1 Computational Geometry 19

1.1 SmallestCircle

```
using PT=point<T>; using CPT=const PT;
PT circumcenter(CPT &a,CPT &b,CPT &c){
     PT u=b-a, v=c-a;
T c1=u.abs2()/2,c2=v.abs2()/2;
     T d=u.cross(v);
return PT(a.x+(v.y*c1-u.y*c2)/d,a.y+(u.
           x*c2-v.x*c1)/d);
   void solve(PT p[],int n,PT &c,T &r2){
  random_shuffle(p,p+n);
     c=p[0]; r2=0; // c,r2 = 圓心,半徑平方
   for(int i=1;i<n;i++)if((p[i]-c).abs2()>r2
        c=p[i]; r2=0;
   for(int j=0;j<i;j++)if((p[j]-c).abs2()>r2
        ){
          c.x=(p[i].x+p[j].x)/2;
c.y=(p[i].y+p[j].y)/2;
15
          r2=(p[j]-c).abs2();
17
   for(int k=0;k<j;k++)if((p[k]-c).abs2()>r2
             c=circumcenter(p[i],p[j],p[k]);
             r2=(p[i]-c).abs2();
        }
     }
23 }
```

1.2 最近點對

```
template<typename _IT=point<T>* >
T cloest_pair(_IT L, _IT R){
  if(R-L <= 1) return INF;</pre>
      _{\rm IT} mid = L+(R-L)/2;
      \overline{T} x = mid -> x;
     T d = min(cloest_pair(L,mid),
      cloest_pair(mid,R));
inplace_merge(L, mid, R, ycmp);
static vector<point> b; b.clear();
      for(auto u=L;u<R;++u){</pre>
         if((u->x-x)*(u->x-x)>=d) continue;
         for(auto v=b.rbegin();v!=b.rend();++v
           T dx=u->x-v->x, dy=u->y-v->y; if(dy*dy>=d) break;
           d=min(d,dx*dx+dy*dy);
        b.push_back(*u);
18
      return d;
19
   T closest_pair(vector<point<T>> &v){
      sort(v.begin(),v.end(),xcmp);
      return closest_pair(v.begin(),v.end());
23 }
```

2 Data Structure

2.1 01 背包

#include<bits/stdc++.h>

2.2 array simulate linked list

```
#define pll pair<LL,LL>
#define RSIZE 100002
   #define oo 1000000001
   #define good ios_base::sync_with_stdio(0)
         ;cin.tie(0)
   typedef long long LL;
   using namespace std;
   struct {
    LL center,height;
        LL pre, next;
        bool alive;
   }tree[RSIZE];
   queue<LL> 0://check removed tree
15
   void removable(LL index){
16
        if(!tree[index].alive) return;
        LL`s = tree[index].pre,t = tree[index
             ].next;
        if(tree[index].center-tree[index].
    height >= tree[s].center ||
            tree[index].center+tree[index].
20
                 height <= tree[t].center){
                  tree[index].alive = false;
                 Q.push(index);
tree[s].next = t;
23
24
                  tree[t].pre = s;
25
   }
   int main(){
        //freopen("P_2_1_5.in", "r", stdin)
;//input redirection
30
        LL bound, n, total = 0, high = 0;
        cin >> n >> bound;
32
        for(LL i = 1; i <= n; i++)</pre>
             cin >> tree[i].center;
        for(LL i = 1; i <= n; i++)
    cin >> tree[i].height;
36
        for(LL i = 1; i <= n; i++){
    tree[i].pre = i-1;</pre>
37
38
             tree[i].next = i+1;
             tree[i].alive = true;
41
42
        tree[0].center = 0.tree[0].height =
             00;
        tree[n+1].center = bound, tree[n+1].
        height = oo;
for(LL i = 1; i <= n; i++)
             removable(i);
             while (!Q.empty()){
   LL v = Q.front();
46
47
                  Q.pop();
48
                  total++;
                  high = max(high,tree[v].
50
                       height);
                  removable(tree[v].pre);
                  removable(tree[v].next);
52
53
             cout << total << endl << high;</pre>
        return 0;
```

2.3 binary search

2.4 discretization

2.5 half enumeration

```
1 #include < bits / stdc++.h>
   #define good ios_base::sync_with_stdio(0)
        ;cin.tie(0)
   typedef long long LL;
   using namespace std;
   LL sa[1<<18],sb[1<<18],no[1<<18];//subset
         product of a and b
   LL subset(LL num[],LL length,LL product
        [],LL p){//pass by pointer
       LL k = 0,i,j;//count
       11
12
13
                       product times num[i]
            product[k] = num[i];//for num[i]
    itself
15
            k += k+1;
16
17
        return k;//return the size of subset
19
   }
20
  LL exp_modp(LL x,LL y,LL p){
   if(y == 0) return 1;
   if(y % 2) return (exp_modp(x,y-1,p)*x
21
23
             ) % p;
            LL temp = exp_modp(x,y/2,p);
25
26
            return (temp*temp) % p;
27
   int main(){
       //freopen("file name", "r", stdin);
31
            input redirection
       LL i,n,p;
32
       LL a[30],b[30];
33
       cin >> n >> p;
int len_a = n/2,len_b = n - len_a;
       for(i = 0; i < len_a; i++)</pre>
       cin >> a[i];
for(i = 0; i < len_b; i++)
38
            cin >> b[i];
       LL len_sa = subset(a,len_a,sa,p);
LL len_sb = subset(b,len_b,sb,p);
        sort(sb,sb+len_sb);
43
       LL len_sb2 = 1;//len_sb2 followed by
44
             i below
45
       no[0] = 1;//assume not empty(check
        for(i = 1;
                    i < len_sb; i++){
            if(sb[i] != sb[i-1]){//new
                  eLement
                 sb[len_sb2] = sb[i];
no[len_sb2] = 1;
                 len_sb2++;
            else//old element
52
                 no[len_sb2-1]++;
       LL ans = (sb[0] == 1) ? no[0]\%p : 0;
       for(i = 0; i < len_sa; i++){
   if(sa[i] == 1) ans = (ans+1) % p;</pre>
            LL y = exp_modp(sa[i],p-2,p);//
module inverse
```

int it = lower_bound(sb,sb+
 len_sb2,y) - sb;

2.6 LCS

2.7 LIS

2.8 skew heap

```
node *merge(node *a,node *b){
    if(!a||!b) return a?a:b;
    if(b->data<a->data) swap(a,b);
    swap(a->1,a->r);
    a->l=merge(b,a->l);
    return a;
}
```

2.9 sliding window

```
for(i = 0; i < m; i++){//making first</pre>
         window
        LL color = discret[a[right]];
        cnt[color]++;
        if(cnt[color] == 1) n_color++;
        right++;
   while(right < n){
   if(n_color == m)</pre>
             ans++;
        LL l_remove = discret[a[left]];
        cnt[l_remove]--;//remove left one
13
        if(cnt[1_remove] == 0) n_color--;
        LL add = discret[a[right]];
cnt[add]++,right++;//add next one
if(cnt[add] == 1) n_color++;
15
16
17
```

