Lecture 6
The New Keynesian model

Rhys Bidder

KBS/QCGBF

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Disclaimer

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Key features of the New Keynesian model

Key features of the New Keynesian model

The New Keynesian model shares many common features with 'classical' models but. . .

1. The NK model has monopolistically competitive firms
   - Firms have pricing power (face downward sloping demand curve)
   - Household consumes a bundle of different consumption goods
   - Willingness to substitute between these goods is the source of firms' market power

2. NK model features price stickiness
   - Various ways to model this
   - *Common assumption:* A fraction of firms are randomly 'allowed' to change prices in each period

Key features of the New Keynesian model

Key equations within any NK model are:

1. Dynamic IS curve
   - Essentially this is an Euler equation
   - Captures consumption (and here output) behavior and connects to current and future real rates
   - Part of 'demand' block for economy ($\approx$ IS in IS-LM but micro-founded)
2. New Keynesian Phillips Curve
   - Captures optimal price setting by firms
   - 'Supply' block for the economy
   - Level of (current and future) real activity relevant for price setting, so feedback from 'demand' block
3. Monetary policy (Taylor) rule
   - Combined with DIS, completes the 'demand block'
   - But policy assumed to respond to prices (inflation) so there is feedback from the 'supply' block

There may be other equations but something like these 3 elements will essentially always be present

Households

Households - Preferences

Objective function of a household (see here for summation notation)

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U \left(C_t, N_t; Z_t \right) \right] \tag{1}$$

Looks quite familiar but...

- Infinite horizon (rather than our two period examples in the technical notes earlier in term)
- C_t is a bundle of different goods (see next slide)

$$C_t \equiv \left(\int_0^1 C_t \left(i \right)^{\frac{\varepsilon - 1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon - 1}}$$

- Utility also depends (negatively) on labor supply, N_t
- There is a mysterious 'preference shock', Z_t (will discuss)

Households - Preferences

$$C_t \equiv \left(\int_0^1 C_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Sometimes this is referred to as a 'Dixit-Stiglitz' aggregator

- It aggregates (adds up) contributions from different types of goods
- Each good is indexed by $i \in [0, 1]$ (and will be produced by a monopolistically competitive firm)
- $C(i)$ is a particular type of consumption good
- C_t is 'overall' consumption

Households - Preferences

Following Gali we will assume (again, with log utility as a limiting case of the CRRA consumption component)

$$U(C_t, N_t; Z_t) \equiv \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t \tag{2}$$

where $z_t \equiv \log Z_t$ follows an AR(1)

$$\begin{aligned} z_t &= \rho_z z_{t-1} + \epsilon_{z,t} \\ \epsilon_{z,t} &\sim N(0, \sigma_z^2) \end{aligned}$$

WARNING: σ is the coefficient of relative risk aversion

- Not its inverse (the elasticity of intertemporal substitution) as in earlier notes

Households - Preferences

Comparing lifetime expected utility (1) with the period utility function (2) we see that the **effective** time discount factor is $\beta^t Z_t$, from the perspective of time 0

- Thus, Z_t is a preference shock that affects how 'patient' the household is
- High (low) Z_t means more (less) patient
- Thus a positive (negative) Z_t surprise will imply a negative (positive) demand shock

Households - Budget constraint

The budget constraint is

$$\int_0^1 P_t(i)C_t(i)di + Q_{n,t}B_t \leq B_{t-1} + W_tN_t + D_t$$

where we note that each consumption good has its own price, $P_t(i)$

- Note households earn nominal wage, W_t for labor supplied, N_t
- They receive payments from riskless nominal bonds purchased in the previous period
- They also receive dividend income (there will be monopolist firms owned by the households)

Households - Multiple goods

At any optimum, the household must maximize C_t for any amount spent on consumption goods

- The $C_t(i)$ must be chosen appropriately, given the prices the household faces
- Optimal allocation across goods (see Galí Ch. 3 appendix) yields the following condition...

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t \tag{3}$$

Households - Multiple goods

Optimal demand across goods

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

This is associated with a price index *defined* as

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

Why is this an appropriate definition of a price index?

