

$$\lambda_1 = -21.125, \quad \boxed{\lambda_2 = -5.604}, \quad \lambda_3 = 2.675$$

$$\lambda_4 = 11.054$$

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}, \quad V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

• Calculate matrix $A - \lambda_2 I$

$$A - \lambda_2 I = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix} - (-5.604) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A - \lambda_2 I = \begin{bmatrix} 4 - (-5.604) & 8 & -1 & -2 \\ -2 & -9 - (-5.604) & -2 & -4 \\ 0 & 10 & 5 - (-5.604) & -10 \\ -1 & -13 & -14 & -13 - (-5.604) \end{bmatrix}$$

$$A - \lambda_2 I = \begin{bmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{bmatrix}$$

• System of equations

$$(A - \lambda_2 I)v = 0$$

$$\begin{bmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

\Rightarrow

$$\begin{aligned} \textcircled{1} \quad & 9.604v_1 + 8v_2 - v_3 - 2v_4 = 0 \\ \textcircled{2} \quad & -2v_1 - 3.396v_2 - 2v_3 - 4v_4 = 0 \\ \textcircled{3} \quad & 10v_2 + 10.604v_3 - 10v_4 = 0 \\ \textcircled{4} \quad & -v_1 - 13v_2 - 14v_3 - 7.396v_4 = 0 \end{aligned}$$

• Solving the above system using Gaussian elimination.

$$\begin{bmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

* Swap Row 1 and Row 4

$$R_1 \leftrightarrow R_4 \Rightarrow \begin{bmatrix} -1 & -13 & -14 & -7.396 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 9.604 & 8 & -1 & -2 & 0 \end{bmatrix}$$

* Make the pivot 1

$$R_1 \rightarrow -1 \times R_1 \begin{bmatrix} 1 & 13 & 14 & 7.396 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 9.604 & 8 & -1 & -2 & 0 \end{bmatrix}$$

* eliminate other entries in Col 1

$$R_2 \rightarrow R_2 + 2 \times R_1$$

$$R_4 \rightarrow R_4 - 9.604 \times R_1$$

New R_2 :

$$\begin{aligned} & [-2 + (2)(1)] \quad [-3.396 + 2(13)] \quad [-2 + 2(14)] \\ & [-4 + 2(7.396)] \\ & [0] \quad [22.604] \quad [26] \quad [10.792] \end{aligned}$$

New R_4 :

$$\begin{aligned} & [9.604 - 9.604(1)] \quad [8 - 9.604(13)] \quad [-2 - 9.604(14)] \\ & [-2 - 9.604(7.396)] \end{aligned}$$

$$\cdot [0] [-116.852] [-135.456] [-73.021]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 13 & 14 & 7.396 & 0 \\ 0 & 22.604 & 26 & 10.792 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 0 & -116.852 & -135.45 & -73.021 & 0 \end{array} \right]$$

Step 4: Get a 1 in the pivot position (Row 2, Col 2)
use Row 3.

$$R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{cccc|c} 1 & 13 & 14 & 7.396 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 0 & 22.604 & 26 & 10.792 & 0 \\ 0 & -116.852 & -135.45 & -73.021 & 0 \end{array} \right]$$

• Make the pivot 1

$$R_2 \rightarrow R_2 / 10 \quad \left[\begin{array}{cccc|c} 1 & 13 & 14 & 7.396 & 0 \\ 0 & 1 & 1.0604 & -1 & 0 \\ 0 & 22.604 & 26 & 10.79 & 0 \\ 0 & -116.8 & -135.45 & -73.021 & 0 \end{array} \right]$$

• Eliminate other entries.

$$R_2 \rightarrow R_1 - 13 * R_2$$

$$R_2 \rightarrow R_3 - 22.604 * R_2$$

$$R_4 \rightarrow R_4 + 116.852 * R_2$$

• new R_1 : $[1] [0] [0.2148] [20.396]$

• New R_3 : $[0] [0] [2.0307] [33.396]$

$$[0] [0] [-11.5457] [-189.87]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0.2148 & 20.39 & 0 \\ 0 & 1 & 1.0604 & -1 & 0 \\ 0 & 0 & 2.0307 & 33.39 & 0 \\ 0 & 0 & -11.5457 & -189.873 & 0 \end{array} \right]$$

• Create third pivot in Col 3

• Make the pivot 1

$$R_3 \rightarrow R_3 / 2.0307$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0.2149 & 20.396 & 0 \\ 0 & 1 & 1.0604 & -1 & 0 \\ 0 & 0 & 1 & 16.4455 & 0 \\ 0 & 0 & -11.5457 & -189.87 & 0 \end{array} \right]$$

• Eliminate other entries

$$R_1 \rightarrow R_1 - 0.2148 * R_3$$

$$R_2 \rightarrow R_2 - 1.0604 * R_3$$

$$R_4 \rightarrow R_4 + 11.5457 * R_3$$

• New R_1 : $[1] \ [0] \ [0] \ [16.862]$

$$[0] \ [1] \ [0] \ [-18.439]$$

$$[0] \ [0] \ [0] \ [-189.87 + 189.87]$$

Row echelon form

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 16.862 & 0 \\ 0 & 1 & 0 & -18.439 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Final RREF from calculator

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1.065 & 0 \\ 0 & 1 & 0 & -1.585 & 0 \\ 0 & 0 & 1 & 0.505 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$1. V_1 - 1.065 V_4 = 0$$

$$2. V_2 - 1.585 V_4 = 0$$

$$3. V_3 + 0.505 V_4 = 0$$

$$V_1 = 1.065 V_4$$

$$V_2 = 1.585 V_4$$

$$V_3 = -0.505 V_4$$

\Rightarrow

$$\text{let } V_4 = 1$$

$$V_2 = 1.585$$

$$V_2 = \begin{bmatrix} 1.065 \\ 1.585 \\ -0.505 \\ 1 \end{bmatrix}$$