2.10 undo disjoint set

```
i struct DisjointSet {
      // save() is like recursive
// undo() is like return
      int n, fa[MXN], sz[MXN];
vector<pair<int*,int>> h;
      vector<int> sp;
      void init(int tn) {
        n=tn;
         for (int i=0; i<n; i++) sz[fa[i]=i</pre>
                1=1:
        sp.clear(); h.clear();
11
      void assign(int *k, int v) {
12
        h.PB({k, *k});
13
         *k=v;
15
      void save() { sp.PB(SZ(h)); }
void undo() {
16
17
        assert(!sp.empty());
         int last=sp.back(); sp.pop_back();
         while (SZ(h)!=last) {
20
21
           auto x=h.back(); h.pop_back();
            *x.F=x.S;
        }
      int f(int x) {
  while (fa[x]!=x) x=fa[x];
         return x;
      void uni(int x, int y) {
29
        bid uni(int x, int y, t
x=f(x); y=f(y);
if (x==y) return;
if (sz[x]<sz[y]) swap(x, y);
assign(&sz[x], sz[x]+sz[y]);</pre>
32
33
34
        assign(&fa[y], x);
35
36 }djs;
```

3 Graph

3.1 bellman ford

| #include < bits / stdc++.h>

```
#define good ios_base::sync_with_stdio(0)
         ;cin.tie(0)
   #define RSIZE 101
   #define pll pair<LL,LL>
#define lc 2*index
#define rc 2*index+1
   #define maxdis 1000000009
typedef long long LL;
   using namespace std;
struct Node{
        LL in, out, weight;
12
   LL dis[RSIZE];
void bellman_ford(vector<Node> &edges,LL
13
          v,LL e,LL start){//v for vertex, e
          for edge
         //vector<LL> dis(v,LLONG_MAX);
         dis[start] = 0;
16
17
         //relaxation
         for(LL i = 0; i < v-1; i++){
    for(LL j = 0; j < e; j++){
        LL x = edges[j].in,y = edges[</pre>
18
19
20
                          j].out;
                    LL w = edges[j].weight;
21
                   if(dis[x] != maxdis && dis[x
]+w < dis[y])</pre>
22
                         dis[y] = dis[x] + w;
              }
24
         27
               is:" << endl;
         for(LL i = 0; i < v; i++){
    cout << i << "\t\t" << dis[i] <</pre>
28
29
                     endl:
30
31
   bool checkinf(vector<Node> &edges,LL e){
   for(LL i = 0; i < e; i++){
      LL x =edges[i].in,y = edges[i].</pre>
32
33
34
                    out;
              36
                      < dis[y])
37
                    return true;
```

```
return false;
39
40
   int main(){
42
        good;
        LL vertex,edge;
43
        cin >> vertex >> edge;
44
        memset(dis,maxdis,sizeof(dis));
45
        for(LL i = 0; i < edge; i++)
    cin >> graph[i].in >> graph[i].
46
47
                  out >> graph[i].weight;
        LL source;
49
50
        cin >> source:
        bellman_ford(graph, vertex, edge, source
51
        if(checkinf(graph,edge))
   cout << "found negative round" <</pre>
52
                    end1:
54
             cout << "not found negative round
55
                     << end1;
```

3.2 BFS

```
LL val;//unnecessary
bool visited[5000] = {false};
   vector<LL> graph[5000];
void BFS(LL start) {
        queue<tL> q;
        q.push(start);
        visited[start] = true;
        while (!q.empty()){
   LL curr = q.front();
             q.pop();
             for(auto it: graph[curr]){
11
                   if(!visited[it]){
                        q.push(it);
                        visited[it] = true;
14
15
                  }
             }
        }
```

3.3 **DFS**

```
#include < bits / stdc++.h>
   #define good ios_base::sync_with_stdio(0)
    ;cin.tie(0);cout.tie(0)
typedef long long LL;
using namespace std;
   int fa[100000],d[100000] = {0};//
          unnecessary
   bool visit[100000] = {false};
   vector<LL> v[100000];
void dfs(LL now,LL depth){
         for(auto x:v[now]){
              if(!visit[x]){
                    cout << x << ' ';
visit[x] = true;
                    d[x] = depth;
fa[x] = now;
13
14
                    dfs(x,depth+1);
15
         }
   int main(){
19
20
         good;
         LL i,n,a,b;
21
         cin >> n;
         for(i = 0; i < n; i++){</pre>
              cin >> a >> b;
               v[a].push_back(b);
25
               v[b].push_back(a);
27
         dfs(0,1);
```

3.4 dijkstra

```
#include<bits/stdc++.h>
   #define good ios_base::sync_with_stdio(0)
        ;cin.tie(0)
   #define N 10002
  #define oo 1000000001//1e9+1
typedef long long LL;
   using namespace std;
   vector<pair<LL,LL>> adjacent[N];//out
        neighbor, weight of edge
   LL dis[N],parent[N];
   bool visit[N] = {false};
  int main(){
12
       LL i,n,m;
cin >> n >> m;
for(i = 0; i < m; i++){
            LL x,y,w;
             adjacent[x].push_back({y,w});
            adjacent[y].push_back({x,w});
20
       //initial
21
       LL source = 0;
       memset(dis,oo,sizeof(dis));
memset(parent,-1,sizeof(parent));
23
24
       priority_queue<pair<LL,LL>> PQ;//-dis
             [],vertex·技巧性讓最小值pop
       PQ.push({dis[source] = 0,source});
27
       //dijkstra
       while (!PQ.empty()){
            auto p = PQ.top();
            PQ.pop();
             LL v = p.second;//vertex
            if(visit[v]) continue;
32
            visit[v] = true;
for(auto it : adjacent[v]){
33
                 LL e = it.first,w = it.second
35
                 if(w + dis[v] < dis[e]){</pre>
                      dis[e] = w + dis[v];
parent[e] = v;
PQ.push({-dis[e],e});
37
                 }
            }
43
       LL maxd = -1, cnt = 0, far;
       for(i = 0; i < n; i++){
    if(dis[i] < oo){</pre>
45
                 if(dis[i] > maxd)
                      maxd = dis[i],far = i;
                 cnt++;//for can't reach
50
       cout << maxd << endl << cnt;</pre>
52
53
       return 0;
```

3.5 topology sort

```
1 int main(){
        good;
        LL indeg[1002] = {0};
        vector<LL> graph[1002];
       LL n,m,a,b;
       cin >> n >> m;
for(LL i = 0; i < m; i++){
             cin >> a >> b;
             graph[a].push_back(b);
             indeg[b]++;
        LL topo[1002], head = 0, tail = 0;//??
             queue
        for(LL i = 0; i < n; i++)
       if(indeg[i] == 0)
    topo[tail++] = i;
while(head < tail){</pre>
15
             LL v = topo[head++];//get data
                  and pop
             for(LL u : graph[v]){
    if(--indeg[u] == 0)
                      topo[tail++] = u;
             }
21
        if(tail < n) cout << "not a DAG" <<</pre>
              endl;
             for(LL i = 0; i < n; i++)</pre>
25
                  cout << topo[i] <<</pre>
```

3.6 union and find

#define good ios_base::sync_with_stdio(0)

| #include < bits / stdc++.h>

;cin.tie(0)

typedef long long LL;

using namespace std;

LL parent[503*503];

int graph[503*503] = {0};
int dxy[4] = {1,-1};
LL now_area = 0,max_area = 0;

#define RSIZE 101

return 0;