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t \tag{4}$$

Households - Multiple goods

Relative to overall demand (C_t), demand for good i decreases in its relative price

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

The elasticity of demand, ε, controls strength of this effect

- Large $\varepsilon \Rightarrow$ large decline in demand for good i
- Looking ahead - this will affect pricing power of firms
- High (low) elasticity $\Rightarrow$ low (high) market power
- $\varepsilon \to \infty$ represents price taking / perfect competition

Note: This is the demand structure discussed in the monopolistic competition pre-recording and accompanying note.

Households - Optimality

Equation (4) means that the budget constraint can be re-written as

$$P_t C_t + Q_{n,t} B_t \leq B_{t-1} + W_t N_t + D_t$$

Consequently, we have the following optimality conditions:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{5}$$

$$Q_t = \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right] \tag{6}$$

Equation (5) captures optimal labor supply, while equation (6) is our (now familiar) Euler equation

- Recall the nominal riskless rate is Q_t^{-1}

Households - Optimality

Where does this come from?

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

Rewrite and use our 'marginal benefit' = 'marginal cost' intuition for optimality

$$-U_{n,t} = U_{c,t}\frac{W_t}{P_t}$$

- **LHS (marginal cost of more work):** The negative of the marginal utility of marginally more work (marginally more N will lower utility so $U_{n,t}$ will be negative)
- **RHS (marginal benefit of more work):** If I work marginally more, I get paid the real wage (W/P) for that, which yields consumption goods, which I value, on the margin, using marginal utility of consumption (U_c)

Households - Optimality

The Euler equation is

$$Q_t = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \Pi_{t+1}^{-1} \right]$$

or

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \Pi_{t+1}^{-1} R_t^{Nom} \right]$$

where we have noted the relationship between the gross riskless nominal rate and the price of a nominally riskless one period bond (also $\Pi_t \equiv P_t / P_{t-1}$ is the gross inflation rate)

- Consulting the technical notes - what is the nominal SDF here?

Households - Optimality

The real SDF is

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right)$$

The nominal SDF is

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \Pi_{t+1}^{-1}$$

(so the real SDF but with a tweak to convert nominal payoffs to real)

Households - Optimality

Approximating around the zero inflation steady state we obtain (lower case means logs and we denote the nominal riskless net interest rate associated with R_t^{Nom} with i_t)

$$
\begin{aligned}
c_t &= E_t[c_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - \rho) + \frac{1}{\sigma}(1 - \rho_z)z_t \\
\rho &\equiv -\log \beta
\end{aligned}
$$

(**you have to trust me on this as I don't want to teach you Taylor approximations**)

This the log-linearized (i.e. approximate) Euler equation

- It looks very much like the DIS from the 3-equation NK model
- What is the difference? Consumption vs Output?
- Note the role of z_t as a now (sort of) micro-founded demand shock

Firms

Firms - Monopolistic Competition

There is a continuum of firms indexed by $i \in [0,1]$

- They each produce a different good
- Identical production function
- Common technology (assume $\alpha \in (0,1)$)

$$Y_t = A_t N_t(i)^{1-\alpha}$$

The technology process, $a_t \equiv \log A_t$, follows an AR(1)

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_t^a \\ \varepsilon_t^a &\overset{iid}{\sim} N\left(0, \sigma_a^2\right) \end{aligned}$$

Firms - Monopolistic Competition

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

Implies a demand curve for firm i's good, given C_t and P_t

- Note: **Aggregate** price level and consumption are taken as given
- Firm can choose **its** price and (thus) **its** quantity

The firms thus operate in a monopolistically competitive environment

- 'Monopolistic' - they are the only producer of their good i and can set their price
- 'Competitive' - goods partly substitutable (limits market power)
- Only producer of energy drink x but if 'too expensive' people will shift to energy drink y

Firms - Monopolistic Competition

In a 'standard' monopolistically competitive situation

- Firm sets their price as a markup over marginal cost
- Given demand curve $\Rightarrow$ picking output and employment
- $\varepsilon \to \infty$ shows no markup in perfectly competitive limit

$$P_t(i)^* = \frac{\varepsilon}{\varepsilon - 1} MC_t \tag{7}$$

$$MC_t = \frac{W_t}{(1-\alpha) A_t N_t(i)^{-\alpha}} \tag{8}$$

But in the NK model the firm may be unable to set their price as they wish

- See pre-record (though most material repeated here)

