What do c^*, C_f, η_{c^*}, and η_{C_f} mean?

Saaketh Ramoju

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Definitions

Here are a few important definitions that will matter:

Characteristic Velocity

$$c^*_{ideal} = \frac{P_c A_t}{\dot{m}_{ideal}}$$

c^* is a measurement of how much thermal energy is released in the combustion chamber that is available to be used as momentum to produce thrust. The higher your characteristic velocity, the higher your potential for thrust.

c^* Efficiency

$$\eta_{c^*} = \frac{c^*_{actual}}{c^*_{ideal}}$$

This efficiency is *only* defined for a known pressure and throat area. Therefore, the definition can be simplified as:

$$\eta_{c^*} = \frac{\frac{P_c A_t}{\dot{m}_{actual}}}{\frac{P_c A_t}{\dot{m}_{ideal}}} = \frac{\dot{m}_{ideal}}{\dot{m}_{actual}}$$

c^* efficiency simply measures how much mass flow we used compared to what we wanted to use *for a given pressure and throat area*. Therefore, it is not a bulk knockdown of chamber pressure. For example, if we want a P_c of 300 psia, but we only got 250 psia, η_{c^*} would not include $\frac{P_{c,actual}}{P_{c,ideal}}$. Same with throat area.

Choked Mass Flow Rate

If you assume choked, isentropic flow, the mass flow through the engine is:

$$\dot{m} = \frac{P_c A_t}{\sqrt{T_c}} \sqrt{\frac{\bar{M}\gamma}{\bar{R}}} \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

So mass flow is a function of P_c, A_t, molecular weight ($\bar{M}$), and T_c.

Thrust

$$\tau = \dot{m}V_e + A_t\epsilon(P_e - P_{amb})$$

V_e is the exhaust velocity and ϵ is the expansion ratio. This equation comes from steady-state momentum conservation. While $\dot{m}$ is in this equation, the thrust in the chamber actually comes from the pressure distribution, so we can actually remove $\dot{m}$ and write everything in terms of pressures:

$$\tau = P_c A_t \gamma \sqrt{\frac{2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} + A_t\epsilon(P_e - P_{amb})$$

Thrust Coefficient

$$C_f = \frac{\tau}{P_c A_t}$$

Also only defined for a given pressure and throat area. This tells us how well the nozzle amplifies the momentum available in the chamber.

Thrust Coefficient Efficiency

$$\eta_{C_f} = \frac{C_{f,actual}}{C_{f,ideal}}$$

Again, only defined for a given pressure and throat area. Therefore, this equation can be simplified to:

$$\eta_{C_f} = \frac{\tau_{actual}}{\tau_{ideal}}$$

η_{C_f} is a measure of how much thrust we actually got versus what we wanted. But if our thrust dropped because the chamber pressure dropped, we can't define a η_{C_f} for that.

Why are c^* and η_{c^*} important?

Relation to mass flow

Solve the c^* equation for $\dot{m}$:

$$\dot{m}_{actual} = \frac{P_c A_t}{c^*_{actual}}$$

Compare this to the isentropic mass flow equation

$$\dot{m} = \frac{P_c A_t}{\sqrt{T_c}}\sqrt{\frac{\bar{M}\gamma}{\bar{R}}}\left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

and you'll see that c^* is just the part of the mass flow equation without P_c and A_t.

$$(c^*)^{-1} = \left(\frac{1}{\sqrt{T_c}}\sqrt{\frac{\bar{M}\gamma}{\bar{R}}}\left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}})\right)^{-1}$$

So, for a given P_c and A_t, c^*_{actual} tells if you reached the T_c and $\bar{M}$ you want (basically a measurement of combustion efficiency). This is why c^* is measurement of energy release.

Usually, you can get the ideal c^* if you know the P_c and mixture ratio, MR. So then, $c^*_{actual} = \eta_{c^*} c^*_{ideal}$, and you know exactly how much more mass flow rate you'll get, *for an existing throat area!*.

Relation to V_e and I_{sp}

By definition:

$$c^*_{actual} C_{f,actual} = V_{e,actual} = g I_{sp,actual}$$

Extending this to efficiencies gives:

$$\eta_{c^*} \eta_{C_f} c^*_{ideal} C_{f,ideal} = \eta_{I_{sp}} V_{e,ideal} = \eta_{I_{sp}} g I_{sp,ideal}$$

For given P_c and MR, all of the above ideal values can be derived from CEA. Now, for example, if we know our P_c, MR, and η_{c^*}, assuming a perfect C_f efficiency, we can say

$$V_{e,actual} = \eta_{c^*} V_{e,ideal}$$

Relation to speed of sound and momentum

Looking at the c^* equation, we can see:

$$c^* = \frac{\sqrt{\gamma R T_c}}{\Gamma(\gamma)}$$

where $\Gamma(\gamma)$ is just some function of γ. $\sqrt{\gamma R T_c}$ represents the speed of sound in the combustion chamber, so c^* is simply proportional to the chamber gases' speed of sound, offset by a compressibility fact, $\Gamma(\gamma)$. This makes sense as both values represent the ability of energy to propagate through the nozzle.

Although not exactly this, c^* kind of represents the amount of momentum for thrust we would get out of the combustion chamber, per unit mass. We can see this in the fact that

$$V_e = c^* C_f$$

Both V_e and c^* have units of $\frac{m}{s}$, while C_f is just a dimensionless coefficient. V_e represents the momentum per unit mass at the exit, so C_f basically represents the amplification of momentum to get to V_e.