28 | 29 | }

10

11

```
LL sfind(LL dots){//find leader,leader's parent = size of set
12
        if(parent[dots] < 0)</pre>
             return dots;
14
        return parent[dots] = sfind(parent[
15
              dots]);
   LL BFS(LL now,LL root){//find root and
17
         return size
        parent[now] = root;
        LL cnt = 1;
for(int k = 0; k < 4; k++){//4
20
                                                         14
             directions
                                                         15
             int u = now+dxy[k];
21
                                                         16
             if(graph[u] == 1 && parent[u] ==
22
                                                         17
                   -1)//unvisited
                  cnt += BFS(u,root);
24
25
        return cnt:
26
   void combine(LL u,LL v){//merge two sets
        LL set1 = sfind(u), set2 = sfind(v);
        if(set1 == set2) return;//same set
30
        max_area = max(max_area,-parent[set1
     ]-parent[set2]);
now_area--;//merge -> 2 pools become
31
                                                         26
32
        if(parent[set1] < parent[set2]){//1</pre>
              is larger
                                                         31
             parent[set1] += parent[set2];
parent[set2] = set1;
                                                         32
35
                                                         33
                                                         34
                                                         35
             parent[set2] += parent[set1];
             parent[set1] = set2;
40
                                                         38
41
        return:
                                                         39
42
                                                         40
   int main(){
                                                         41
43
        //freopen("file name", "r", stdin);
                                                         43
              //input redirection
        LL i,j,m,n,k;
cin >> m >> n >> k;
                                                         45
47
        memset(parent,-1,sizeof(parent));
        for(i = 1; i <= m; i++){
    for(j = 1; j <= n; j++)
        cin >> graph[i*(n+2)+j];
                                                         49
                                                         50
       n += 2;
dxy[2] = n,dxy[3] = -n;
LL mn = (m+1)*n;
                                                         51
                                                         52
        58
                                                         56
                         point consider as root
                                                         57
                  now_area++;
                  max_area = max(max_area,-
                        parent[x]);
61
             }
                                                         61
62
        LL ans = now area.max ans = max area:
63
        while(k--){
64
             LL x,y,temp;
65
             cin >> x >> y;
temp = x*n+y;
67
                                                         66
             if(graph[temp] == 1) continue;
68
             graph[temp] = 1;
69
                                                         68
             now_area++;
70
             max_area = max(max_area,(LL)1);
71
             for(i = 0; i < 4; i++){
    if(graph[temp+dxy[i]] == 0)
                        continue:
                  combine(temp,temp+dxy[i]);
74
```

4 Number Theory

ans += now_area;

max_ans += max_area;

