Advanced Monetary Policy

Technical note

Euler equation and inflation

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Spring 2024

Abstract

This (brief) note is a follow up to the Euler equation and asset pricing note, to extend our analysis into the realm of pricing nominal payoffs.

This is a monetary policy course and we should therefore generalize our analysis to allow for that. Most assets are denominated in dollars rather than consumption units (except things like inflation linked government bonds) and yet, ultimately, our model doesn't need much to be done to it. Why? Because it's still the case that people make decisions based on real implications for their welfare. So, in a sense, all we have to do is convert nominal objects to real, and apply our existing Euler equation.

- Suppose an asset pays off $\mathcal{P}_2^{\$}$ in dollars next period.
- Let its price today, in dollars, be denoted $Q_1^{\$}$.
- Denote the overall price level in the economy (price in dollars of a unit of consumption) in t by P_t
- Define gross inflation in t as

$$\Pi_t \equiv \frac{P_t}{P_{t-1}}$$

Suppose I can reduce my consumption slightly today and invest in the asset using the additional savings. Again, we can use a marginal cost = marginal (expected) benefit intuition to derive an Euler equation.

If I increase my savings by \$1, then that means I must have reduced my real consumption (today) by $\frac{1}{P_1}$, right?[1] What is the marginal cost of that to me, in terms of foregone welfare? That would be $u'(c_1) \times \frac{1}{P_1}$, where c_1 is *real* consumption today.

If I invest that extra dollar in buying an amount of the asset, I end up buying $\frac{1}{Q_1^{\$}}$ units of the asset (obviously). What does that get me tomorrow, in terms of a nominal payoff? It gets me $\frac{1}{Q_1^{\$}} \times \mathcal{P}_2^{\$}$ dollars next period which, will buy me $\frac{\frac{1}{Q_1^{\$}} \times \mathcal{P}_2^{\$}}{P_2}$ units of consumption.[2] How do I value that additional small (as we're implicitly still thinking about marginal changes) consumption from the perspective of next period? That would be $u'(c_2) \times \frac{\frac{1}{Q_1^{\$}} \times \mathcal{P}_2^{\$}}{P_2}$. And from today's perspective I will discount it with β and, of course, evaluate things with an expectations operator, taking randomness into account. So the marginal benefit, as perceived from today is $\beta E_1 \left[u'(c_2) \times \frac{\frac{1}{Q_1^{\$}} \times \mathcal{P}_2^{\$}}{P_2} \right]$.

Summing up, we have

$$\underbrace{u'(c_1) \times \frac{1}{P_1}}_{MC} = \underbrace{\beta E_1 \left[u'(c_2) \times \frac{\frac{1}{Q_1^{\$}} \times \mathcal{P}_2^{\$}}{P_2} \right]}_{MB}$$

but we can rewrite this

$$1 = \beta E_1 \left[\frac{u'(c_2)}{u'(c_1)} \times \frac{\mathcal{P}_2^{\$}}{Q_1^{\$}} \times \frac{P_1}{P_2} \right]$$

[1] Don't overthink it! What is the nominal value of a $\frac{1}{P_1}$ amount of real consumption (remember P_1 is just a number)? Well it's $P_1 \times \frac{1}{P_1} = 1$ (obviously).

[2] The price of consumption tomorrow is P_2 so...

But this is just

$$1 = E_1 \left[\Lambda_{1,2}^{Real} \frac{\mathcal{P}_2^{\$}}{Q_1^{\$}} \Pi_2^{-1} \right]$$

where $\Lambda_{1,2}$ is the real SDF, from our earlier 'real' analysis:

$$\Lambda_{1,2}^{Real} \equiv \beta \frac{u'(c_2)}{u'(c_1)}$$

though I now give it a 'Real' superscript.

This should make sense when you notice that $\frac{Q_1^{\$}}{P_1}$ is the real price of buying a unit of the asset today (its nominal price, scaled by the price level today) and $\frac{\mathcal{P}_2^{\$}}{P_2}$ is the real payoff from the asset tomorrow (its nominal payoff, scaled by the price level tomorrow). Since the real gross return (real payoff over real price) on the asset is thus

$$R_2^{Real} \equiv \frac{\frac{\mathcal{P}_2^{\$}}{P_2}}{\frac{Q_1^{\$}}{P_1}}$$

we can rewrite the EE as

$$1 = E_1 \left[\Lambda_{1,2}^{Real} R_2^{Real} \right]$$

which is an EE in purely real terms as before. **The point is that whether you are expressing things in consumption units or in dollar terms, the fact that the agent is optimizing and cares about how much she 'eats' (as that's what influences utility) doesn't change.**

Having said that, it's typically the case that we have data in nominal terms, so it's useful to define a *nominal* SDF

$$\Lambda_{1,2}^{Nom} \equiv \Lambda_{1,2}^{Real} \Pi_2^{-1}$$

so basically there is an adjustment for relative nominal prices (i.e. inflation) between the two periods, and then we're in real terms. We can then write

$$1 = E_1 \left[\Lambda_{1,2}^{Nom} \frac{\mathcal{P}_2^{\$}}{Q_1^{\$}} \right] \equiv E_1 \left[\Lambda_{1,2}^{Nom} R_2^{Nom} \right]$$

where I make the obvious definition of R_2^{Nom} as the ratio of *nominal* payoff, to *nominal* price.

As an exercise for yourself, write down a two period optimization (like in the first note on consumption choice) where everything is in nominal terms. Get the EE that way (rather than the MC=MB approach we took here).