Stochastic Simulation Discrete simulation/Ferry simulation case

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Rødby Puttgarden Simulation study

- Two ferry operating companies
  - One existing
  - A potential competitor wanting access
- Legally the new operator should not disturb the current
  - This is within reasonable limits
  - Real world experimentation is unrealistic

Capacity and security constraints

- Access to harbour facilities
- A number of rules regarding
  - ◇ The harbour channel
  - ◇ The ferry berths
- Data for
  - ◇ Sailing times
  - ◇ unload/load times

The system events

- arrive_harbour;
- depart_channel
- loaded
- ready_sail
- arrive_channel
- arrive_berth

Global (resource) variables


```
short channel[2],berth[2][4];
```

Ferry data structures

```
class ferry_type {

public:
  short type;
  short id;
  short status;
  short event;
  short harbour;
  short berth;
  time_type event_time;
  time_type scheduled_time;
  ferry_type * previous;
  ferry_type * next;
};
```

Main program - initialisation


```
void main()
{
  ferry_type * ferry;

  time.minutes=0;
  time.hours=0;
  event_list = 0;
  Initialization();
  Print_ferries(event_list);
```

Main program simulation loop

```
while (time.hours<100)
{
  ferry = event_list;
  time = ferry->event_time;
  event_list = event_list->next;
  if (event_list != 0) event_list->previous =0;
  switch(ferry->event)
  {
    case arrive_harbour : Arrive_harbour(ferry); break;
    case depart_channel : Depart_channel(ferry); break;
    case loaded         : Loaded(ferry);         break;
    case ready_sail     : Ready_sail(ferry);     break;
    case arrive_channel : Arrive_channel(ferry); break;
    case arrive_berth   : Arrive_berth(ferry); break;
    default             : break;
  } /* End switch */
} /* End main loop */
Print_statistics();
```

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Sample event procedure


```
void Arrive_harbour(ferry_type * ferry)
{
  if ((Request_channel(ferry)>0) &&
      (Request_berth(ferry)>0))
  {
    ferry->event = arrive_channel;
    ferry->event_time = time;
    Insert_in_event_list(ferry);
  }
  else
    Wait_for_arrive(ferry);
}
```

Another event procedure


```
void Depart_channel(ferry_type * ferry)
{
  channel[ferry->harbour] = vacant;
  ferry->event = arrive_harbour;
  ferry->event_time = time + Sailing_time(ferry);
  Check_waiting_ferries(ferry->harbour);
  ferry->harbour = New_harbour(ferry->harbour);
  Insert_in_event_list(ferry);
}
```

Print_ferry procedure, animation/debug


```
void Print_ferry(ferry_type * ferry)
{
  cout << " " << Print_time(ferry->event_time);
  cout << " " << Print_ferry_type(ferry->type)
       << " " << Print_ferry_id(ferry->id)
       << " " << Print_harbour(ferry->harbour)
       << " " << Print_event(ferry->event)
       << " " << Print_time(ferry->scheduled_time)
       << " " << Print_ferry_status(ferry->status)
       << endl;
}
```

Initialisation and first time step

```
 00:00.50 eal     1 Puttgarden arrive_harbour 00:06.00 active
 00:01.00 slcargo 1 Roedby     ready_sail     00:01.00 active
 00:06.00 eal     3 Roedby     ready_sail     00:06.00 active
 00:15.00 slpr97  2 Puttgarden ready_sail     00:15.00 active
 00:15.00 slpr97  1 Roedby     ready_sail     00:15.00 active
 00:30.00 eal     2 Roedby     arrive_harbour 00:36.00 active
 00:45.00 slpr97  4 Puttgarden ready_sail     00:45.00 active
 00:45.00 slpr97  3 Roedby     ready_sail     00:45.00 active

Time is 00:00.50
 00:00.50 eal     1 Puttgarden arrive_harbour 00:06.00 active
 00:01.00 slcargo 1 Roedby     ready_sail     00:01.00 active
 00:06.00 eal     3 Roedby     ready_sail     00:06.00 active
 00:15.00 slpr97  2 Puttgarden ready_sail     00:15.00 active
 00:15.00 slpr97  1 Roedby     ready_sail     00:15.00 active
 00:30.00 eal     2 Roedby     arrive_harbour 00:36.00 active
 00:45.00 slpr97  4 Puttgarden ready_sail     00:45.00 active
 00:45.00 slpr97  3 Roedby     ready_sail     00:45.00 active
```