cout << max_ans << endl << ans;</pre>

4.1 basic

76

77

78

81

```
template<typename T>
void gcd(const T &a,const T &b,T &d,T &x,
      T &y){
  if(!b) d=a,x=1,y=0;
  else gcd(b,a%b,d,y,x), y-=x*(a/b);
long long int phi[N+1];
void phiTable(){
  for(int i=1;i<=N;i++)phi[i]=i;</pre>
  for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=
        i)phi[x]-=phi[i];
void all_divdown(const LL &n) {// all n/x
  for(LL a=1;a<=n;a=n/(n/(a+1))){
     // dosomething;
}
const int MAXPRIME = 1000000;
int iscom[MAXPRIME], prime[MAXPRIME],
     primecnt;
int phi[MAXPRIME], mu[MAXPRIME];
void sieve(void){
  memset(iscom,0,sizeof(iscom));
  primecnt = 0;
  phi[1] = mu[1] = 1;
   for(int i=2;i<MAXPRIME;++i) {</pre>
     if(!iscom[i]) {
       prime[primecnt++] = i;
       mu[i] = -1;
phi[i] = i-1;
     for(int j=0;j<primecnt;++j) {
  int k = i * prime[j];
  if(k>=MAXPRIME) break;
       iscom[k] = prime[j];
if(i%prime[j]==0) {
         mu[k] = 0;
         phi[k] = phi[i] * prime[j];
         break;
       } else {
         mu[k]
                = -mu[i];
         phi[k] = phi[i] * (prime[j]-1);
    }
  }
}
for(int i=0;i<v.size();++i)</pre>
     if(modexp(g,(p-1)/v[i],p)==1)
       return false;
  return true;
LL primitive_root(const LL &p) {
  if(p==2) return 1;
  vector<LL> v;
  Factor(p-1,v);
  v.erase(unique(v.begin(), v.end()), v.
        end());
  for(LL g=2;g<p;++g)
   if(g_test(g,p,v))</pre>
       return g;
  puts("primitive_root NOT FOUND");
   return -1;
int Legendre(const LL &a, const LL &p) {
    return modexp(a%p,(p-1)/2,p); }
LL inv(const LL &a, const LL &n) {
  LL d,x,y;
  gcd(a,n,d,x,y);
  return d==1 ? (x+n)%n : -1;
int inv[maxN];
LL invtable(int n,LL P){
  inv[1]=1;
for(int i=2;i<n;++i)</pre>
     inv[i]=(P-(P/i))*inv[P%i]%P;
```

```
return ;
                                                     163
                                                                                                                      is_prime[j]=1;
                                                                                                          12
                                                                                                                      euler[j]=euler[j]/i*(i-1);
77
   LL log_mod(const LL &a, const LL &b,
                                                     164
                                                                                                          13
      const LL &p) {
// a ^ x = b ( mod p )
                                                          while(true){
                                                                                                          14
                                                     165
                                                             g2=a1.multiply(h1).substract(g1);
                                                                                                          15
                                                     166
      int m=sqrt(p+.5), e=1;
                                                             h2=N.substract(g2.pow(2)).divide(h1);
                                                                                                               }
                                                     167
                                                                                                          16
      LL v=inv(modexp(a,m,p), p);
                                                             a2=g2.add(a0).divide(h2);
     map<LL,int> x;
                                                             p=a1.multiply(p2).add(p1);
                                                                                                          18
                                                                                                             LL pow(LL a, LL b, LL mod){//a^b%mod
                                                     169
                                                             q=a1.multiply(q2).add(q1);
if(p.pow(2).substract(N.multiply(q.
     x[1]=0:
                                                     170
                                                                                                          19
                                                                                                               LL ans=1;
      for(int i=1;i<m;++i) {</pre>
                                                                                                               for(;b;a=a*a%mod,b>>=1)
83
                                                     171
                                                                                                          20
          = LLmul(e,a,p);
                                                                  pow(2))).compareTo(BigInteger.
                                                                                                                 if(b&1)ans=ans*a%mod;
                                                                                                          21
        if(!x.count(e)) x[e] = i;
                                                                  ONE)==0)break;
                                                             g1=g2;h1=h2;a1=a2;
                                                                                                          23
      for(int i=0;i<m;++i) {</pre>
                                                             p1=p2;p2=p;
                                                                                                             bool isless(LL *a,int n,int k){
                                                     173
        if(x.count(b)) return i*m + x[b];
88
                                                     174
                                                             q1=q2;q2=q;
                                                                                                          25
                                                                                                               if(*a==1)return k>1:
                                                                                                               if(--n==0)return *a<k;</pre>
        b = LLmul(b, v, p);
                                                     175
                                                                                                          26
                                                          System.out.println(p+" "+q);
                                                                                                               int next=0;
                                                     176
     return -1:
                                                                                                               for(LL b=1;b<k;++next)</pre>
   }
                                                                                                                 b*=*a;
93
                                                                                                          30
                                                                                                               return isless(a+1,n,next);
94
   LL Tonelli_Shanks(const LL &n, const LL &
                                                                                                          31
                                                               bit set
                                                                                                             LL high_pow(LL *a,int n,LL mod){
   if(*a==1||--n==0)return *a%mod;
                                                        4.2
     p) {
// x^2 = n \pmod{p}
                                                                                                          32
                                                                                                          33
      if(n==0) return 0;
                                                                                                               int k=0,r=euler[mod];
     for(LL tma=1;tma!=pow(*a,k+r,mod);++k)
                                                        void sub set(int S){
                                                                                                                 tma=tma*(*a)%mod;
                                                          int sub=S;
                                                                                                               int S = 0;
                                                          do{
                                                                                                          37
     LL Q = p-1;
while(!(Q&1)) { Q>>=1; ++5; }
                                                            //對某集合的子集合的處理
                                                                                                               int tmd=high_pow(a+1,n,r), t=(tmd-k+r)%
                                                                                                          38
100
                                                             sub=(sub-1)&S;
      if(S==1) return modexp(n\%p,(p+1)/4,p);
101
                                                          }while(sub!=S);
                                                                                                               return pow(*a,k+t,mod);
      for(;Legendre(z,p)!=-1;++z)
103
                                                        void k_sub_set(int k,int n){
  int comb=(1<<k)-1,S=1<<n;</pre>
                                                                                                          40
     LL c = modexp(z,Q,p);
LL R = modexp(n%p,(Q+1)/2,p), t =
                                                                                                             LL a[1000005];
104
105
                                                          while(comb<S){</pre>
                                                                                                          42
                                                                                                             int t, mod;
                                                     10
           modexp(n%p,Q,p);
                                                                                                             int main(){
                                                                                                          43
                                                             //對大小為k的子集合的處理
                                                     11
                                                                                                               init_euler();
106
                                                            int x=comb&-comb,y=comb+x;
comb=((comb&~y)/x>>1)|y;
                                                     12
                                                                                                               scanf("%d",&t);
10
      while(1) {
       if(t==1) return R;
                                                                                                               #define n 4
109
        LL b = modexp(c,1L << (M-i-1),p);
                                                                                                          47
                                                                                                               while(t--){
       R = LLmul(R,b,p);
t = LLmul( LLmul(b,b,p), t, p);
                                                                                                                 for(int i=0;i<n;++i)scanf("%lld",&a[i</pre>
110
                                                                                                                 ]);
scanf("%d",&mod);
printf("%lld\n",high_pow(a,n,mod));
111
        c = LLmul(b,b,p);
112
113
                                                               matrix exponential
114
     return -1;
                                                                                                               return 0;
115
116
   }
117
                                                        void exp(LL m[2][2], LL x){
   template<tvpename T>
118
                                                            LL c[2][2] = \{\{1,1\},\{1,0\}\},n[2][2];

n[0][0] = m[0][0]*c[0][0] + m[0][1]*c
   T Euler(T n){
119
                                                                                                             4.6 大數取模
     T ans=n;
                                                                  [1][0];
121
      for(T i=2;i*i<=n;++i){</pre>
                                                             n[0][1] = m[0][0]*c[0][1] + m[0][1]*c
       if(n%i==0){
  ans=ans/i*(i-1);
122
                                                                   [1][1];
                                                                                                           1 LL exp(LL x,LL y,LL p){
2     if(y == 0) return 1;
3     if(y & 1) return (exp(x,y-1,p)*x) % p
123
                                                                      = m[1][0]*c[0][0] + m[1][1]*c
                                                             n[1][0]
          while(n%i==0)n/=i;
124
                                                                  [1][0];
125
                                                            n[1][1] = m[1][0]*c[0][1] + m[1][1]*c
[1][1];
126
                                                                                                                       ;//y is odd
      if(n>1)ans=ans/n*(n-1);
                                                             if(x != 1)
                                                                                                                      LL temp = exp(x,y/2,p);
128
      return ans;
                                                                 exp(n,x-1);
129
   }
                                                                                                                      return (temp*temp) % p;
                                                             else
130
                                                                  cout << n[0][0];
   //Chinese_remainder_theorem
131
                                                     12
   template<typename T>
                                                                                                             LL calcmod(LL index,LL p){
132
                                                     13
                                                        int main(){
   T pow_mod(T n,T k,T m){
                                                                                                                 if(index == 0) return base[index]-'0'
133
                                                             LL u[2][2] = \{\{1,1\},\{1,0\}\},n;
     T ans=1;
                                                     14
134
                                                     15
                                                            cin >> n; cout << "90\muRC^{2}B" << n+2 << "9\mu\gamma^{\circ}";
135
      for(n=(n)=m?n\%m:n);k;k>>=1){
                                                                                                                  LL single = calcmod(index-1,p)*10;
        if(k&1)ans=ans*n%m;
                                                                                                                  return (single%p + base[index]-'0')%p
136
                                                                                                          12
                                                     17
                                                             exp(u,n);
137
        n=n*n%m;
                                                                                                          13 }
138
139
     return ans;
140
   template<typename T>
141
                                                        4.4 SpeedExpo
   T crt(vector<T> &m,vector<T> &a){
                                                                                                             4.7
                                                                                                                    模逆元
142
      T M=1,tM,ans=0;
143
      for(int i=0;i<(int)m.size();++i)M*=m[i</pre>
144
                                                      I LL expo(LL a,LL b,LL p){
           1;
                                                                                                             #include < bits / stdc++.h>
                                                            if(b == 0) return 1;
if(b & 1) return (expo(a,b-1,p)*a)%p;
      for(int i=0;i<(int)a.size();++i){</pre>
                                                                                                             #define good ios_base::sync_with_stdio(0)
145
                                                                                                                  ;cin.tie(0)
146
        tM=M/m[i];
                                                                  //b is odd
        ans=(ans+(a[i]*tM%M)*pow_mod(tM,Euler
                                                                                                             typedef long long LL;
147
                                                             LL temp = expo(a,b/2,p);
             (m[i])-1,m[i])%M)%M;
                                                                                                             using namespace std;
                                                             return (temp*temp)%p;
        /*如果m[i]是質數・Euler(m[i])-1=m[i
148
                                                                                                             LL mod_inverse_by_speed_exp(LL x,LL y,LL
             ]-2,就不用算Euler了*/
                                                                                                                  p){
                                                                                                                 if(y == 0) return 1;
150
     return ans;
                                                                                                                 if(y % 2) return (
    mod_inverse_by_speed_exp(x,y-1,p)
   }
151
                                                        4.5
                                                               外星模運算
152
                                                                                                                       )*x)%p;
   //java code
                                                                                                                 else{
   //求 sqrt(N)的 連分數
154
                                                        //a[0]^(a[1]^a[2]^.
                                                                                                                      LL temp =
155
   public static void Pell(int n){
                                                        #define maxn 1000000
                                                                                                                            mod_inverse_by_speed_exp(x,y
      BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2
156
                                                        int euler[maxn+5];
                                                                                                                            /2,p);
           ,h1,h2,p,q;
                                                        bool is_prime[maxn+5];
void init_euler(){
                                                                                                                      return (temp*temp)%p;
                                                                                                          11
      g1=q2=p1=BigInteger.ZERO;
157
                                                                                                                 }
                                                                                                          12
     h1=q1=p2=BigInteger.ONE;
a0=a1=BigInteger.valueOf((int)Math.sqrt
                                                          is_prime[1]=1;//一不是質數
158
                                                                                                          13
159
                                                          for(int i=1;i<=maxn;i++)euler[i]=i;
for(int i=2;i<=maxn;i++){</pre>
                                                                                                             int main(){
           (1.0*n));
                                                                                                                 good;
      BigInteger ans=a0.multiply(a0);
160
                                                            if(!is_prime[i]){//是質數
                                                                                                          16
                                                                                                                 LL n,i,p,x;
      if(ans.equals(BigInteger.valueOf(n))){
                                                                                                                 cin >> n >> p;
for(i = 0; i < n; i++){
                                                               euler[i]--
                                                                                                          17
        System.out.println("No solution!");
                                                               for(int j=i<<1;j<=maxn;j+=i){</pre>
```

```
cin >> x;
                                                                  now*=tmp[i];
for(int j=0;j<len;++j)</pre>
20
             cout << mod_inverse_by_speed_exp( 82</pre>
                   x, p-2, p) <<'
                                                          83
                                                          84
                                                                     v.push_back(v[j]*now);
        return 0;
                                                          85
                                                               }
23 }
```