Firms - Price Stickiness

Each firm may only reset its price with probability $1 - \theta$

- See Calvo (1983)
- Same across firms in each period
- Independent of time since the firm last was able to reset its price

This means that in each period a fraction θ of firms keep prices unchanged

- Continuum of firms $i \in [0, 1]$
- Law of large numbers

Average duration of a given price $= \frac{1}{1-\theta}$

- Natural to interpret θ as an index of price rigidity or 'stickiness'

Firms - Price Stickiness

In every period the distribution of prices across firms is a mixture of

1. The price of the $1 - \theta$ of firms who get to reoptimize
   - All set the same price since they face the same optimization problem
2. The prices of the θ fraction of firms whose prices were reoptimized before t but are now fixed
   - Among these prices, a fraction $1 - \theta$ were reoptimized in $t - 1$ and a fraction θ were reoptimized before $t - 1$
   - Among those prices reoptimized before $t - 1$, a fraction $1 - \theta$ were reoptimized in $t - 2$ and a fraction θ were reoptimized before $t - 2$
   - Continue the logic...

Firms - Optimal Pricing

In t we have prices prevailing that were reoptimized in the current period *and all previous periods*

- The fraction of prices in t that were set in $t-j$ is declining (to zero) as $j \to \infty$

When firms set their prices they do so acknowledging that...

- There is a distribution of prevailing prices now and in the future
- Their own price will prevail for a random length of time into the future

The problem is thus very different from standard 'static' monopolistic competition that implied

$$P_t(i)^* = \frac{\varepsilon}{\varepsilon - 1} MC_t$$

Firms - Optimal Pricing

Since a firm's price will prevail (with some probability) for several periods after it is set, the firms must consider the implications of that price for *future* profits in those contingencies

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t} \right) \right]$$

This looks (and is quite) complicated but we will go through it carefully and see that it is very intuitive after all...

- **Not as scary as it looks!**

Firms - Optimal Pricing

Contribution to the value of a firm, in t, of profits *in periods and contingencies in which* P_t^* *prevails:*

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t} \right) \right]$$

Note that profits from periods/contingencies in which P_t^* has been superseded are not relevant for the choice of P_t^*

- Those aren't in the sum, as the choice of P_t^* is irrelevant
- When the *next* chance to reoptimize comes they are free to choice whatever

Firms - Optimal Pricing

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t} \right) \right]$$

- $\sum_{k=0}^{\infty}$ is just notation meaning we are adding up terms involving k allowing k to take values from 0 to ∞ (again see summation notation)
- θ_k is the probability of P_t^* still prevailing k periods from t
- $\Lambda_{t,t+k}$ values the stream of real profits (k-step household SDF because households are the shareholders)
  - Will involve β^k and marginal utility adjustments for risk
- $Y_{t+k|t}$ and $N_{t+k|t}$ are the output and associated employment in $t+k$ **for firms who last reset their price in t**
- $P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t}$ are nominal profits
- Dividing by P_{t+k} converts nominal profits to real

Firms - Optimal Pricing

Useful to clarify components of the maximand

$$
\begin{aligned}
W(P_t^*) &\equiv \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(\mathcal{R}(Y_{t+k|t}, P_t^*) - \mathcal{C}(Y_{t+k|t}) \right) \right] \\
\mathcal{R}(Y_{t+k|t}, P_t^*) &\equiv P_t^* Y_{t+k|t} \\
\mathcal{C}(Y_{t+k|t}) &\equiv W_{t+k} \mathcal{N}(Y_{t+k|t})
\end{aligned}
$$

where we define the employment level induced by $Y_{t+k|t}$ as

$$
\mathcal{N}(Y_{t+k|t}) \equiv \left(\frac{Y_{t+k|t}}{A_{t+k}} \right)^{\frac{1}{1-\alpha}}
$$

