A concrete example: EE in Triangular Lattices

Entanglement Entropy in Condensed Matter Systems

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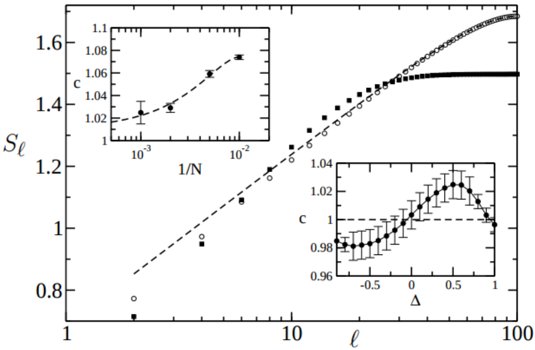
The back story: EE in 1D systems

- General Area Law: $S_l \propto l^{d-1}$
- $d = 1 \Rightarrow$ EE is constant
- EE variations imply phase transitions due to broken symmetries

Quantum Criticality in 1D

- At criticality ($\xi \to \infty$), $S_l = \frac{c}{3}\log_2\left[\frac{L}{\pi a}\sin\left(\frac{\pi}{L}l\right)\right] + A$
- Away from criticality ($\xi >> a$ but finite), $S_l = \frac{c}{3}\log_2\left(\frac{\xi}{a}\right)$



Model spin systems

- The XXZ model:

$$H_{XXZ} = \sum_{<i,j>} \left\{ J_x(\theta)\left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y\right) + J_z(\theta)\sigma_i^z \sigma_j^z \right\}$$

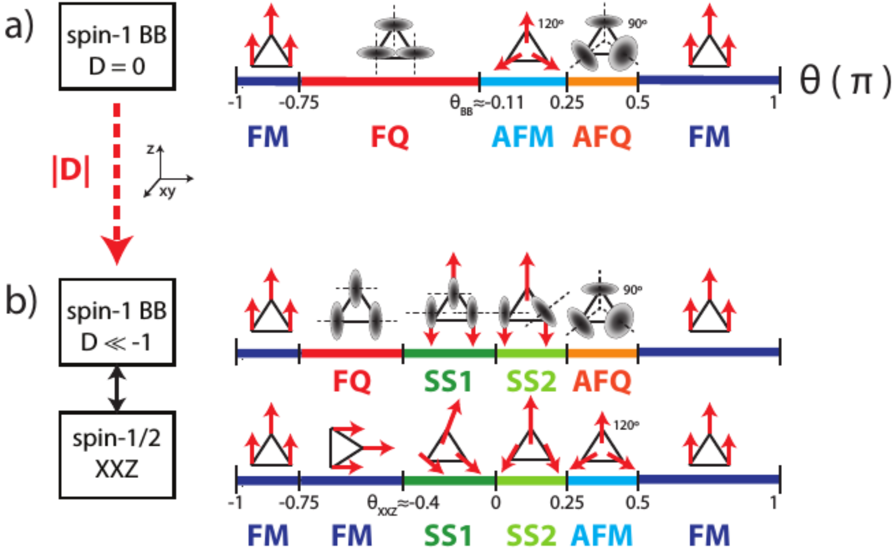
with $J_x = \cos\theta - \frac{\sin\theta}{2}$ and $J_z = \frac{\sin\theta}{2}$

- The bilinear biquadratic model:

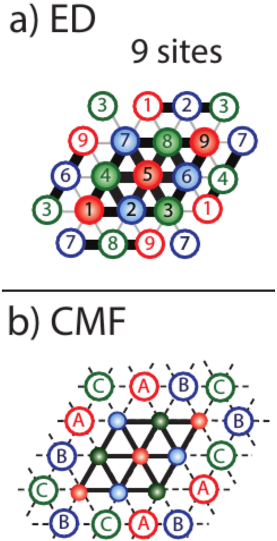
$$H_{BB} = \sum_{<i,j>} \left\{ \cos\theta \vec{\mathbf{S_i}} \cdot \vec{\mathbf{S_j}} + \sin\theta \left(\vec{\mathbf{S_i}} \cdot \vec{\mathbf{S_j}}\right)^2 \right\}$$

Model spin systems

Phase diagram



The Triangular Lattice



How does one compute EE? - A computational approach

Exact Diagonalization (ED)

1. Generate the entire Hilbert Space
   $\mathcal{H}_N = span\left\{|j\rangle = \prod_{i=1}^{N} \left|s^z_{i,j}\right\rangle, s^z_{i,j} = \uparrow, \downarrow\right\}$, with $dim(\mathcal{H}_N) = 2^N$

