Pre-recording: *Heterogeneity*

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Disclaimer

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Aggregation and the representative agent

Aggregation and the representative agent

In all of our NK discussions we have implicitly been using models where the economies behaved as if there were a single household

- Actually, this is consistent with there being many underlying firms or households
- But the structure of the economy (complete markets, absence of distortions) means that they 'behave the same'
- If we think of households as initially being identical (same wealth, say) then with complete financial markets they will insure away any idiosyncratic shocks and end up consuming (and supplying labor) in the same way
- Even if they differ in initial wealth, then they will consume different amounts but (in our simple models) consumption **growth** will be the same, so the Euler equation will hold when we use aggregate consumption growth instead of individual households

Aggregation and the representative agent

We can also introduce heterogeneity to firms (and banks) in richer models

- In fact, we have a form of heterogeneity arising from price stickiness
- The household can insure away any firm-specific risk though, by holding a diversified portfolio of firms
- The household doesn't care which firms get which shocks - overall there will always be a share θ whose prices are stuck
- Calvo doesn't introduce aggregate risk

Heterogenous firm models are heavily used - and heterogenous **bank** models are an active area of research

- But I will focus on the household dimension

Aggregation and the representative agent

Where are all the households?

- In our analysis, there could be multiple households, indexed by i
- We have been working with C_t notation rather than $C_{i,t}$ etc. - are we missing something?
- If we assume N households, the aggregate variables are just the sum over $i \in \{1, ..., N\}$

$$C_t \equiv \sum_{i=1}^{N} C_{i,t}$$

- But we like to think of 'lots' of 'small' households, so typical instead to say $i \in [0, N]$ (there is a 'measure N' of households)

$$C_t \equiv \int_0^N C_{i,t} di$$

- Household 'small' in the sense that C_t unaffected by any single $C_{i,t}$

Aggregation and the representative agent

How many households?

- There are uncountably many but the 'mass' or 'measure' of them is N

$$\text{Mass of households} = \int_0^N 1\, di = N$$

- What is 'average' consumption?

$$\bar{C}_t \equiv \frac{\int_0^N C_{i,t} di}{N}$$

But there are uncountably many households in $[0,1]$ or in $[0,N]$ so let's make algebra easier and say $N{=}1$

- It's not as if we care if it's N or 1
- This is just a normalization

Aggregation and the representative agent

The average is equal to the sum (aggregate consumption) *with 'unit mass' of households*

$$\bar{C}_t \equiv \frac{\int_0^1 C_{i,t} di}{1} \equiv \int_0^1 C_{i,t} di$$

So let's just drop the bar and use the aggregate notation

$$C_t \equiv \int_0^1 C_{i,t} di$$

Aggregation and the representative agent

'Traditional' NK models assume 'all households are the same', due to complete markets allowing insurance of any idiosyncratic risk

- Literally the same (including wealth), so they all choose same $C_{i,t}$
- Or, they could differ in wealth, but their consumption growth (or ratios of marginal utilities across time and contingencies) are the same
- Loosely, you can think of households consuming shares, λ_i of aggregate consumption ($C_{i,t} = \lambda_i C_t$)
- Poor (rich) households have low (high) λ_i but they all share

$$g_{i,c,t} = \frac{\lambda_i C_t}{\lambda_i C_{t-1}} \equiv \frac{C_t}{C_{t-1}} \equiv g_{c,t}$$

Only need the latter for Euler equations and other marginal optimal conditions to hold in terms of aggregate variables

Aggregation and the representative agent

If the economy behaves as if everyone is like the average agent, then it means we can basically just think about the aggregate.