質因數分解 4.8

```
| LL func(const LL n.const LL mod.const int
     return (LLmul(n,n,mod)+c+mod)%mod;
  }
  LL pollorrho(const LL n, const int c) {//
        循環節長度
     LL a=1, b=1;
     a=func(a,n,c)%n;
b=func(b,n,c)%n; b=func(b,n,c)%n;
     while(gcd(abs(a-b),n)==1) {
       a=func(a,n,c)%n;
       b=func(b,n,c)%n; b=func(b,n,c)%n;
12
     return gcd(abs(a-b),n);
13
  }
14
15
  void prefactor(LL &n, vector<LL> &v) {
     for(int i=0;i<12;++i) {</pre>
       while(n%prime[i]==0)
         v.push_back(prime[i]);
19
         n/=prime[i];
20
21
    }
  }
23
24
  void smallfactor(LL n, vector<LL> &v) {
    if(n<MAXPRIME) {</pre>
26
       while(isp[(int)n]) {
         v.push_back(isp[(int)n]);
         n/=isp[(int)n];
30
31
       v.push_back(n);
     } else {
32
       for(int i=0;i<primecnt&&prime[i]*</pre>
33
            prime[i]<=n;++i) {</pre>
          while(n%prime[i]==0)
35
           v.push_back(prime[i]);
           n/=prime[i];
37
38
       if(n!=1) v.push_back(n);
39
    }
  }
42
43
  void comfactor(const LL &n, vector<LL> &v
     if(n<1e9) {
       smallfactor(n,v);
       return;
47
     if(Isprime(n)) +
48
49
       v.push back(n);
50
       return;
51
     LL d:
53
     for(int c=3;;++c) {
       d = pollorrho(n,c);
55
       if(d!=n) break;
56
     comfactor(d,v);
57
     comfactor(n/d,v);
  }
60
  void Factor(const LL &x, vector<LL> &v) {
61
    LL n = x;
if(n==1) { puts("Factor 1"); return; }
prefactor(n,v);
62
63
     if(n==1) return;
     comfactor(n,v);
67
     sort(v.begin(),v.end());
  }
68
69
  void AllFactor(const LL &n, vector<LL> &v)
70
     vector<LL> tmp;
     Factor(n,tmp);
     v.clear():
73
     v.push_back(1);
     int len;
     LL now=1;
     for(int i=0;i<tmp.size();++i) {
  if(i==0 || tmp[i]!=tmp[i-1]) {</pre>
78
         len = v.size();
79
         now = 1;
80
```

String

manacher(最小回文字串) 5.1

```
ı //原字串: asdsasdsa
2 //要先把字串變成這樣: @#a#s#d#s#a#s#d#s#a
  void manacher(char *s,int len,int *z){
    int l=0,r=0;
for(int i=1;i<len;++i){</pre>
      z[i]=r>i?min(z[2*1-i],r-i):1;
while(s[i+z[i]]==s[i-z[i]])++z[i];
       if(z[i]+i>r)r=z[i]+i,l=i;
    }//ans = max(z)-1
```

Tree Problem

findLCA 6.1

```
1 LL findLCA(LL u,LL v){
       while(depth[u] > depth[v])
    u = father[u];
       while(depth[v] > depth[u])
v = father[v];
       while(u != v){
            u = father[u]:
            v = father[v];
       return u;//or return v
```

6.2 kruskal(MST)

| #include < bits / stdc++.h>

```
2 #define good ios_base::sync_with_stdio(0)
  ;cin.tie(0)
#define RSIZE 10002
   #define pll pair<LL,LL>
typedef long long LL;
   using namespace std;
   struct EDGE{
       LL u,v,w;
10 };
11
   vector<EDGE> adjacent;//out neighbor,
13 LL fa[RSIZE];
14
   bool cmp(EDGE &a, EDGE &b){//sort by
15
        weight
        return a.w < b.w;</pre>
   LL sfind(LL now){//find root,root's
18
        father=set size
if(fa[now] < 0)</pre>
            return now;
        return fa[now] = sfind(fa[now]);
   bool merge(LL u,LL v){//find two root,
        comparing size(by root's father)
LL set1 = sfind(u), set2 = sfind(v);
24
        if(set1 == set2) return false;//same
25
             root-> no need to merge
        if(fa[set1] < fa[set2]){ //set1 is</pre>
             Larger
             fa[set1] += fa[set2];
            fa[set2] = set1;
28
29
             fa[set2] += fa[set1];
            fa[set1] = set2;
33
        return true;
34
35 }
```

```
37
   int main(){
38
        good;
        //freopen("file name", "r", stdin);
39
             //input redirection
        LL i,n,m;
        cin >> n >> m;
for(i = 0; i < m; i++){</pre>
41
42
            LL x,y,weight;
cin >> x >> y >> weight;
43
44
             adjacent.push_back({x,y,weight});
46
        memset(fa,-1,sizeof(fa));//unvisited
48
        sort(adjacent.begin(),adjacent.end(),
             cmp);//sort by weight
        LL cost = 0, now_edge = 0;
for(EDGE e : adjacent){
             if(merge(e.u,e.v)){//connect edge
52
                 cost += e.w;
                 now_edge++;
53
            }
54
55
56
        if(now_edge < n-1)//not a MST</pre>
            cout << -1 << endl;
57
58
59
            cout << cost << endl;</pre>
        return 0:
60
61 }
```

6.3 LCA

11

25

13

14

16

19

20

```
1 const int MAXN=100000; // 1-base
   const int MLG=17; //Log2(MAXN)+1;
int pa[MLG+2][MAXN+5];
   int dep[MAXN+5];
   vector<int> G[MAXN+5];
void dfs(int x,int p=0){//dfs(root);
     pa[0][x]=p;
     for(int i=0;i<=MLG;++i)</pre>
       pa[i+1][x]=pa[i][pa[i][x]];
     for(auto &i:G[x]){
       if(i==p)continue;
       dep[i]=dep[x]+1;
12
13
       dfs(i,x);
     }
14
16
   inline int jump(int x,int d){
     for(int i=0;i<=MLG;++i)</pre>
       if((d>>i)&1) x=pa[i][x];
18
     return x;
19
   inline int find_lca(int a,int b){
     if(dep[a]>dep[b])swap(a,b);
23
     b=jump(b,dep[b]-dep[a]);
24
     if(a==b)return a;
     for(int i=MLG;i>=0;--i){
       if(pa[i][a]!=pa[i][b]){
26
          a=pa[i][a];
          b=pa[i][b];
       }
30
     return pa[0][a];
```

