Lecture 7
Policy in the New Keynesian model

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Disclaimer

The views expressed in this presentation, and all errors and omissions, should be regarded as those solely of the author, and are not necessarily those of the Bank of England or Qatar Central Bank.

Inefficiency in the NK model

Inefficiency in the NK model

- We have shown (pre-record on effect of shocks) that monetary policy can have real effects in the New Keynesian model
- We also note that the New Keynesian model features distortions that lead to inefficiencies
- Ability to affect an economy featuring inefficiencies $\Rightarrow$ possible role for policy interventions

Steady state (in)efficiency in the NK model

Recall from the previous lecture:

- Steady state value of y_t^n (and y_t) was lower than in Classical model
- Related to the pricing power of monopolistically competitive firms
- Markup: $\mu \equiv \log(\mathcal{M}) \equiv \log(\frac{\varepsilon}{\varepsilon-1})$

Natural rate of output in NK model

$$\begin{aligned} y_t^n &= \psi_{yn} + \psi_{yn,a} a_t \\ \psi_{yn} &\equiv -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} \end{aligned}$$

Output in Classical model

$$\begin{aligned} y_t^c &= \psi_{yc} + \psi_{yc,a} a_t \\ \psi_{yc} &\equiv \frac{(1-\alpha)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \end{aligned}$$

An efficient benchmark

Loosely speaking, efficiency $\Rightarrow$ MRS $=$ MRT which in this case implies

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

But we know in the flex price equilibrium

$$-\frac{U_{n,t}}{U_{c,t}} \overset{HHOLD}{=} \frac{W_t}{P_t} \overset{FIRM}{=} \frac{MPN_t}{\mathcal{M}} < MPN_t$$

An efficient benchmark

Additional conditions that must be satisfied by an efficient allocation are. . .

- All goods (indexed by i) should be consumed (and thus produced) in the same quantities
$$C_t(i) = C_t \ \forall i \in [0, 1]$$
- And all firms (each identified with a good, i) should employ the same amount of labor
$$N_t(i) = N_t \ \forall i \in [0, 1]$$

Let us consider the first. . .

Digression on Dixit-Stiglitz aggregation

Recall our household preferences are defined with consumption being made up of a Dixit-Stiglitz bundle of different consumption goods

$$C_t \equiv \left(\int_0^1 C_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

All the goods enter in the same way and the aggregator is concave

- This means that, for a given amount of expenditure, the household would *ideally* like to consume equal amounts of each good
- They won't if the relative prices of the goods differ (they respond to some goods being relatively expensive/inexpensive)
- Why price dispersion is **'a bad thing'**
- Price dispersion will arise if there is inflation, combined with staggered (Calvo) price setting by firms

Digression on Dixit-Stiglitz aggregation

Consider a two good case:

$$
\begin{aligned}
C_t &= \left(\sum_{i=1}^{2} C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
&\equiv \left(C_t(1)^{\frac{\varepsilon-1}{\varepsilon}} + C_t(2)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}
$$

Digression on Dixit-Stiglitz aggregation

Imagine a baseline where the same amount is consumed of both goods

$$C_t = \left(\bar{C}^{\frac{\varepsilon-1}{\varepsilon}} + \bar{C}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} = 2^{\frac{\varepsilon}{\varepsilon-1}} \bar{C}$$

Versus a proportional reduction in one and an increase in the other

(assume $\Delta > 0$) so that $C_t(1) = (1+\Delta)\bar{C}$ and $C_t(2) = (1-\Delta)\bar{C}$ in which case

$$C_t = \left((1+\Delta)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\Delta)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \bar{C}$$

Digression on Dixit-Stiglitz aggregation

So we need to compare

$$2 \qquad vs \qquad (1+\Delta)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\Delta)^{\frac{\varepsilon-1}{\varepsilon}}$$

Which is larger? Well, note that since $\varepsilon > 1$

$$\frac{\varepsilon - 1}{\varepsilon} \equiv 1 - \frac{1}{\varepsilon} \in (0,1)$$

Let's consider $f(x) = x^{\theta}$ for $\theta \in (0,1)\ldots$

Digression on Dixit-Stiglitz aggregation

Recalling our math note and the discussion on differentiation, we have the function, its first derivative and its second derivative as

$$\begin{aligned} f(x) &\equiv x^{\theta} \\ f'(x) &= \theta x^{\theta-1} \\ f''(x) &= (\theta-1)\theta x^{\theta-2} \end{aligned}$$

We note that (for $x > 0$ which is the relevant set of values) f and f' are positive functions but f'' is negative (as $\theta - 1 < 0$)