Firms - Optimal Pricing

Using the demand curve implied by optimal allocation across goods

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k}$$

It is useful to define nominal marginal cost

$$\Psi_{t+k|t} \equiv \frac{d\mathcal{C}(Y_{t+k|t})}{dY_{t+k|t}}$$

Firms - Optimal Pricing

As stated in Galí (p. 56) the FOC for the choice of P_t^* can be rearranged to be

$$\sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} Y_{t+k|t} \frac{1}{P_{t+k}} \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right] = 0 \tag{9}$$

If $\theta = 0$ then we are back in the static monopolistic competitive case (**as in the pre-record**)

- Convention is $0^0 \equiv 1$
- Recover static optimal markup of $P_t^* = \mathcal{M}\Psi_t$
- Call $\mathcal{M}$ the 'desired' or 'natural' or 'flex-price' markup

Firms - Optimal Pricing

If $\theta \in (0, 1)$ then the static condition will (generically) not hold

$$P_t^* \neq \mathcal{M}\Psi_t$$

However, firms are setting prices to try to keep the deviations from this condition 'small' in all periods

- The optimality condition is a weighted sum of deviations of the firm's price in $t + k$ from $\mathcal{M}\Psi_{t+k|t}$
- Intuition for weights...
  - $\theta^k \Rightarrow$ particularly care about near future
  - $\Lambda_{t,t+k} \Rightarrow$ particularly care about reduced profits when MU is high
  - $Y_{t+k|t} \Rightarrow$ static suboptimality more concerning if producing a lot

Note we implicitly assume firms always provide what is demanded at their prevailing price

Firms - Optimal Pricing

Approximating around the zero inflation steady state we obtain (lower case means logs)

$$
\begin{aligned}
p_t^* &= \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\psi_{t+k|t}\right] \qquad (10) \\
\mu &\equiv \log \mathcal{M}
\end{aligned}
$$

(**you have to trust me on this as I don't want to teach you Taylor approximations**)

- Firms markup by $\mathcal{M}$ but not over current marginal cost
- Instead they markup over a weighted average of future marginal costs
- Weights $\propto$ time discount (β^k) and probability of price prevailing (θ^k)

Thus, firms set prices in a **forward looking** manner

Firms - Optimal Pricing

'Unwinding' the logs we get

$$
\begin{aligned}
P_t^* &= \exp\left\{\mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\psi_{t+k|t}\right]\right\} \\
&\equiv \exp\{\mu\} \times \exp\left\{(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\psi_{t+k|t}\right]\right\} \\
&\equiv \mathcal{M} \times \exp\left\{(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\psi_{t+k|t}\right]\right\}
\end{aligned}
$$

So it's the 'standard' markup, but not **only** on current marginal cost, but a combination (suitably weighted) of current **and** future marginal cost

Firms - Aggregate Price Dynamics

As shown in the text...

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

where, recall, $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate

If we take a log-linear approximation and rearrange we obtain

$$p_t = \theta p_{t-1} + (1-\theta)p_t^* \tag{11}$$

Thus the current price level is a weighted average of last period's price level and the new reset price

- Price level evolves as p_t^* typically $\neq p_{t-1}$
- Weights are intuitively connected to θ
- Another example of difficult algebra leading to an intuitive outcome

Firms - Aggregate Price Dynamics

What ingredients do we now have?

- An expression of optimal reset price in terms of future marginal costs
- A firm production function that can link marginal cost to output
- An expression of the overall price level as a combination of the lagged price level (reflecting a chunk of firms not resetting) and the current optimal reset price (reflecting a chunk of firms resetting)

You can probably **already see** how these - suitably combined - can lead to the NKPC connecting inflation (the change in the overall price level) to current and future activity

Price setting and inflation

Price setting and inflation

This stuff gets quite messy

- Don't despair - you don't need to know much in detail
- I will be emphasizing the intuition and the *end point* of the derivations

Price setting and inflation

Recall that firm marginal cost is wage over marginal product of labor (this is from taking logs of equation (8) using - again - the log tricks from the math note):

$$\psi_t(i) = w_t - (a_t - \alpha n_t(i) + \log(1-\alpha))$$

Then, using the approximation $n_t = \int_0^1 n_t(i)di$ (average firm employment is sum of all firms) we can show

$$\psi_{t+k|t} = \psi_{t+k} + \alpha(n_{t+k|t} - n_{t+k})$$

If $\alpha > 0$, firms who haven't set prices since t have MC higher than the average if their employment levels are relatively high (why?)