6.4 Prim(MST)

```
#include < bits / stdc++.h>
  #define good ios_base::sync_with_stdio(0)
  ;cin.tie(0)
#define RSIZE 502
  #define oo 1000000001 //1e9+1
typedef long long LL;
  using namespace std;
   vector<pair<LL,LL>> adjacent[RSIZE];//out
  neighbor, weight of edge
LL dis[RSIZE],fa[RSIZE];//dis for weight
        of two vertexes
  bool visit[RSIZE] = {false};
  int main(){
       good;
       //freopen("file name", "r", stdin);
             //input redirection
       LL i,n,m;
       cin >> n >> m;
       for(i = 0; i < m; i++){</pre>
            LL x,y,w;
            cin >> x >> v >> w:
            adjacent[x].push_back({y,w});
21
            adjacent[y].push_back({x,w});
```

```
48 l
       //initial
23
24
                                                       49
       LL start = 0;
       memset(dis,oo,sizeof(dis));
25
       memset(fa,-1,sizeof(fa));
27
       priority_queue<pair<LL,LL>> PQ;//-dis
             [].vertex
       PQ.push({dis[start] = 0,start});
29
       //prim
        while (!PQ.empty()){
            auto pt = PQ.top();
            PQ.pop();
            LL v = pt.second;
if(visit[v]) continue;
                                                       56
34
            visit[v] = true;
for(auto it : adjacent[v]){
35
                 LL neibor = it.first,w = it.
                       second;
                 if(visit[neibor]) continue;
                                                       61
                 if(w < dis[neibor]){ //new</pre>
39
                       edge is shorter
                      dis[neibor] = w;
fa[neibor] = v;
                                                       63
                      PQ.push({-dis[neibor],
42
                            neibor});
                 }
                                                       65
            }
44
                                                       66
45
                                                       67
       LL cost = 0, cnt = 0;
       //count cost and check if MST exists
48
        for(i = 0; i < n; i++){</pre>
                                                       70
            if(dis[i] < oo)</pre>
                cost += dis[i];
50
                                                       71
            else
                 cnt++:
                                                       73
       if(cnt)
55
            cout << -1 << endl;</pre>
                                                       75
       else
56
                                                       76
            cout << cost << endl;</pre>
                                                       77
57
       return 0;
58
                                                       78
59 }
```

6.5 segment tree

```
#include < bits / stdc++.h>
   #define good ios_base::sync_with_stdio(0)
   ;cin.tie(0)
#define RSIZE 100000
   #define pll pair<LL,LL>
#define lc 2*index //c means child
   #define rc 2*index+1
   typedef long long LL;
   using namespace std;
LL tree[4*RSIZE];//saving range maximum
LL lazy[4*RSIZE] = {0};
LL num[RSIZE],cnt = 1;
   //using range maximum as example
void build(LL L,LL R,LL index){
   LL temp = index;
15
        if(L == R){
17
              tree[index] = num[cnt];
20
              return:
        LL M = (L+R)/2:
22
        build(L,M,lc);
        build(M+1,R,rc);
        tree[index] = max(tree[lc],tree[rc]);
   //single point modify
27
   void modify(LL x,LL v,LL L,LL R,LL index) 18 }
28
29
        if(L == R){
              tree[index] = v;
              return;
        LL M = (L+R)/2;
if(x <= M)//left side
33
35
             modify(x,v,L,M,lc);
        modify(x,v,M+1,R,rc);
tree[index] = max(tree[lc],tree[rc]);
38
   //a range including index has to add tag
40
   void addtag(LL tag,LL index){
    tree[index] += tag;
41
43
        lazy[index] += tag;
  //transfering tag to child
void push(LL index){
45
        addtag(lazy[index],lc);
```

```
50 }
51 //Lower variables are queried range, UPPER
52 ones are full range
53 Lt query(Lt 1,Lt r,Lt L,Lt R,Lt index){
54    if(1 <= L && R <= r) return tree[
        index];
55    push(index);//if use single point
        modify, no need
55    Lt M = (L+R)/2:</pre>
```

lazy[index] = 0;//tag is transfered

addtag(lazy[index],rc);

to child

7 default

}

int main(){

7.1 8 queen

7.2 debug

7.3 IncStack

10 #endif

```
1 //Magic
  #pragma GCC optimize "Ofast"
  //stack resize, change esp to rsp if 64-
        bit system
  asm("mov %0,%%esp\n" :: "g"(mem+10000000))
   -Wl,--stack,214748364 -trigraphs
  #pragma comment(linker, "/STACK
        :1024000000,1024000000")
  //linux stack resize
#include<sys/resource.h>
  void increase_stack(){
  const rlim_t ks=64*1024*1024;
     struct rlimit rl;
     int res=getrlimit(RLIMIT_STACK,&rl);
     if(!res&&rl.rlim_cur<ks){</pre>
       rl.rlim_cur=ks;
       res=setrlimit(RLIMIT_STACK,&rl);
15
16
```

7.4 input

7.5 randomize

7.6 sweepline

```
#include < bits / stdc++.h>
  #define good ios_base::sync_with_stdio(0)
        :cin.tie(0)
  typedef long long LL;
  using namespace std;
  struct Seg{
      LL left, right;
  bool cmp(Seg &a,Seg &b){
       return a.left < b.left;</pre>
  }
13
14
15
  int main(){
16
      good:
17
       //freopen("P_2_1_5.in", "r", stdin)
18
            ;//input redirection
       LL n;
      21
                right;
       sort(line,line+n,cmp);
       Seg last = line[0];
LL total = 0;
for(LL i = 1; i < n; i++){</pre>
25
26
```

```
if(line[i].left > last.right){
29
               total += last.right - last.
                    left;
               last = line[i];
               continue;
31
           last.right = max(last.right,line[
33
                i].right);//merge last and
                line[i]
      total += last.right - last.left;
      cout << total;</pre>
       return 0;
38
         模板
  7.7
  #include<bits/stdc++.h>
  #define good ios_base::sync_with_stdio(0)
       ;cin.tie(0)
  #define RSIZE 101
  #define pll pair<LL,LL>
  #define lc 2*index
#define rc 2*index+1
  typedef long long LL;
  using namespace std;
  int main(){
      good;
11
      //freopen("file name", "r", stdin);
12
           //input redirection
13
      return 0;
15 }
```

other

8.1 1D0D dp

```
#include<bits/stdc++.h>
   #define good ios_base::sync_with_stdio(0)
        ;cin.tie(0)
   typedef long long LL; using namespace std;
   int main(){
        good;
        //freopen("file name", "r", stdin);
             //input redirection
        LL i,n;
        LL money[100002],dp[100002] ={0};//dp
for last i day max profit
        cin >> n;
        for(i = 1; i <= n; i++)</pre>
12
        cin >> money[i];
dp[0] = 0,dp[1] = money[1],dp[2] =
    max(money[1],money[2]);
13
14
        for(i = 3; i <= n; i++)
             dp[i] = max(dp[i-2]+money[i],dp[i]
16
                   -1]);//choose,discard
        cout << dp[i-1];
        return 0:
18
19 }
```