A sequence of events - 1

```
Time is 01:11.64
 01:11.64 eal     2 Roedby     depart_channel 02:06.00 active
 01:12.42 slpr97  1 Puttgarden loaded         01:15.00 active
 01:12.79 slpr97  2 Roedby     loaded         01:15.00 active
 01:13.40 slcargo 1 Puttgarden arrive_berth   01:14.00 active
 01:17.39 eal     3 Puttgarden loaded         01:36.00 active
 01:17.74 slpr97  4 Roedby     arrive_harbour 01:30.00 active
 01:20.05 slpr97  3 Puttgarden arrive_harbour 01:30.00 active
 01:23.73 eal     1 Roedby     arrive_harbour 01:36.00 active

Time is 01:12.42
 01:12.42 slpr97  1 Puttgarden loaded         01:15.00 active
 01:12.79 slpr97  2 Roedby     loaded         01:15.00 active
 01:13.40 slcargo 1 Puttgarden arrive_berth   01:14.00 active
 01:17.39 eal     3 Puttgarden loaded         01:36.00 active
 01:17.74 slpr97  4 Roedby     arrive_harbour 01:30.00 active
 01:20.05 slpr97  3 Puttgarden arrive_harbour 01:30.00 active
 01:23.73 eal     1 Roedby     arrive_harbour 01:36.00 active
 01:55.19 eal     2 Puttgarden arrive_harbour 02:06.00 active
```

Sequence of events - 2

```
Time is 01:12.79
 01:12.79 slpr97  2 Roedby     loaded          01:15.00 active
 01:13.40 slcargo 1 Puttgarden arrive_berth    01:14.00 active
 01:15.00 slpr97  1 Puttgarden ready_sail      01:15.00 active
 01:17.39 eal     3 Puttgarden loaded          01:36.00 active
 01:17.74 slpr97  4 Roedby     arrive_harbour  01:30.00 active
 01:20.05 slpr97  3 Puttgarden arrive_harbour  01:30.00 active
 01:23.73 eal     1 Roedby     arrive_harbour  01:36.00 active
 01:55.19 eal     2 Puttgarden arrive_harbour  02:06.00 active

Time is 01:13.40
 01:13.40 slcargo 1 Puttgarden arrive_berth    01:14.00 active
 01:15.00 slpr97  2 Roedby     ready_sail      01:15.00 active
 01:15.00 slpr97  1 Puttgarden ready_sail      01:15.00 active
 01:17.39 eal     3 Puttgarden loaded          01:36.00 active
 01:17.74 slpr97  4 Roedby     arrive_harbour  01:30.00 active
 01:20.05 slpr97  3 Puttgarden arrive_harbour  01:30.00 active
 01:23.73 eal     1 Roedby     arrive_harbour  01:36.00 active
 01:55.19 eal     2 Puttgarden arrive_harbour  02:06.00 active
```

Sequence of events - 3

```
Time is 01:15.00
 01:15.00 slpr97  2 Roedby     ready_sail     01:15.00 active
 01:15.00 slpr97  1 Puttgarden ready_sail     01:15.00 active
 01:17.39 eal     3 Puttgarden loaded         01:36.00 active
 01:17.74 slpr97  4 Roedby     arrive_harbour 01:30.00 active
 01:20.05 slpr97  3 Puttgarden arrive_harbour 01:30.00 active
 01:23.73 eal     1 Roedby     arrive_harbour 01:36.00 active
 01:27.38 slcargo 1 Puttgarden loaded         01:31.00 active
 01:55.19 eal     2 Puttgarden arrive_harbour 02:06.00 active

Time is 01:15.00
 01:15.00 slpr97  1 Puttgarden ready_sail     01:15.00 active
 01:17.39 eal     3 Puttgarden loaded         01:36.00 active
 01:17.74 slpr97  4 Roedby     arrive_harbour 01:30.00 active
 01:20.05 slpr97  3 Puttgarden arrive_harbour 01:30.00 active
 01:20.85 slpr97  2 Roedby     depart_channel 02:00.00 active
 01:23.73 eal     1 Roedby     arrive_harbour 01:36.00 active
 01:27.38 slcargo 1 Puttgarden loaded         01:31.00 active
 01:55.19 eal     2 Puttgarden arrive_harbour 02:06.00 active
```