- f' being positive means f is increasing in x
- But negative f'' means it is increasing at a slower rate, as x increases
- So we lose more from subtracting Δ from 1, than we gain from adding Δ to 1
- Thus $2 > (1+\Delta)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\Delta)^{\frac{\varepsilon-1}{\varepsilon}}$

An efficient benchmark

Employment subsidy at rate $\tau \Rightarrow$ effective wage paid by firms is $(1-\tau)W_t$

- Then firm optimality implies

$$\frac{W_t(1-\tau)}{P_t} = \frac{MPN_t}{\mathcal{M}}$$

- So that, if τ is set to be $= \varepsilon^{-1}$ we recover

$$\frac{W_t}{P_t} = MPN_t$$

An efficient benchmark

Under the flex-price equilibrium or the zero inflation **steady state** of the sticky price model

- All firms are producing the same amounts
  - *Why?* They face the same technology and prices
- Each (identical) household consumes the same amount of each good
  - *Why?* All goods have the same price (steady state means any dispersion has worked its way out of the system) and enter symmetrically in the concave utility function

Thus, the employment subsidy restores efficiency in the flex-price equilibrium and **the steady state** of the sticky price model

- **Note:** The employment subsidy is not something the central bank can implement but monetary policy can attempt to reduce price dispersion

An efficient benchmark

Even if the *steady state* of the NK model is efficient under the subsidy this does not mean it is efficient in any given period

- Due to price stickiness, the average markup will vary over time and differ from $\mathcal{M}$
  - Average marginal cost will vary with average scale of production
  - Prices do not adjust fully to reflect this
  - Would need a time varying subsidy but only have constant $\tau = \varepsilon^{-1}$

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \frac{\mathcal{M}}{\mathcal{M}_t}$$

- Due to price stickiness there will be dispersion in prices
  - Leads to dispersion in consumption and employment across firms/goods
  - Violates efficiency conditions for consumption and resource allocation

Optimal Allocation

Optimal Allocation

Suppose we start from a steady state situation

- All firms were setting the same price in the previous period
- Price was at desired markup over (subsidy adjusted) marginal cost
- All firms operate on the same scale
- Goods are consumed in the same quantity
- Output is at its natural level

Optimal Allocation

If shocks hit the economy, how should policy respond?

- The aim is to preserve $y_t = y_t^n$ (or $\tilde{y}_t = 0$) since y_t^n is efficient
- Since this must be part of an equilibrium, the NKPC must hold
- Iterating the NKPC forwards $\Rightarrow$ if $\tilde{y}_t = 0\ \forall t$ then $\pi_t = 0\ \forall t$

$$
\begin{aligned}
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t \\
&= \beta E_t[\beta E_{t+1}[\pi_{t+2}] + \kappa \tilde{y}_{t+1}] + \kappa \tilde{y}_t \\
&\ldots \\
&= \kappa \sum_{j=0}^{\infty} \beta^j E_t[\tilde{y}_{t+j}]
\end{aligned}
$$

Optimal Allocation

From an alternative perspective. . .

- Assume $\pi_t = 0 \; \forall t$ and all firms are initially at their desired markup
- If policy is such that marginal cost is stabilized, then the existing price will continue to be optimal
  - Since the markup is already at desired level and the marginal cost to which the markup is applied is unchanged
- No firm (*even the* $1-\theta$ *who* **can** *reset*) will want to change price
  - Thus inflation will be zero
  - Price stickiness irrelevant (like being in flex price)
- Output is equal to natural $\Rightarrow$ constant real marginal cost
  - Thus, under zero inflation, we have constant nominal marginal cost
  - Justifies firms not changing prices

Optimal Allocation

If policy achieves price stability then it also coincidentally achieves $y_t = y_t^n$

- **'Divine coincidence'**
- No trade-off between price stability and goals for y_t
- If you pursue price stability then 'coincidentally' you will keep $y_t = Y_t^n$

Obviously this is a stark result in a very particular stylized model

Optimal Allocation

Note that efficiency does not imply constant activity

- MC_t is stabilized such that the desired markup $\Rightarrow$ constant P_t
- But *output* can still vary in an efficient allocation
  - $\tilde{y}_t = 0 \Rightarrow y_t = y_t^n$
  - y_t^n depends on a_t

This reflects one of the **main insights** of the RBC literature (business cycles $\not\Longrightarrow$ market failure)

- Take a moment to grasp this

Optimal Allocation

Nice quote from Galí p. 104

> *The intuition behind the desirability of zero inflation in the case of an efficient natural allocation can be conveyed as follows: if price stability is attained, then it must be the case that no firm is adjusting its price even when having the option to do so, from which it follows that the constraints on price setting are not binding and, hence, that the equilibrium allocation corresponds to that of an economy with flexible prices (which is, under the assumptions made here, efficient).*

Optimal Policy

Optimal Policy

What interest rate policy is consistent with the optimal allocation as an *equilbrium* outcome?

