
- Catastrophic policy errors can be (and were and still are) made from ignoring this intuition
- The Lucas critique is a big part of why we use structural models in policy (though that comes with costs - **do you have the right model?**)