Sequence of events - 4

```
Time is 01:17.39
 01:17.39 eal     3 Puttgarden loaded         01:36.00 active
 01:17.74 slpr97  4 Roedby     arrive_harbour 01:30.00 active
 01:20.05 slpr97  3 Puttgarden arrive_harbour 01:30.00 active
 01:20.85 slpr97  2 Roedby     depart_channel 02:00.00 active
 01:21.24 slpr97  1 Puttgarden depart_channel 02:00.00 active
 01:23.73 eal     1 Roedby     arrive_harbour 01:36.00 active
 01:27.38 slcargo 1 Puttgarden loaded         01:31.00 active
 01:55.19 eal     2 Puttgarden arrive_harbour 02:06.00 active

Time is 01:17.74
 01:17.74 slpr97  4 Roedby     arrive_harbour 01:30.00 active
 01:20.05 slpr97  3 Puttgarden arrive_harbour 01:30.00 active
 01:20.85 slpr97  2 Roedby     depart_channel 02:00.00 active
 01:21.24 slpr97  1 Puttgarden depart_channel 02:00.00 active
 01:23.73 eal     1 Roedby     arrive_harbour 01:36.00 active
 01:27.38 slcargo 1 Puttgarden loaded         01:31.00 active
 01:36.00 eal     3 Puttgarden ready_sail     01:36.00 active
 01:55.19 eal     2 Puttgarden arrive_harbour 02:06.00 active
```

Delay summaries

Average delay

	slcargo	slpr97	eal
Rødby	0.65	0.30	0.26
Puttgarden	0.74	0.06	0.04

Standard deviation of delay

	slcargo	slpr97	eal
Rødby	1.13	0.64	0.58
Puttgarden	1.18	0.27	0.19

Delay distributions

Delay (Δ) Distribution for Rødby

Ferry type	$\Delta \leq 1$	$1 \leq \Delta \leq 3$	$4 \leq \Delta \leq 6$
slcargo	79%	18%	3%
slpr97	90%	9%	1%
eal	91 %	9%	0%

Delay (Δ) Distribution for Puttgarden

Ferry type	$\Delta \leq 1$	$1 \leq \Delta \leq 3$	$4 \leq \Delta \leq 6$
slcargo	70%	27%	3%
slpr97	98%	2%	0%
eal	99%	1%	0%

Exercise 4

Write a discrete event simulation program for a blocking system, i.e. a system with m service units and no waiting room. The offered traffic A is the product of the mean arrival rate and the mean service time.

1. The arrival process is modelled as a Poisson process. Report the fraction of blocked customers, and a confidence interval for this fraction. Choose the service time distribution as exponential. Parameters: m $=$ 10, mean service time $=$ 8 time units, mean time between customers $=$ 1 time unit (corresponding to an offered traffic of 8 Erlang), 10 x 10.000 customers.

   This system is sufficiently simple such that the analytical solution is known. See the last slide for the solution. Verify your simulation program using this knowledge.

2. The arrival process is modelled as a renewal process using the same parameters as in Part 1 when possible. Report the fraction of blocked customers, and a confidence interval for this fraction for at least the following two cases
   (a) Experiment with Erlang distributed inter arrival times The Erlang distribution should have a mean of 1
   (b) hyper exponential inter arrival times. The parameters for the hyper exponential distribution should be $p_1 = 0.8, \lambda_1 = 0.8333, p_2 = 0.2, \lambda_2 = 5.0$.
3. The arrival process is again a Poisson process like in Part 1. Experiment with different service time distributions with the same mean service time and m as in Part 1 and Part 2.
   (a) Constant service time
   (b) Pareto distributed service times with at least $k = 1.05$ and $k = 2.05$.

(c) Choose one or two other distributions.

4. Compare confidence intervals for Parts 1, 2, and 3 then interpret and explain differences if any.

Exercise 4 - exact solution

- With arrival intensity λ and mean service time s
- Define $A = \lambda s$
- Erlang's B-formula

$$B = P(m) = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}$$

- Valid for all service time distributions
- But arrival process has to be a Poisson process