- $y_t = y_t^n$ combined with the DIS curve implies $r_t = r_t^n$
- Zero inflation $\forall t$ implies $i_t = r_t$ (by the Fisher equation)

Thus under optimal policy, in equilibrium,

$$i_t = r_t^n \tag{1}$$

But is this an adequate *rule* for how the interest rate should be set in all contingencies?

Optimal Policy

$i_t = r_t^n$ holds in our desired equilibrium with $\tilde{y}_t = \pi_t = 0$

- But it *also* can hold in other less desirable equilibria
- In these equilibria we do not have $\tilde{y}_t = \pi_t = 0$
- Thus we lose the desired efficiency properties
  - We can have $i_t = r_t^n$ but if $\pi_t \neq 0\ \forall t$ then r_t will deviate from r_t^n
  - Then we cannot guarantee that $y_t = y_t^n\ \forall t$

Equation (1) derived 'assuming' optimal allocation ($\tilde{y}_t = \pi_t = 0\ \forall t$)

- Doesn't allow for possibility of deviations from the optimal allocation
- This 'opens the door' to alternative allocations
- Needs to be augmented with response to 'off equilibrium' outcomes

Optimal Policy

Consider instead two alternative rules

- A rule that responds to realized inflation and activity

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \tilde{y}_t$$

- A rule that responds to forecasts/expectations of inflation and activity

$$i_t = r_t^n + \phi_\pi E_t[\pi_{t+1}] + \phi_y E_t[\tilde{y}_t]$$

Let us simplify these rules further

$$\begin{aligned} i_t &= r_t^n + \phi_\pi \pi_t \\ i_t &= r_t^n + \phi_\pi E_t[\pi_{t+1}] \end{aligned}$$

Optimal Policy

Explicit adjustments to the simple ($i_t = r_t^n$) policy if π_t not as desired

- $i_t = r_t^n$ **if** $\pi_t = 0$ or $E_t[\pi_{t+1}] = 0$, respectively, but...
  - $\pi_t > 0$ or $E_t[\pi_{t+1}] > 0 \implies i_t > r_t^n$
  - $\pi_t < 0$ or $E_t[\pi_{t+1}] < 0 \implies i_t < r_t^n$

Assume that $\phi_\pi > 1$ (and, for the forecast rule, that ϕ_π is not 'too big')

- In this case the **only** equilibrium possible is the desired one
- $\tilde{y}_t = \pi_t = 0 \ \forall t$ so in equilibrium the adjustments never get made and $i_t = r_t^n$ after all!
- But the 'threat' of those adjustments eliminates other equilibria

A plan for rates should specify actions **even in contingencies that should not occur** under the plan!

Optimal Policy

Some intuition for the importance of $\phi_\pi > 1$ can be gained from the simplified forecast rule (where we ignore the response to the output gap)

$$i_t = r_t^n + \phi_\pi E_t[\pi_{t+1}]$$

Using the Fisher equation this implies that

$$r_t = r_t^n + (\phi_\pi - 1)E_t[\pi_{t+1}]$$

$\phi_\pi >$ or < 1 determines whether r_t rises or falls with $E_t[\pi_{t+1}]$

- Consider an example of an 'inflation scare' to illustrate implications of this. . .

Optimal Policy

If $\phi_\pi > 1$, an increase in expected inflation $\implies r_t \uparrow$, all else equal

- But we know $r_t \uparrow$ is contractionary and drives inflation down, so $\pi_{t+1} \downarrow$ in expectation
- But that **contradicts** assumption of higher inflation expectations!
- $\implies$ only $\pi_t = 0 \ \forall t$ is consistent with equilibrium

If $\phi_\pi < 1$, an increase in expected inflation $\implies r_t \downarrow$, all else equal

- But we know $r_t \downarrow$ is expansionary and drives inflation up, so $\pi_{t+1} \uparrow$ in expectation
- That is **consistent with** assumption of higher inflation expectations
- $\implies \pi_t \neq 0$ is consistent with equilibrium

Optimal Policy

The ‘Taylor principle’ ($\phi_\pi > 1$) is desirable partly because it ensures policy responds ‘sufficiently strongly’ to inflationary pressure

- Note that $\phi_\pi > 1$ ensures zero inflation in equilibrium
- But then we recover $i_t = r_t^n$ and the expectation response term is ‘dormant’ in equilibrium
- Nevertheless, its presence is vital to eliminate other equilibria

Simple Policy Rules

One problem with specifying policy simply as $i_t = r_t^n$ is that it allows 'multiple equilibria'

