

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{bmatrix}$$

$$\lambda_1 = 11.05, \lambda_2 = 2.67, \lambda_3 = -21.12, \lambda_4 = -5.60$$

$$\lambda_1 = 11.05$$

$$A - 11.05I = \begin{bmatrix} 4-11.05 & 8 & -1 & -2 \\ -2 & -9-11.05 & -2 & -4 \\ 0 & 10 & 5-11.05 & -10 \\ -1 & -13 & -14 & -13-11.05 \end{bmatrix}$$

$$\begin{pmatrix} -7.05 & 8 & -1 & -2 \\ -2 & -20.05 & -2 & -4 \\ 0 & 10 & -6.05 & -10 \\ -1 & -13 & -14 & -24.05 \end{pmatrix}$$

$$\begin{pmatrix} -7.05 & 8 & -1 & -2 \\ -2 & -20.05 & -2 & -4 \\ 0 & 10 & -6.05 & -10 \\ -1 & -13 & -14 & -24.05 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -7.05 & 8 & -1 & -2 & : & 0 \\ -2 & -20.05 & -2 & -4 & : & 0 \\ 0 & 10 & -6.05 & -10 & : & 0 \\ -1 & -13 & -14 & -24.05 & : & 0 \end{pmatrix}$$

Divide Row 1 by -7.05

$$\begin{pmatrix} 1 & -1.135 & 0.142 & 0.284 & : & 0 \\ -2 & -20.05 & -2 & -4 & : & 0 \\ 0 & 10 & -6.05 & -10 & : & 0 \\ -1 & -13 & -14 & -24.05 & : & 0 \end{pmatrix}$$

Eliminate the first column below the pivot.

$$\begin{pmatrix} 1 & -1.135 & 0.142 & 0.284 & : & 0 \\ 0 & -22.32 & -1.716 & -3.432 & : & 0 \\ 0 & 10 & -6.05 & -10 & : & 0 \\ -1 & -13 & -14 & -24.05 & : & 0 \end{pmatrix} \quad R2 + 2R1$$

$$\begin{pmatrix} 1 & -1.135 & 0.142 & 0.284 & : & 0 \\ 0 & -22.32 & -1.716 & -3.432 & : & 0 \\ 0 & 10 & -6.05 & -10 & : & 0 \\ 0 & -14.135 & -13.858 & -23.766 & : & 0 \end{pmatrix} \quad R4 + R1$$

Divide Row 2 by -22.32

$$\begin{pmatrix} 1 & -1.135 & 0.142 & 0.284 & : & 0 \\ 0 & 1 & 0.077 & 0.154 & : & 0 \\ 0 & 10 & -6.05 & -10 & : & 0 \\ 0 & -14.135 & -13.858 & -23.766 & : & 0 \end{pmatrix}$$

Eliminate the second column

$$R_1 + 1.135R_2:$$

$$\begin{pmatrix} 1 & 0 & 0.229 & 0.459 & 1 & 0 \\ 0 & 1 & 0.077 & 0.154 & 1 & 0 \\ 0 & 10 & -6.05 & -10 & 1 & 0 \\ 0 & -14.135 & -13.858 & -23.766 & 1 & 0 \end{pmatrix}$$

$$R_3 - 10R_2$$

$$\begin{pmatrix} 1 & 0 & 0.229 & 0.459 & 1 & 0 \\ 0 & 1 & 0.077 & 0.154 & 1 & 0 \\ 0 & 0 & -6.82 & -11.54 & 1 & 0 \\ 0 & -14.135 & -13.858 & -23.766 & 1 & 0 \end{pmatrix}$$

$$R_4 + 14.135R_2$$

$$\begin{pmatrix} 1 & 0 & 0.229 & 0.459 \\ 0 & 1 & 0.077 & 0.154 \\ 0 & 0 & -6.82 & -11.54 \\ 0 & 0 & -12.770 & -21.589 \end{pmatrix}$$

Make the Third Pivot equal to 1

Divide Row 3 by -6.82

$$\begin{pmatrix} 1 & 0 & 0.229 & 0.459 & 1 & 0 \\ 0 & 1 & 0.077 & 0.154 & 1 & 0 \\ 0 & 0 & 1 & 1.692 & 1 & 0 \\ 0 & 0 & -12.770 & -21.589 & 1 & 0 \end{pmatrix}$$

ELIMINATE THE THIRD COLUMN

$$R_1 - 0.229R_3$$

$$\begin{pmatrix} 1 & 0 & 0 & 0.71 & 1 & 0 \\ 0 & 1 & 0.077 & 0.154 & 1 & 0 \\ 0 & 0 & 1 & 1.692 & 1 & 0 \\ 0 & 0 & -12.770 & -21.589 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0.71 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.692 & 0 \\ 0 & 0 & -12.770 & -21.589 & 0 \end{pmatrix} \quad R_2 - 0.077R_3$$

$$R_4 + 12.770 \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0.71 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.692 & 0 \\ 0 & 0 & 0 & 0.16 & 0 \end{pmatrix}$$

$$V_{1x} = -0.071$$

$$V_{1y} = -0.024$$

$$V_{1z} = -1.692$$

$$V_{1z} = \text{free parameter}$$

$$V_{1x} = -0.071, V_{1y} = -0.024, V_{1z} = -1.692, V_{1z} = 1$$

Normalize

$$V_1 = \begin{bmatrix} -0.071, -0.024, -1.692, 1 \end{bmatrix}^T$$

$$\text{Magnitude: } \|V_1\| = \sqrt{(-0.071)^2 + (-0.024)^2 + (-1.692)^2 + 1^2}$$

$$\text{magnitude} \approx 1.967$$

$$\text{Final Answer} = \begin{bmatrix} 0.036, -0.012, -0.860, 0.509 \end{bmatrix}^T$$

$$(-0.036, -0.012, -0.860, 0.509) = V_1$$

Determine the importance of the eigen values %

$$\text{Importance of } \lambda_i = \frac{|\lambda_i|}{\sum |\lambda_i|} \times 100\%$$

$$|\lambda_1| = 11.05, \quad |\lambda_2| = 21.125$$

$$|\lambda_3| = 5.604, \quad |\lambda_4| = 2.675$$

$$\text{Total} = 11.05 + 21.125 + 5.604 + 2.675 \\ \Rightarrow 40.454$$

Percentage importance of $\lambda_1 =$

$$\frac{11.05}{40.454} \times 100\% = 27.31\%$$