- Why? We have just shown that aggregate and average can be trivially aligned by careful notation and (trivial) choice of unit mass

Suppose the representative agent consumes $C_{t,rep}$ and the economy allows us to think that the households all behave like this agent then

$$C_t \equiv \int_0^1 C_{rep,t} di = C_{rep,t} \int_0^1 1 di = C_{rep,t}$$

So you might as well do all the algebra etc. without worrying about i

- If you solve for all the households and *then* aggregate, you will get the same implications for the macroeconomy

Aggregation and the representative agent

Let us consider an Euler equation for a household

$$1 = E_t \left[\beta \frac{u'(C_{i,t+1})}{u'(C_{i,t+1})} R_t \right]$$

or, assuming a particular u, perhaps

$$1 = E_t \left[\beta \left(\frac{C_{i,t+1}}{C_{i,t+1}} \right)^{-\sigma} R_t \right]$$

but that implies

$$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_{t+1}} \right)^{-\sigma} R_t \right]$$

and if we (in our simple model) recall $C_t = Y_t$ we get the dynamic IS

$$1 = E_t \left[\beta \left(\frac{Y_{t+1}}{Y_{t+1}} \right)^{-\sigma} R_t \right]$$

Heterogeneity in macro

Heterogeneity in macro

In the real world the economy likely *doesn't* allow us to ignore heterogeneity

- So solving as if there is one representative agent who consumes the average **will not give you the same result** in the aggregate as explicitly solving for all the different households and then aggregating/averaging

$\Rightarrow$ Even if you only care about the aggregate dynamics, you need to consider heterogeneity

Heterogeneity is also interesting **in its own right** - for questions over inequality, understanding individuals' behavior, testing models of market incompleteness, asset pricing, labor supply...

Heterogeneity in macro

We will (briefly) discuss two dimensions:

- The role of monetary policy in inequality
- Heterogeneous Agent New Keynesian (HANK) models

The role of monetary policy in inequality

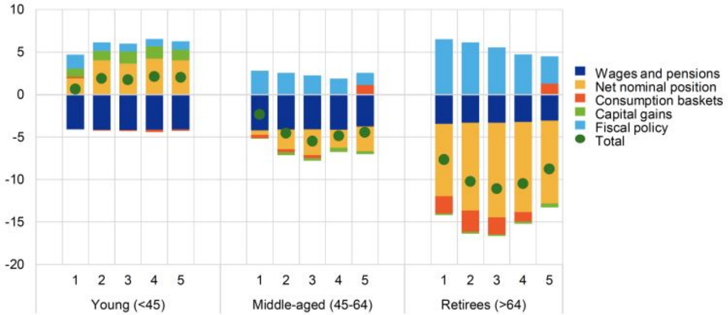
Frequently (especially after QE) people claim that monetary policy exacerbates inequality due to its positive impact on asset prices

- Logic is that rich households already hold the most assets
- Central banks (rightly or wrongly) have begun to respond to this (though not necessarily in terms of setting policy differently)

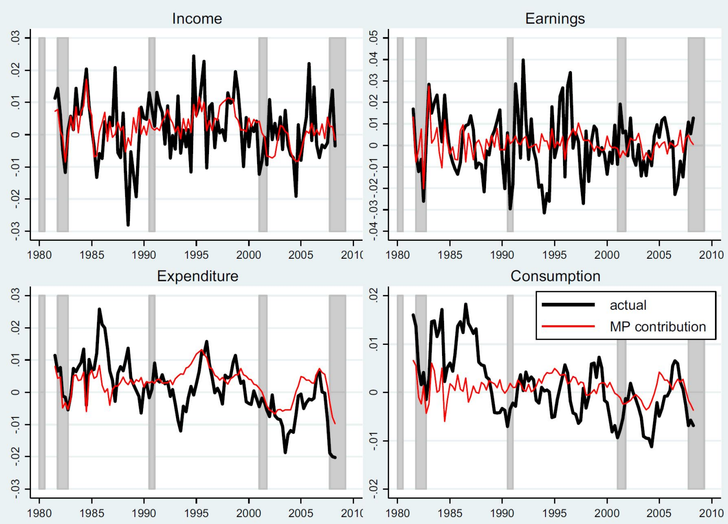
But this is only one channel

- To begin with, 'rich' households are often 'old' and thus rely disproportionately on fixed incomes (and hold a lot of nominal assets)
- Expansionary policy may erode their real income through inflation (see here for a nuanced analysis of inflation's distributional effects)
- In contrast those in debt (esp. with mortgages) may benefit from (unexpected) inflation, as it reduces the debt burden