Relation to combustion efficiency

η_{c^*} is an overall measure of how much energy we get out of the propellants at a given pressure.

$$\eta_{c^*} = \frac{\eta_{ERE}}{C_d}$$

Here, η_{ERE} is the energy release efficiency and C_d is the discharge coefficient of the nozzle. Energy release depends on propellant mixing, vaporization losses, and heat losses. Again, these values are all defined for a single chamber pressure.

In general, if your combustion is bad, but not horrible, you may not see a horrible drop in chamber pressure, but you may see an a decrease in chamber temperature. Because of this, the density of the chamber gases will increase, and there will be more mass flow out of the throat. (You can see this by lowering temperature in the isentropic mass flow equation). This is why lower η_{c^*} causes more mass flow for a given P_c.

If your combustion gets way worse, you simply won't reach the same P_c that you targeted, so the η_{c^*} you had before will not apply. With this new chamber pressure, assuming the tanks remain at the same pressure and the system flow resistance is roughly the same, we can check the following formula:

$$\dot{m} = C_d A \sqrt{2\rho \Delta p}$$

With the higher Δp, even though the chamber pressure decreased, the mass flow through the system, including the engine, will increase. So, we can pretty confidently say that the c^* efficiency went down too. What this basically means is that if you're not operating an engine with a fixed geometry at its design conditions, you will use more mass flow than you want. Moreover, we can use the following equation:

$$\tau = C_f P_c A_t$$

For the same MR, as P_c decreases, C_f also tends to decrease. And since P_c itself is getting lower, thrust will go down. So, with higher $\dot{m}$ and lower thrust, I_{sp} is bound to go down by a bunch. V_e will also go down since the engine can no longer physically push gases out at the same momentum.

Just remember, whatever η_{c^*} we found for the design pressure does not account for the drop in pressure. Just because η_{c^*} measure combustion efficiency does not mean it accounts for combustion so bad that it drops our chamber pressure.

Relation to thrust

Okay let's say we know the chamber pressure, throat area, and c^* efficiency. How will this impact thrust? (Assume $\eta_{C_f} = 1$ for now)

$$\tau = \dot{m}_{actual} V_{e,actual} + A_t \epsilon (P_e - P_{amb})$$

We know that $\dot{m}_{actual} = \frac{\dot{m}_{ideal}}{\eta_{c^*}}$ and $V_{actual} = \eta_{c^*} V_{e,ideal}$. Putting this together gives:

$$\tau = \frac{\dot{m}_{ideal}}{\eta_{c^*}} \eta_{c^*} V_{e,ideal} + A_t \epsilon (P_e - P_{amb})$$

So,

$$\tau = \dot{m}_{ideal} V_{e,ideal} + A_t \epsilon (P_e - P_{amb})$$

So it seems like thrust is pretty much "immune" to η_{c^*}. The biggest factors that play into thrust are P_c, MR, and A_t. Things like T_c and $\bar{M}$, which play a big role in c^*, don't really impact thrust. In fact, in the C_f equation:

$$C_f = \gamma \sqrt{\frac{2}{\gamma - 1} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]} + \frac{\epsilon}{P_c} (P_e - P_{amb})$$

we can see that the thrust coefficient is not a function of $\bar{M}$ and T_c.

In reality, the changing T_c and $\bar{M}$ will affect γ, which will change thrust, but this effect should be encapsulated in η_{C_f}.

How should we go about getting mass flow?

If P_c and MR are kept constant, but our η_{c^*} changes, then we can simply do:

$$\dot{m} = \frac{P_c A_t}{\eta_{c^*} c^*_{ideal}}$$

where c^*_{ideal} comes from CEA. This is equivalent to doing:

$$\dot{m}_{new} = \frac{\tau_{design}}{\eta_{c^*} V_{e,ideal}}$$

This is assuming that A_t stays constant. **DO NOT** use this to size throat area from thrust!

How should we go about getting throat area?

If you know design thrust, P_c, and MR, use:

$$A_t = \frac{\tau_{design}}{\eta_{C_f} C_{f,ideal} P_c}$$

$C_{f,ideal}$ can be determined from CEA, and η_{C_f} can usually be estimated. 95% is usually a decent estimate (if you're confident that you can make your nozzle correctly).

The following documents are helpful:

References

[1] Brian J. Cantwell. *AA284A Advanced Rocket Propulsion: Session 2 — Review of Rocket Performance Parameters, Thrust, Impulse, Efficiency.* Lecture slides. Jan. 2020. URL: `https://web.stanford.edu/~cantwell/AA284A_Course_Material/AA284A_Course_Org%2C%20lecture%201%20topic%20assignments%20and%20themes%2C%20Lecture%202%20performance%20parameters/AA284A1_Session_02_Review_of_Rocket_performance_Brian_Cantwell.pdf` (visited on 09/30/2025).

[2] Jerry M. Seitzman. *Thrust Coefficient, Characteristic Velocity and Ideal Nozzle Expansion.* AE4451 Rocket Propulsion lecture notes. 2020. URL: `https://seitzman.gatech.edu/classes/ae4451/thrust_coefficient.pdf` (visited on 09/30/2025).