- $\Leftrightarrow$ their output is relatively high (why?)
- $\Leftrightarrow$ their price is relatively low (why?)

$$\psi_{t+k|t} = \psi_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k})$$

Price setting and inflation

We derived an expression for p_t^* earlier in terms of expected $\psi_{t+k|t}$ - combining that with the expression for $\psi_{t+k|t} \dots$

$$\pi_t = \beta E_t[\pi_{t+1}] - \lambda \hat{\mu}_t$$

with

$$\begin{aligned} \lambda &\equiv \theta^{-1}(1-\theta)(1-\beta\theta)\Theta > 0 \\ \Theta &\equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \end{aligned}$$

and where

$$\begin{aligned} \hat{\mu}_t &\equiv \mu_t - \mu \\ \mu_t &\equiv p_t - \psi_t \end{aligned}$$

Price setting and inflation

Inflation reflects expected path of markup 'gaps'

$$\pi_t = -\lambda \sum_{k=0}^{\infty} \beta^k E_t[\hat{\mu}_{t+k}]$$

- Markups expected to be below desired $\Rightarrow \pi_t > 0$
- Markups expected to be at desired level $\Rightarrow \pi_t = 0$
- Markups expected to be above desired level $\Rightarrow \pi_t < 0$

Price setting and inflation

Expressed more intuitively perhaps. . .

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k E_t[\hat{mc}_{t+k}]$$

where $\hat{mc}_t$ is the deviation of real marginal cost from desired

- Marginal cost expected to be below desired $\Rightarrow \pi_t < 0$
- Marginal cost expected to be at desired level $\Rightarrow \pi_t = 0$
- Marginal cost expected to be above desired level $\Rightarrow \pi_t > 0$

Intuition: pricing decisions of reoptimizing firms tends to restore (but not immediately/completely) the desired markup and these adjustments drive inflation

Triumph of NK theory - micro-founded price stickiness

Price setting and inflation

We want to connect the markup to the level of activity (y_t) in the economy

- Note that $\mu_t = -mc_t$ where mc_t is **real** marginal cost
- Use this to derive an expression for μ_t in terms of y_t and a_t

$$
\begin{aligned}
\mu_t &= p_t - \psi_t \\
&= -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -(\sigma y_t + \varphi n_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)
\end{aligned}
$$

Price setting and inflation

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\begin{aligned}
\mu_t &= p_t - \psi_t \\
&\overset{\psi_t}{=} -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&\overset{HHOLD}{=} -(\sigma c_t + \varphi n_t^S) + (a_t - \alpha n_t^D + \log(1-\alpha)) \\
&= -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)
\end{aligned}
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Price setting and inflation

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&= -(\sigma y_t + \varphi n_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)
\end{aligned}$$

Yeah, it's ugly but all this is saying is that marginal cost is related to output and the technology level

Flexible price economy and ‘natural’ variables

Flexible price economy and 'natural' variables

A very important concept to grasp is the 'natural' value of a (real) variable

- It is the value that prevails in the 'flexible price' form of this model
- We obtain this by setting $\theta = 0$
  - No price stickiness
  - All firms can reset price in each period
  - Markups are always = desired ($P_t = \mathcal{M}\Psi_t$)
  - Firms are always operating at desired scale

Not equivalent to a '**steady** state'

- Fluctuations in technology (and more generally, supply-side factors, will shift the natural rate over time
- **It isn't constant**
- Compare to the BoE trying to figure out how much of the Brexit shock was demand vs supply

Flexible price economy and 'natural' variables

Since $\theta = 0$ is just a special case of the economy we have been discussing, all our equations still apply

$$\mu = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t^n + \left(\frac{1 + \varphi}{1 - \alpha}\right) a_t + \log(1 - \alpha)$$