Some oft-cited research suggests that *contractionary* policy might increase inequality



Welfare costs of the inflation surge across euro area households, by age groups and income quintiles (percentages of 2021-22 income). Source: VoxEU - Pallotti et al (2024)



The contribution of monetary policy shocks to historical variation in U.S. inequality. Source: Coibion et al (2017)

The role of monetary policy in inequality

Different groups consume different bundles of goods

- Poor people disproportionately consume 'necessities' and may have difficulty substituting to other goods
- In the recent inflationary period, this cause very different 'personal' inflation rates
- But is this because of monetary policy, primarily?

Inflation can lead to tax creep

- Nominally defined tax brackets are only infrequently adjusted
- Higher inflation pushes poorer people into the higher brackets
- But adjustments do eventually take place...

The role of monetary policy in inequality

Monetary policy likely contributes to *reducing* inequality as a by-product of pursuing its standard mandates

- Flexible inflation targeting with a concern for real activity is thought to reduce overall volatility
- Recessions are *particularly* devastating for the poor, and can lead to long term scarring
- Avoiding them likely makes an enormous contribution to reducing inequality

The role of monetary policy in inequality

Monetary policy's ability to affect long run trends is limited

- Increases in inequality in many western countries are such trends
- Implausible that monetary policy could be the main driver
- Particularly since loosening is followed by tightening, over the business cycle

Desired degree of inequality is clearly a political/fiscal issue - and multi-dimensional

- Central bank has limited tools and already has multiple goals
- Hard to square democratic legitimacy of a CB with redistribution
- There are many other levers governments can pull

HANK models

We have been dealing with RANK models

- **Representative** agent New Keynesian models

Increasingly popular (even among policymakers - but still mainly among academics) are HANK models

- **Heterogeneous** agent New Keynesian models

HANK models

Heterogeneous agent models with purely *real* elements have been advanced over the past few decades

- See seminal work of Huggett (1993), Aiyagari (1994), Krusell and Smith (1998) - and earlier work by Bewley
- Complicated models that require more sophisticated (global) solution methods (but conceptually we still seek 'policy functions', like dynare)
- Came to great popularity in recent times partly due to a) computational/algorithmic advances and b) a greater emphasis on microfoundations allied with microeconomic data

Build upon more realistic 'Bewley' or 'income fluctuation' problems

- Households do not have access to a complete of securities
- Typically assumed to have access to a bank account
- Assumed they face limits to their borrowing (as in the real world)

HANK models

Household i solves the following problem

$$\max_{\{C_{i,t}\}_{t=0}^{T}} E_0 \sum_{t=0}^{T} \beta^t u(C_{i,t})$$

subject to

$$C_{i,t} + B_{i,t+1} = RB_{i,t} + Y_{i,t}$$

and

$$B_{i,t+1} \geq -\bar{B}$$

with $Y_{i,t}$ some random income process.

HANK models

When studying the NK model we 'hid' the fact that households implicitly had access to a complete set of securities

- We went so far as to ignore any idiosyncratic uncertainty so there was no need to mention them - we proceeded as if there was 1 household
- Had we started out looking at distinct households we would have had a budget constraint 'something like'

$$C_{i,t} + B_{i,t+1} + \int_{s_{i,t+1} \in \mathcal{S}} Q_t(s_{i,t+1}) a_{i,t+1}(s_{i,t+1}) ds_{i,t+1} = RB_{i,t} + Y_{i,t} + a(s_{i,t})$$

where $a(s_{i,t})$ are holdings of 'Arrow Securities'

HANK models

$$C_{i,t} + B_{i,t+1} + \int_{s_{i,t+1} \in \mathcal{S}} Q_t(s_{i,t+1}) a_{i,t+1}(s_{i,t+1}) ds_{i,t+1} = RB_{i,t} + Y_{i,t} + a(s_{i,t})$$

