Problem x.2: Jacobi Eigenvalue Algorithm

Learning Objectives:

- Understand the Jacobi Eigenvalue Algorithm.
- Learn to code this algorithm.
- Use Jacobi Eigenvalue Algorithm for computing the eigenvalues and the corresponding eigenvectors of a matrix.

Jacobi Eigenvalue Algorithm:

This method determines the eigenvalues and eigenvectors of a real symmetric matrix **A**, by converting **A** into a diagonal matrix **D** by similarity transformation

$$\boldsymbol{D}^{(1)} \leftarrow \boldsymbol{A}, \quad \boldsymbol{D}^{(2)} \leftarrow \boldsymbol{S}^{(1)T} \boldsymbol{D}^{(1)} \boldsymbol{S}^{(1)}, \quad \boldsymbol{D}^{(3)} \leftarrow \boldsymbol{S}^{(2)T} \boldsymbol{D}^{(2)} \boldsymbol{S}^{(2)} = \boldsymbol{S}^{(2)T} \boldsymbol{S}^{(1)T} \boldsymbol{D}^{(1)} \boldsymbol{S}^{(1)} \boldsymbol{S}^{(2)},$$

$$\Rightarrow \boldsymbol{D}^{(n)} \leftarrow \left(\boldsymbol{S}^{(1)} \boldsymbol{S}^{(2)} \cdots \boldsymbol{S}^{(n-1)}\right)^T \boldsymbol{D}^{(1)} \boldsymbol{S}^{(1)} \boldsymbol{S}^{(2)} \cdots \boldsymbol{S}^{(n-1)}$$

Algorithm:

1. Get/initialize the matrix **A**
2. Initialize **D** ← **A** and **S** ← **I**, a unit matrix.
3. Find the largest off-diagonal element (in magnitude) from **D** and let it be d_{pq}
4. Find the rotational angle
5. Compute the matrix $\mathbf{S}^{(1)} = [s^{(1)}_{ij}]$
6. Find $\boldsymbol{D} \leftarrow \boldsymbol{S}^{(1)T} \boldsymbol{D} \boldsymbol{S}^{(1)}$, and $\boldsymbol{S} \leftarrow \boldsymbol{S} \boldsymbol{S}^{(1)}$
7. Repeat steps 3 to 6 until D becomes diagonal
8. Diagonal elements of D are the eigenvalues and the columns of S are the corresponding eigenvectors.

Sample Input/output:

```
user@host:~
user@host:~$ ./a.out

Enter the size of the matrix 2
Enter the elements row wise
17
32
32
65
The given matrix is
 17.0000  32.0000
 32.0000  65.0000

The eigenvalues are
  1.0000  81.0000
The corresponding eigenvectors are
(  0.8944, -0.4472 )^T
(  0.4472,  0.8944 )^T
```

Tasks:

1. Write a program to find the eigenvalues and the corresponding eigenvectors of the matrix

$$\boldsymbol{A} = \begin{bmatrix} 17 & 32 \\ 32 & 65 \end{bmatrix}$$