Thus by imposing that the markup is constantly at the desired level, we can *define* the natural rate of output

$$\begin{aligned} y_t^n &= \psi_y + \psi_{y,a} a_t \\ \psi_y &\equiv -\frac{(1 - \alpha)(\mu - \log(1 - \alpha))}{\sigma(1 - \alpha) + \varphi + \alpha} \\ \psi_{y,a} &\equiv \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} \end{aligned}$$

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\psi_y &\equiv -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha) + \varphi + \alpha} \\
\psi_{y,a} &\equiv \frac{1+\varphi}{\sigma(1-\alpha) + \varphi + \alpha}
\end{aligned}$$

Flexible price economy and 'natural' variables

Note connection of markups with SS of y_t^n...

- To see this, put a_t to its steady state / unconditional mean of zero

$$
\begin{aligned}
y_t^n &= \psi_{yn} + \psi_{yn,a} a_t \\
\psi_{yn} &\equiv -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} \\
\psi_{ny,a} &\equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}
\end{aligned}
$$

This means that even eliminating price stickiness won't get us back to full efficiency

- Emphasizes that NK model differs from classical models in having price stickiness **and** in having competitive distortions

Flexible price economy and 'natural' variables

Returning to the expression for the natural rate of output...

$$y_t^n = \psi_{yn} + \psi_{yn,a} a_t$$

The natural rate of output **does not depend on**

- z_t
- Monetary policy

Gaps between natural and actual versions of variables reflect price stickiness

- We will see price stickiness implies real effects of monetary policy
- Monetary policy can influence these gaps

Flexible price economy and 'natural' variables

We define the '*output gap*'

$$\tilde{y}_t \equiv y_t - y_t^n$$

The phrase 'output gap' is used by various people with various meanings

- In the NK model it means something *very* specific
- y_t^n is a particular theoretical object and not some smooth 'trend' or 'moving average'
- y_t^n emerges from an imaginary world with flexible prices and a very particular structure

An empirical question whether commonly used 'trends' are similar to y_t^n

- Remember, our model is only a simplification of reality
- Using a statistical rather than model-based gap may still be useful

Flexible price economy and 'natural' variables

Using (for y_t **and** y_t^n)...

$$\mu_t = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t + \left(\frac{1 + \varphi}{1 - \alpha}\right) a_t + \log(1 - \alpha)$$

combined with...

$$\pi_t = \beta E_t[\pi_{t+1}] - \lambda \hat{\mu}_t$$

we obtain

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t \tag{12}$$

where

$$\kappa \equiv \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t$$

Flexible price economy and ‘natural’ variables

This relation is called the ‘New Keynesian Phillips Curve’ (NKPC)

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t$$

- Phillips (1958) observed an (apparent) relationship between (wage) inflation and a measure of unemployment
- Investigating the relationship between (wage and/or price) inflation and activity (employment, growth, output gap . . .) has been a central macro question since. . . forever!
- For a long time a relationship was asserted (particularly by Keynesians) but with little or no theoretical foundations
- The New Keynesian model proposes microeconomic foundations that are internally consistent, within a General Equilibrium framework

May not be an ideal model - **but a huge intellectual achievement!**

Price setting and inflation

To obtain an 'Dynamic IS' relationship we use the household intertemporal optimality condition (with $y_t = c_t$ implicit)

$$y_t = E_t\left[y_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \frac{1}{\sigma}\left(\rho + (1-\rho_z)\, z_t\right)$$

Setting $y_t = y_t^n$, and using our solution for y_t^n, we define

$$\begin{aligned} r_t^n &\equiv -\sigma(1-\rho_a)\psi_{y,a} a_t + \rho + (1-\rho_z) z_t \\ &\equiv \psi_{rn} + \psi_{rn,a} a_t + \psi_{rn,z} z_t \end{aligned} \quad (13)$$