- We imagine that the household faces some idiosyncratic risk captured in the state $s_{i,t}$ and the state can take values in the set $\mathcal{S}$
- There is an Arrow Security associated with each possible realization and they pay of 1 unit if that value is realized and nothing otherwise
- If households trade these securities, in equilibrium they can eliminate all idiosyncratic risk
- In which case, it has no effect on their behavior or the behavior of the economy in the aggregate, hence starting with

$$C_{i,t} + B_{i,t+1} = RB_{i,t} + Y_{i,t}$$

HANK models

By reducing the set of securities to simply a riskless bond (a bank account) and imposing borrowing constraints, we change the problem **massively**

- Euler equations may not hold with equality (when the borrowing constraint binds)
- Agents try to build precautionary savings until they feel ‘safe’
- The general desire to self-insure by ‘over saving’ drives down the riskless interest rate in equilibrium
- When agents have low savings or are at their borrowing constraint, they may respond very differently to additional income or to interest rates

Because agents’ marginal propensities to consume depend on their wealth level, the whole distribution of wealth becomes relevant for determining how the aggregate economy behaves

HANK models

In our models we have had (for a real riskless asset)

$$u'(c_t) = \beta E_t[u'(c_{t+1})]R_t$$

But suppose one is borrowing constrained, and can't raise c_t 'enough'

$$u'(c_t) > \beta E_t[u'(c_{t+1})]R_t$$

The modeling of consumption growth (and implicitly asset pricing) becomes far more complex

- Implicitly, we also have multiple stochastic discount factors
- We no longer have complete markets **and** we have restricted borrowing

HANK models

In RANK models, average agent determines what happens in aggregate

- I could reallocate wealth arbitrarily among households and it would not affect the aggregate
- The representative agent is unaffected, even if some households are better or worse off

In HANK models, economies with the same average wealth different inequality will respond differently to policy

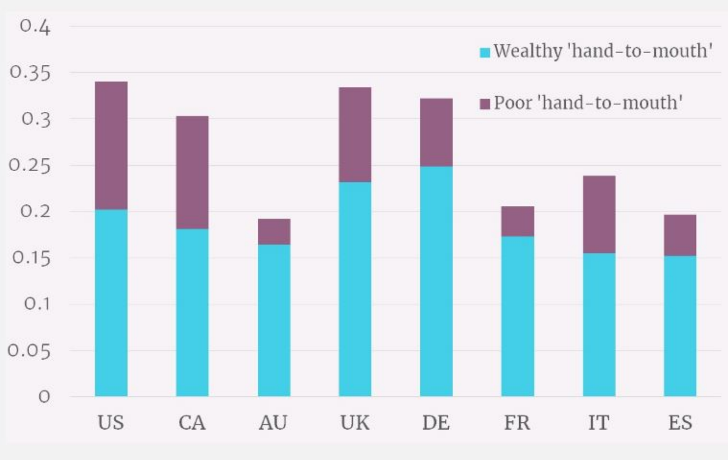
- They will also be associated with different asset prices and aggregate consumption, among other things. . .
- Imagine a policy that takes funds from the rich and gives to the poor - that is likely to raise consumption overall because the poor have higher MPCs

Even if you don't care *per se* about heterogeneity/inequality, ignoring it will undermine modeling of the aggregate

HANK models

In HANK models there can be a weakening of the impact of monetary policy's *direct* transmission through responses to the interest rate

- Other channels, operating through income and wealth can become more influential
- Fiscal policy and the differential effects of taxes also come into play more prominently
- Poor households **and** the 'wealthy hand to mouth' have a high MPC out of income - implying different dynamics from RANK models in response to, say, tax cuts and other income innovations
- Binding borrowing constraints violates assumptions underlying Ricardian equivalence, changing the impact of deficit vs. tax financing



Fraction of poor hand-to-mouth and wealthy hand-to-mouth in various countries. Source: Kaplan and Violante (2016)