Assignment 1

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1. a. The linear regression model can be written as,

$$P = \beta_0 + \beta_1 C + \epsilon$$

where,

- P is the prime interest rate
- C is the core inflation rate
- β_0 is the intercept of the regression line
- β_1 is the slope of the regression line
- ϵ is the error term

Assumptions of the linear regression model are,

i. **Linearity:** The relationship between P and C is linear
ii. **Independence:** The error term ϵ is independent of C
iii. **Normality:** The error term ϵ is normally distributed
iv. **Homoscedasticity:** The error term ϵ has a constant variance across all levels of C

2. If the simple linear regression model can be written as $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, where $E[\epsilon_i] = 0$ and $E[\epsilon_i^2] = \sigma^2$ and where the ϵ's are independent, the ith fitten value is denoted by $\hat{y}_i$ and the ith residual is denoted by e_i $(i = 1, 2, \ldots, n)$, we can show that:

(a) $\sum_{i=1}^{n} e_i = 0$

Proof. Let e_i be the ith residual term. Then for each observation, we can see that the residual for each observation i is defined as:

$$e_i = y_i - \hat{y}_i$$

where,

- y_i is the actual observed value of the dependent variable.
- $\hat{y}_i$ is the predicted value obtained from the regression model.

then we can say that the predicted value $\hat{y}_i$ is equivalent to

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

Where,

- $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$
- $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$
- $S_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$
- $S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2$

We want to show that:

$$\sum_{i=1}^{n} e_i = 0$$

LHS:

$$\begin{aligned}
\sum_{i=1}^{n} e_i &= \sum_{i=1}^{n}(y_i - \hat{y}_i) \\
&= \sum_{i=1}^{n}(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) \\
&= \sum_{i=1}^{n}(y_i - \bar{y} + \hat{\beta}_1 \bar{x} - \hat{\beta}_1 x_i) \\
&= \sum_{i=1}^{n} y_i - n\bar{y} + \hat{\beta}_1 n\bar{x} - \hat{\beta}_1 \sum_{i=1}^{n} x_i \\
&= n\bar{y} - n\bar{y} + \hat{\beta}_1 n\bar{x} - \hat{\beta}_1 n\bar{x} \\
&= 0
\end{aligned}$$

$$LHS = 0 = RHS$$

$\therefore$ The sum of residuals e_i is zero.

$\square$