- We saw a way to 'fix' this was to specify 'off equilibrium path' behavior

But all of these approaches require knowledge of r_t^n

- In practice, that's not easy (in fact, it's effectively impossible)
- See recent debates about 'r^*' (pronounced r-star)

Knowing r_t^n requires exact knowledge of

- The exact structure of the economy's 'true model'
- The values taken by all its parameters (likely changing over time)
- The realized value of all the shocks that influence r_t^n

Simple Policy Rules

The previous rules are too ‘complicated’ - hence people have proposed the use of ‘simple’ rules

- Informed by some of the same logic but...
  - Depend only on observable variables
  - Don’t require deep knowledge of (all the) structural parameters and shocks
- Will not be ‘optimal’ but should perform ‘reasonably well’
  - The rules considered earlier were optimal but infeasible
- Should be robust to a range of parameter values and sources of shocks
  - If being slightly wrong about a parameter is disastrous - then this is a bad rule!

Simple Policy Rules

We consider a simple 'Taylor rule' (inspired by Taylor (1993))

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t \tag{2}$$

where $\hat{y}_t \equiv y_t - y$ (log deviation from steady state - **not natural**) and ϕ_π and ϕ_y are set to ensure a unique equilibrium

Requires relatively little knowledge about the structure of the economy

- Still assumes approximate knowledge of β (ρ) and $\bar{y}$
- But see Levin *et al* (1998) and Orphanides and Williams (2002, 2006) for 'difference rules' that address this issue
- A related and very readable discussion of the role of rules is this speech (hyperlink to Williams (2016))

Simple Policy Rules

To assess the performance of the rule for a given parameterization we use

$$\mathcal{L} \propto \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) var(\tilde{y}_t) + \frac{\varepsilon}{\lambda} var(\pi_t)$$

Welfare loss arises from $\pi_t \neq 0$ and $y_t \neq y_t^n$

- Derived via approximation to the welfare of representative household (Rotemberg and Woodford (1999))
- Weights are functions of the deep parameters
- Loss will be > 0 (unless the rule replicates optimal policy)

Simple Policy Rules

Suppose technology shocks (a_t) are the only shocks hitting the economy

- Tradeoff: stabilizing y_t vs. stabilizing π_t and the (welfare-relevant) $\tilde{y}_t$
- $\phi_y \Uparrow \implies$ Less volatile y_t but more volatile π_t and $\tilde{y}_t$
- Welfare declines as $\phi_y \Uparrow$
- Losses reduced if only respond to π_t ($\phi_y = 0$) and decline as $\phi_\pi \uparrow$

Loose intuition for this tension...

- Recall that technology shocks tend to move output and inflation in *opposite* directions...
- ...but the output *gap* and inflation in the *same* direction
- Policy should loosen after a positive shock - but a (big enough) positive ϕ_y means policy tightens

Simple Policy Rules

Suppose demand shocks (z_t) are the only shocks hitting the economy

- No tradeoff: stabilizing $y_t \Leftrightarrow$ stabilizing π_t and $\tilde{y}_t$
- $\phi_y \Uparrow \Longrightarrow$ Less volatile y_t **and** less volatile π_t and $\tilde{y}_t$
- Welfare improves as $\phi_y \Uparrow$ or as $\phi_\pi \Uparrow$

Why the absence of a tradeoff?

- y_t^n is unaffected by demand shock (z_t) so output gap moves 1:1 with output

These results suggest a sensible rule would be to respond fairly aggressively to inflation

- Inflation response superior if there are supply shocks
- Indifferent if there are no supply shocks

Caveat

Why not simply set $\phi_\pi \to \infty$?

- Sometimes called the 'inflation nutter' approach to policy
- In this simple model it essentially implements optimal policy

Beyond the scope of this course (see Ch. 5 if interested) but...

- We have been assuming that y_t^n is efficient
- Means that price stability is consistent with ideal 'activity' outcomes
- In richer models it may not be

There may be reasons to weight price stability against variation in some measure of output or employment (or financial imbalances?)

- Our model seems to be missing something
- No central bank thinks that focusing **purely** on price stability will achieve a desirable outcome

Summary

Summary

- Under the assumption of a subsidy that makes our economy's *steady state* efficient, remaining inefficiencies arise from price stickiness
- This implies a role for policy to set rates such that price stability is ensured - resulting in output being equal to the natural rate in every period (which may vary over time with technology shocks)
- Under optimal policy (featuring zero inflation and output equal to natural) the nominal interest rate equals the natural real rate
- To ensure our desired equilibrium, policy should also specify how it will respond appropriately to deviations from desired outcomes
- But such policies often require an implausible degree of knowledge of the economy
- 'Simple rules' may come close to achieving the same equilibrium and are implementable in the real world
- Supply shocks are more difficult for simple rules to handle than demand shocks