BMED436 Medical Robot
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Spatial Description and Transformation

1 Rotations around X and Y axes

```
theta_x = deg2rad(60);
R_A2B = [
    1, 0, 0;
    0, cos(theta_x), -sin(theta_x);
    0, sin(theta_x), cos(theta_x);
    ]
```

Listing 1: ${}^{A}_{B}R$. Rotate 60° around X-axis.

```
R_A2B =
    1.0000         0         0
         0    0.5000   -0.8660
         0    0.8660    0.5000
```

```
theta_y = deg2rad(90);
R_B2C = [
    cos(theta_y), 0, sin(theta_y);
    0, 1, 0;
    -sin(theta_y), 0, cos(theta_y);
    ]
```

Listing 2: ${}^{B}_{C}R$. Rotate 90° around Y-axis.

```
R_B2C =
    0         0    1.0000
    0    1.0000         0
   -1.0000         0    0
```

```
R_A2C = R_A2B * R_B2C
```

Listing 3: ${}^{A}_{C}R$. Total rotation matrix.

```
R_A2C =
    0.0000         0    1.0000
    0.8660    0.5000   -0.0000
   -0.5000    0.8660    0.0000
```

2 Equivalence of XYZ Fixed and Z′Y′X′Euler

```
frame_ori = [
    1, 0, 0;
    0, 1, 0;
    0, 0, 1
    ];
```

Listing 4: Original frame.

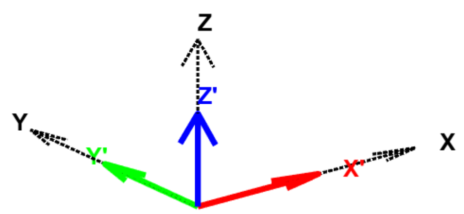


Figure 1: Original frame.

As we will see, Figure 2c, which shows the final orientation after rotations in the fixed frame, and Figure 3c, which shows the final orientation after rotations in the moving frame, are identical. Therefore, the XYZ Fixed Angle rotation and the Z′Y′X′ Euler Angle rotation yield equivalent outcomes.

2.1 XYZ Fixed frame rotation

For the fixed frame, we apply:

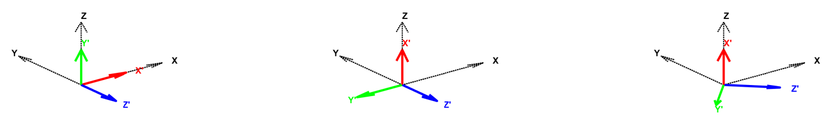
- 90° rotation around X-axis.
- $-90°$ rotation around Y-axis.
- 45° rotation around Z-axis.

```
theta_x = deg2rad(90);
R_X = [
    1, 0, 0;
    0, cos(theta_x), -sin(theta_x);
    0, sin(theta_x), cos(theta_x)
    ];
frame_rot = R_X * frame_ori;

theta_y = deg2rad(-90);
R_Y = [
    cos(theta_y), 0, sin(theta_y);
    0, 1, 0;
    -sin(theta_y), 0, cos(theta_y)
    ];
frame_rot = R_Y * (R_X * frame_ori);

theta_z = deg2rad(45);
R_Z = [
    cos(theta_z), -sin(theta_z), 0;
    sin(theta_z), cos(theta_z), 0;
    0, 0, 1
    ];
frame_rot = R_Z * (R_Y * (R_X * frame_ori));
```

Listing 5: XYZ rotation in fixed frame.



(a) Fixed X 90° (b) Fixed Y -90° (c) Fixed Z 45°

Figure 2: Fixed Angle. (a)-(c) depict the rotations in order.

2.2 Z´Y´X´ Moving frame rotation

For the moving frame, we apply:

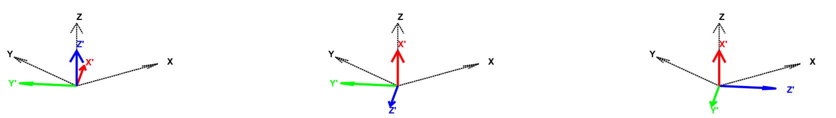
- 45° rotation around Z´-axis.
- −90° rotation around Y´-axis.
- 90° rotation around X´-axis.

```
theta_z = deg2rad(45);
R_A2B = [
    cos(theta_z), -sin(theta_z), 0;
    sin(theta_z), cos(theta_z), 0;
    0, 0, 1
    ];
frame_rot = R_A2B * frame_ori;

theta_y = deg2rad(-90);
R_B2C = [
    cos(theta_y), 0, sin(theta_y);
    0, 1, 0;
    -sin(theta_y), 0, cos(theta_y)
    ];
frame_rot = (R_A2B * R_B2C) * frame_ori;

theta_x = deg2rad(90);
R_C2D = [
    1, 0, 0;
    0, cos(theta_x), -sin(theta_x);
    0, sin(theta_x), cos(theta_x)
    ];
frame_rot = (R_A2B * R_B2C * R_C2D) * frame_ori;
```

Listing 6: Z´Y´X´ rotation in moving frame.



(a) Moving Z´ 45°   (b) Moving Y´ -90°   (c) Moving X´ 90°

Figure 3: Euler Angle. (a)-(c) depict the rotations in order.