This 'natural' real interest rate would prevail in a flexible price economy and we can show

$$\begin{aligned} \tilde{y}_t &= E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - r_t^n\right) \\ &= -\frac{1}{\sigma}\sum_{k=0}^{\infty} E_t[r_{t+k} - r_{t+k}^n] \end{aligned} \quad (14)$$

$$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - r_t^n\right)$$

Price setting and inflation

At this point we have

$$\begin{aligned}
\tilde{y}_t &\equiv y_t - y_t^n && (15)\\
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t && (16)\\
\tilde{y}_t &= E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - r_t^n\right) && (17)\\
y_t^n &= \psi_{yn} + \psi_{yn,a} a_t && (18)\\
r_t^n &= \psi_{rn} + \psi_{rn,a} a_t + \psi_{rn,z} z_t && (19)
\end{aligned}$$

These equations constitute the 'non-policy' block of the NK model

- NKPC determines inflation given an expected path for the output gap
- DIS determines the output gap given an expected path for the natural and actual real interest rates
- The real interest rate reflects expected inflation and the nominal interest rate, which is set by policy

Price setting and inflation

At this point we have

$$\tilde{y}_t \equiv y_t - y_t^n \tag{20}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t \tag{21}$$

$$\tilde{y}_t = E_t[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(i_t - E_t[\pi_{t+1}] - r_t^n\right) \tag{22}$$

$$y_t^n = \psi_{yn} + \psi_{yn,a} a_t \tag{23}$$

$$r_t^n = \psi_{rn} + \psi_{rn,a} a_t + \psi_{rn,z} z_t \tag{24}$$

These equations constitute the ‘non-policy’ block of the NK model

- NKPC determines inflation given an expected path for the output gap
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Equilibrium - Introducing Policy

Equilibrium - Introducing policy

We can't solve for the real variables without specifying monetary policy

- Price stickiness $\Rightarrow$ expected inflation does not move 1:1 with i_t
- $r_t = i_t - E_t[\pi_{t+1}] \Rightarrow r_t$ is affected by policy actions
- Agents' actions influenced by r_t (intertemporal terms of trade)

We imagine i_t being set according to a policy 'rule'...

$$
\begin{aligned}
i_t &= \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t \\
\hat{y}_t &= y_t - y \\
v_t &= \rho_v v_{t-1} + \varepsilon_t^v
\end{aligned}
$$

These sort of rules are called 'Taylor Rules' (after Taylor 1993 and 1999)

- Assume $\phi_\pi > 1$ and it is standard to assume $\phi_y > 0$
- v_t is a policy 'shock'

Equilibrium - Introducing policy

The system we must solve is

$$
\begin{aligned}
i_t &= \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t \\
&= \rho + \phi_\pi \pi_t + + \phi_y \tilde{y}_t + \phi_y \hat{y}_t^n + v_t \\
r_t^n &= \rho - \sigma(1-\rho_a)\psi_{yn,a} a_t + (1-\rho_z) z_t \\
\tilde{y}_t &= -\frac{1}{\sigma}\left(i_t - E_t[\pi_{t+1}] - r_t^n\right) + E_t[\tilde{y}_{t+1}])
\end{aligned}
$$

The solution will be expressions for $\tilde{y}_t$ and π_t in terms of some combination of shocks (a_t, z_t and v_t)

- Recall, we already know r_t^n and y_t^n in terms of shocks

Equilibrium - Introducing policy

Although it is natural to look for equilibrium functions

$$
\begin{aligned}
\pi_t &= \psi_{\pi,a} a_t + \psi_{\pi,z} z_t + \psi_{\pi,v} v_t \\
\tilde{y}_t &= \psi_{\tilde{y},a} a_t + \psi_{\tilde{y},z} z_t + \psi_{\tilde{y},v} v_t
\end{aligned}
$$

Galí looks for expressions in terms of the 'composite' shock, u_t

$$
\begin{aligned}
\pi_t &= \psi_{\pi,u} u_t \\
\tilde{y}_t &= \psi_{\tilde{y},u} u_t \\
u_t &= -\psi_{yn,a} \left(\phi_y + \sigma(1-\rho_a)\right) a_t + (1-\rho_z) z_t - v_t
\end{aligned}
$$

He then makes the assumption that u_t follows an AR(1)

- Given this, he derives $\psi_{\pi,u}$ and $\psi_{\tilde{y},u}$
- Only correct if one shock hits at any one time
- It *works* but it makes things unclear