mergesort

```
#include <bits/stdc++.h>
   using namespace std;
   #define good ios_base::sync_with_stdio(0)
  ;cin.tie(0)
#define NORMALSIZE 8
typedef long long LL;
  void merge(LL *,LL,LL);//prototype
void mergesort(LL *,LL,LL);
   void mergesort(LL* arr,LL 1,LL r){
10
11
       LL m;
       if(1 < r){
12
            m = (1+r)/2;
13
            mergesort(arr,1,m);//sort left
                  subarray
            mergesort(arr,m+1,r);//sort right
15
                   sunarrav
            merge(arr,1,m,r);
```

```
18
        return;
   }
19
   void merge(LL* arr,LL 1,LL m,LL r){
20
        LL left,right,tmp[NORMALSIZE],i;
        left = 1; // left subarr.begin
        right = m+1; //right subarr.begin
23
        i = 1:
24
        while ((left <= m) && (right <= r)){
    //merging left subarr and right</pre>
25
26
              if(arr[left] < arr[right]){ //</pre>
                  left subarr is smaller
tmp[i] = arr[left];
27
                   i++,left++;
28
29
              else{ //the other situation
                   tmp[i] = arr[right];
32
                   i++, right++;
33
             }
34
        while(left <= m){ // right subarr is</pre>
              fully sorted
              tmp[i] = arr[left];
             i++,left++;
37
38
        while(right <= r){ // left subarr is</pre>
39
             fully sorted
tmp[i] = arr[right];
              i++,right++;
42
        for(i = 1; i <= r; i++)
    arr[i] = tmp[i];</pre>
43
44
45 }
```

8.3 WhatDay

```
i int whatday(int y,int m,int d){
    if(m<=2)m+=12,--y;
if(y<1752||y==1752&&m<9||y==1752&&m
         ==9&&d<3)
      return (d+2*m+3*(m+1)/5+y+y/4+5)%7;
    return (d+2*m+3*(m+1)/5+y+y/4-y/100+y
         /400)%7;
```

zformula

9.1 formula

9.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形,面積 = 內部格點 數 + 邊上格點數/2-1

9.1.2 圖論

- 1. 對於平面圖 · $F = E V + C + 1 \cdot C$ 是連通 ²⁰ 分量數
- 2. 對於連通圖 · E < 3V 6 22 3 對於連通圖 · E < 3V 6 32 3 對於連通圖 · B 大獨立點集的大小設為 I(G) · 23 3 3 最大匹配大小設為 M(G) · 最小點覆蓋設為 Cv(G) · 最小邊覆蓋設為 $\acute{Ce}(G)$ · 對於任意連通
 - $\begin{array}{ll} \text{(a)} & I(G)+Cv(G)=|V| \\ \text{(b)} & M(G)+Ce(G)=|V| \end{array}$
- 4. 對於連通二分圖:
 - $\begin{array}{ll} \text{(a)} & I(G) = Cv(G) \\ \text{(b)} & M(G) = Ce(G) \end{array}$
- 5. 最大權閉合圖:
 - $\begin{array}{ll} \text{(a)} & C(u,v) = \infty, (u,v) \in E \\ \text{(b)} & C(S,v) = W_v, W_v > 0 \\ \text{(c)} & C(v,T) = -W_v, W_v < 0 \\ \text{(d)} & \text{ans} = \sum_{W_v > 0} W_v flow(S,T) \end{array}$
- 6. 最大密度子圖:
 - (a) $\not \equiv \max\left(\frac{W_e + W_v}{|V'|}\right), e \in E', v \in V'$
 - (b) $U = \sum_{v \in V} 2W_v + \sum_{e \in E} W_e$
 - (c) $C(u,v) = W_{(u,v)}, (u,v) \in E$ · 雙向邊 (d) $C(S, v) = U, v \in V$
 - (e) $D_u = \sum_{(u,v) \in E} W_{(u,v)}$
 - (f) $C(v,T) = U + 2g D_v 2W_v, v \in V$

- (g) 二分搜 g: $l=0, r=U, eps=1/n^2$ $if((U \times |V| - flow(S, T))/2 > 0) l =$ else r = mid
- (h) ans= $min_cut(S,T)$ (i) |E| = 0 要特殊判斷
- 7. 弦圖:
 - 點數大於 3 的環都要有一條弦 完美消除序列從後往前依次給每個點染
 - 色·給每個點染上可以染的最小顏色 最大團大小 = 色數
 - 最大獨立集: 完美消除序列從前往後能選
 - 最小團覆蓋: 最大獨立集的點和他延伸的
 - (f)
 - 最外國復盛。最外獨立采的和相心經濟的 邊構成 區間圖是弦圖 區間圖的完美消除序列: 將區間按造又端 點由小到大排序 區間圖染色: 用線段樹做

9.1.3 dinic 特殊圖複雜度

1. 單位流: $O\left(\min\left(V^{3/2}, E^{1/2}\right)E\right)$ 2. 二分圖: $O(V^{1/2}E)$

9.1.4 0-1 分數規劃

= $\{0,1\}\cdot x_i$ 可能會有其他限制,求 $\max\left(\frac{\sum B_i x_i}{\sum C_i x_i}\right)$

- 1. $D(i,g) = B_i g \times C_i$
- 2. $f(g) = \sum D(i, g)x_i$
- 3. f(g) = 0 時 g 為最佳解 $\cdot f(g) < 0$ 沒有意義
- 4. 因為 f(g) 單調可以二分搜 g
- 5. 或用 Dinkelbach 通常比較快

```
i binary_search(){
    while(r-l>eps){
      g=(1+r)/2;
      for(i:所有元素)D[i]=B[i]-g*C[i];//D(i
      ,g)
找出一組合法x[i]使f(g)最大;
      if(f(g)>0) l=g;
      else r=g;
    Ans = r:
10
Dinkelbach(){
    g=任意狀態(通常設為0);
12
    do{
13
      for(i:所有元素)D[i]=B[i]-g*C[i];//D(i
15
          ,g)
      找出一組合法x[i]使f(g)最大;
      p=0,q=0;
17
      for(i:所有元素)
18
       if(x[i])p+=B[i],q+=C[i];
19
      g=p/q;//更新解·注意q=0的情况
    }while(abs(Ans-g)>EPS);
    return Ans:
```

9.1.5 學長公式

- 1. $\sum_{d\mid n}\phi(n)=n$
- 2. $g(n) = \sum_{d \mid n} f(d) => f(n) = \sum_{d \mid n} \mu(d) \times$ g(n/d)
- 3. Harmonic series $H_n = \ln(n) + \gamma + 1/(2n) 1/(12n^2) + 1/(120n^4)$
- 4. $\gamma = 0.57721566490153286060651209008240243104215$
- 5. 格雷碼 = $n \oplus (n >> 1)$
- 6. $SG(A+B) = SG(A) \oplus SG(B)$
- 7. 選轉矩陣 $M(\theta) = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$ $-sin\theta$ $cos\theta$