2. Generate all matrix entries $\mathcal{H}_{i,j} = \langle i|\mathcal{H}_N|j\rangle$ ($2^N \times 2^N$ matrix)

3. Compute the ground state $|\Psi_{gs}\rangle = \sum_{j=1}^{2^N} c_j\,|j\rangle$, and $\rho_{j,i} = {c_i}^* c_j$

4. Define lattice partitioning, generate $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

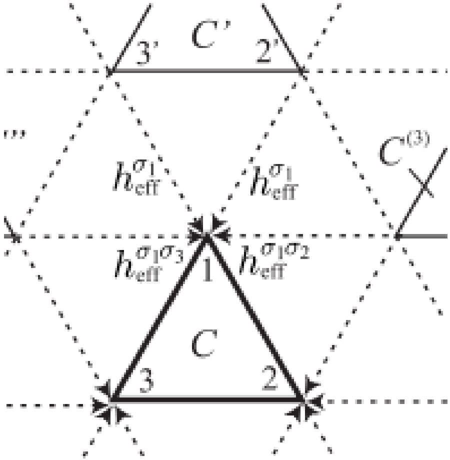
$$\rho_A = Tr_B(\rho) = \sum_{|k\rangle \in \mathcal{H}_B} \langle k|\rho|k\rangle$$

5. Check which entries from $\rho_{i,j} = c_i^* c_j \, |i\rangle\langle j|$ need to be summed into entry (i_A, j_A) of ρ_A ($2^{N_A} \times 2^{N_A}$ matrix)
6. Find its 2^{N_A} eigenvalues λ_i
7. Von Neumann entropy

$$EE(N_A) = -\sum_{i=1}^{2^{N_A}} \lambda_i \log(\lambda_i)$$

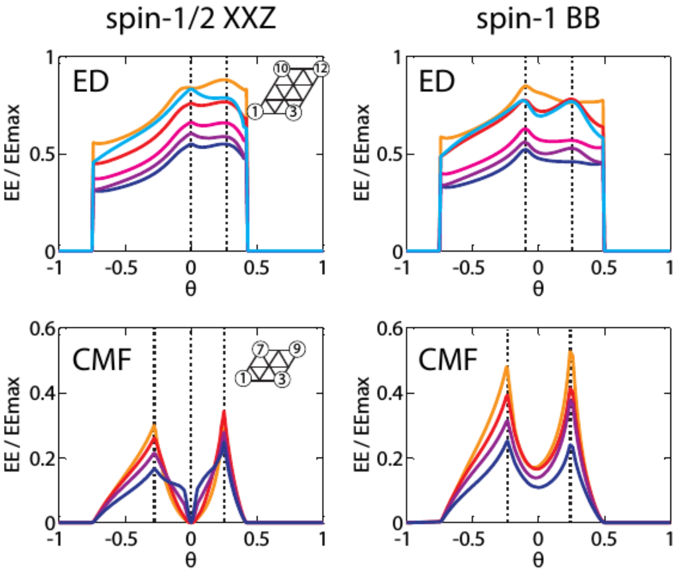
8. Normalization $EE_{max}(N_A) = \log\left(2^{min(N_A,N_B)}\right) \propto min(N_A, N_B)$

The Cluster Mean Field approach



$$H_C = -J \sum_{<i,j>\in C} \sigma_i \sigma_j - J \sum_{i,i'\in C} \left(h_{eff}^{\sigma_i} + h_{eff}^{\sigma_i \sigma_{i'}} \right) \sigma_i$$

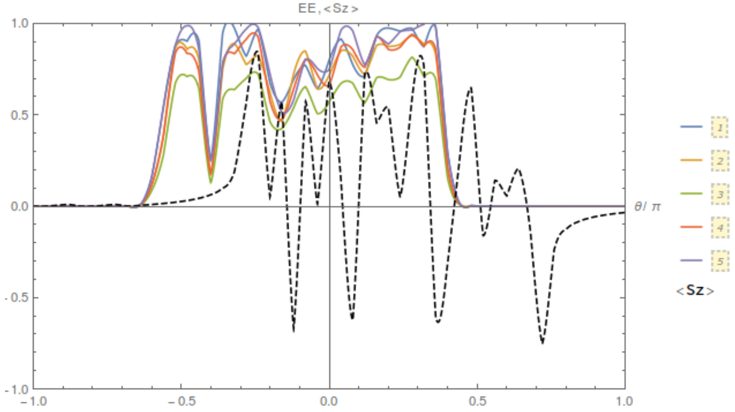
Results



Our simulation: The triangular chain

- N=6 (faster)
- Coordination number $6 \to 4$
- Tests whether the coordination number is relevant to the actual dimensionality of the system

Our results



Closing remarks

- EE opens up a direct path to studying quantum criticality when there is no relevant order parameter
- Dimensionality and partitioning have a clear influence on EE
- Entanglement between partitions is mostly correlated with internal symmetries
- Numerics offer an easy but limited way for computing entanglement

References

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