9.1.6 基本數論

- 1. $\sum_{d|n} \mu(n) = [n == 1]$
- 2. $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m)$ $\sum_{d|m} \mu(d) \times g(m/d)$
- 3. $\sum_{i=1}^{m} \sum_{j=1}^{m} \overline{\Delta} \widetilde{g} \widetilde{\mathbf{y}} \widetilde{\mathbf{z}} = \sum \mu(d) \left\lfloor \frac{n}{d} \right\rfloor \left\lfloor \frac{m}{d} \right\rfloor$ 4. $\sum_{i=1}^{n} \sum_{j=1}^{n} lcm(i,j) = n \sum_{d \mid n} d \times \phi(d)$

9.1.7 排組公式

- 1. k 卡特蘭 $\frac{C_n^{kn}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$
- 2. $H(n,m) \cong x_1 + x_2 \dots + x_n = k, num = C_k^{n+k-1}$
- 3. Stirling number of 2^{nd} ,n 人分 k 組方法數目
 - (a) S(0,0) = S(n,n) = 1
 - (b) S(n,0) = 0
 - (c) S(n,k) = kS(n-1,k) + S(n-1,k-1)
- 4. Bell number, n 人分任意多組方法數目
 - (a) $B_0 = 1$

 - (a) $B_0 = 1$ (b) $B_n = \sum_{i=0}^n S(n,i)$ (c) $B_{n+1} = \sum_{k=0}^n C_k^n B_k$ (d) $B_{p+n} \equiv B_n + B_{n+1} mod p$, p is prime
 - (e) $B_{pm+n} \equiv mB_n + B_{n+1} mod p$, p is prime
 - (f) From $B_0: 1, 1, 2, 5, 15, 52$, 203, 877, 4140, 21147, 115975
- 5. Derangement, 錯排, 沒有人在自己位置上
 - (a) $D_n = n!(1 \frac{1}{1!} + \frac{1}{2!} \frac{1}{3!} \dots +$ (-1)ⁿ $\frac{1}{n!}$) (b) $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 =$
 - $1, D_1 = 0$
 - (c) From $D_0: 1, 0, 1, 2, 9, 44$, 265, 1854, 14833, 133496
- 6. Binomial Equality
 - (a) $\sum_{k} {r \choose m+k} {s \choose n-k} = {r+s \choose m+n}$

 - (b) $\sum_{k} \binom{l}{m+k} \binom{s-k}{n+k} = \binom{l+s}{l-m+n}$ (c) $\sum_{k} \binom{l}{m+k} \binom{s+k}{n} (-1)^{k} \binom{l+m+n}{n-l}$
 - $(-1)^{s+m} {n-l \choose n-l}$ $(d) \sum_{k \le l} {l-k \choose m} {s \choose k-n} (-1)^k$ $(-1)^{l+m} {s-m-1 \choose l-n-m}$ $(e) \sum_{0 \le k \le l} {l-k \choose m} {q+k \choose n} = {l+q+1 \choose m+n+1}$ $(f) {r \choose k} = (-1)^k {k-r-1 \choose k}$ $(g) {r \choose m} {m \choose k} = {r \choose k} {r-k \choose m-k}$

 - (h) $\sum_{k \le n} {r+k \choose k} = {r+n+1 \choose n}$
 - (i) $\sum_{0 \le k \le n} {k \choose m} = {n+1 \choose m+1}$
 - (j) $\sum_{k \le m} {m+r \choose k} x^k y^k$ $\sum_{k \le m} {-r \choose k} (-x)^k (x+y)^{m-k}$

9.1.8 幂次, 幂次和

- 1. $a^{b} \% P = a^{b\% \varphi(p) + \varphi(p)}, b > \varphi(p)$
- 2. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- 3. $1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} \frac{n}{30}$
- 4. $1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} \frac{n^2}{12}$
- 5. $0^k + 1^k + 2^k + \dots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n+1$
- 6. $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^n C_k^{n+1} B_k m^{n+1-k}$
- 7. $\sum_{j=0}^{m} C_j^{m+1} B_j = 0, B_0 = 1$
- 8. 除了 $B_1=-1/2$ · 剩下的奇數項都是 0
- 9. $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} =$ -174611/330,

9.1.9 Burnside's lemma

- 1. $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- 2. $X^g = t^{c(g)}$
- 3. G 表示有幾種轉法 \cdot X^g 表示在那種轉法下 \cdot 有 幾種是會保持對稱的,t 是顏色數,c(g) 是循環 節不動的面數。

9.1.10 Count on a tree

- 1. Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^{n} (i \times a_i \times a_i)$ $\sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
- 2. Unrooted tree:
 - $\begin{array}{ll} \text{(a)} & \text{Odd:} a_n \sum_{i=1}^{n/2} a_i a_{n-i} \\ \text{(b)} & \text{Even:} Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1) \end{array}$
- 3. Spanning Tree
 - (a) 完全圖 n^n-2
 - (b) 一般圖 (Kirchhoff's theorem)M[i][i] = $degree(V_i), M[i][j] = -1, if have E(i, j), 0 if no edge. delete any one row$ and col in A, ans = det(A)

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Codebook - ss

C++ Resource Test

```
#include <bits/stdc++.h>
   using namespace std;
   namespace system_test {
  const size_t KB = 1024;
const size_t MB = KB * 1024;
const size_t GB = MB * 1024;
   size_t block_size, bound;
void stack_size_dfs(size_t depth = 1) {
     if (depth >= bound)
12
13
        return;
     int8_t ptr[block_size]; // 若無法編譯將
     block_size 改成常數
memset(ptr, 'a', block_size);
cout << depth << endl;
     stack_size_dfs(depth + 1);
17
  }
18
19
   void stack_size_and_runtime_error(size_t
20
         block_size, size_t bound = 1024) {
     system_test::block_size = block_size;
system_test::bound = bound;
     stack_size_dfs();
```

```
56 void runtime_error_4() {
57   // free(): invalid pointer
58   int *ptr = new int[7122];
   double speed(int iter num) {
       const int block_size = 1024;
                                                                          ptr += 1;
       volatile int A[block_size];
                                                                          delete[] ptr;
       auto begin = chrono::
             high_resolution_clock::now();
      while (iter_num--)
for (int j = 0; j < block_size; ++j)
    A[j] += j;
auto end = chrono::</pre>
                                                                   void runtime_error_5() {
    // maybe illegal instruction
    int a = 7122, b = 0;
    cout << (a / b) << endl;</pre>
31
32
            high_resolution_clock::now();
                                                                    67
       chrono::duration<double> diff = end -
                                                                       void runtime_error_6() {
  // floating point exception
  volatile int a = 7122, b = 0;
             begin:
                                                                    69
      return diff.count();
35
                                                                          cout << (a / b) << endl;
   void runtime_error_1() {
   // Segmentation fault
   int *ptr = nullptr;
39
                                                                    74
                                                                    75 void runtime_error_7() {
       *(ptr + 7122) = 7122;
                                                                         // call to abort.
                                                                          assert(false);
42
   void runtime_error_2() {
    // Segmentation fault
    int *ptr = (int *)memset;
                                                                    80 } // namespace system_test
                                                                   81
      *ptr = 7122;
                                                                       #include <sys/resource.h>
                                                                    82
48 }
                                                                       void print_stack_limit() { // only work
                                                                    83
                                                                              in Linux
   void runtime_error_3() {
   // munmap_chunk(): invalid pointer
   int *ptr = (int *)memset;
                                                                          struct rlimit 1;
                                                                          delete ptr;
                                                                   87 }
```